

Math 21A
Vogler
Discussion Sheet 5

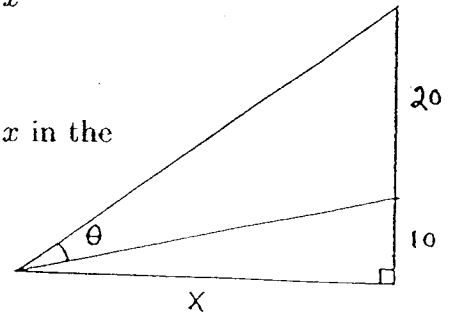
1.) Use any method to differentiate the following functions. You need not simplify answers.

a.) $y = 7 \sec 3x$ b.) $f(x) = \frac{x \sin x}{1 + \tan x}$ c.) $f(x) = \left(\frac{3x + 7}{7x - 9} \right)^{50}$

d.) $g(x) = x^3 \cos x^2$ e.) $f(x) = \sqrt{1 + \sqrt{2 + \sqrt{3 - x}}}$

f.) $y = \cot^5(\sin^3(10x^5))$

2.) Derive a formula for the measure of angle θ as a function of x in the given diagram.



3.) Let $f(x) = x(x - 5)^4$.

- a.) Solve $f'(x) = 0$ for x .
b.) Solve $f''(x) = 0$ for x .

6.) Find all points (x, y) which are equidistant from the three given points $(0, 0)$, $(4, 0)$, and $(3, 2)$.

7.) Derive an equation of the line tangent to the graph of $y = \frac{x}{x^2 + 3}$ at $x = -1$.

8.) Derive equations of all lines which are tangent to the graph of $y = -7 - x^2$ and passing through the point $(3, 0)$.

9.) Assume that a baseball is projected directly upward from the ground with an initial velocity of 112 ft./sec. Assuming only that the acceleration due to gravity is -32 ft./sec.^2 , derive equations for the height of the ball above the ground after t seconds and the velocity of the ball after t seconds. What is the velocity of the ball after $t = 1 \text{ sec.}$, $t = 2 \text{ sec.}$, and $t = 3 \text{ sec.}$? At what time does the ball reach its maximum height? What is the maximum height? In how many seconds does the ball strike the ground? What is the ball's velocity as it strikes the ground?

10.) A spaceship is traveling (left to right) along the curve $y = 3 \cos x$. An object is released from the spaceship at $x = \frac{\pi}{3}$ and travels along a line tangent to the graph of $y = 3 \cos x$ towards the x -axis.

- a.) At what point x will the object strike the x -axis?
b.) At what angle θ will the object strike the x -axis?

1.) Assume that y is a function of x . Compute $y' = \frac{dy}{dx}$ and $y'' = \frac{d^2y}{dx^2}$ (You need not simplify y'' .) for each equation.

a.) $y = x + x^3$

b.) $x = y + y^3$

c.) $y = x^2 \sin x$

d.) $y = x^2 \sin y$

e.) $x^2 + y^3 = xy$

f.) $(x - y)^3 = x^2 - y^2$

g.) $\sin(3y) + \tan^2 y = \cos x$

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The following problem is for recreational purposes only.

11.) A horse is tethered by a rope to the corner of a small shed with a square 10 ft. by 10 ft. floor. If the rope is 40 feet long, sketch the shape of the horse's grazing area. How close can you plant flowers to the shed and keep the horse from eating them ?