1.

(a) State the PRECISE definition of the limit for

$$\lim_{x \to a} f(x) = L$$

(b) Give an ε, δ-proof for the following limit. This is a writing exercise.

$$\lim_{x \to -2} (x^2 + 3) = 7$$

2. Determine the following limits

(a) $\displaystyle\lim_{x\to 1} \frac{x^2+x-2}{x^2-1}$

(b) $\displaystyle\lim_{h\to 0} \frac{\sin 5h}{2h}$

(c) $\displaystyle\lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|}$

(d) $\displaystyle\lim_{x\to 4} \frac{\sqrt{5+x}-3}{x-4}$

(e) $\displaystyle\lim_{x\to\infty} \frac{3+\cos(x)}{x^4}$ (HINT : Use the Squeeze Principle.)

3. Sketch the following graph. Make sure to CLEARLY state intercepts, asymptotes, along with the domain and range.

$$y = \frac{x+2}{x^2(x+1)}$$

4. Use the Intermediate Value Theorem to show that the equation $x^3 = 10 + \sqrt{x}$ is solvable. This is a writing exercise.

5.

(a) Consider the following function

$$f(x) = \begin{cases} \frac{x^2+x-2}{x^2-1} & \text{if } x < 1 \\ 3/2 & \text{if } x = 1 \\ 2x - 1/2 & \text{if } x > 1 \end{cases}$$

Use the three step definition of continuity to determine if f is continuous at $x = 1$.

(b) Consider the following function

$$f(x) = \begin{cases} x^2 + K & \text{if } x < -1 \\ x + K^2 & \text{if } x > -1 \end{cases}$$

Determine all constants K such that the function is continuous for all values of x.

4.) Consider the following function $f(x) = \begin{cases} \dfrac{x^2 - 3x}{x^2 - 9}, & \text{if } x \neq 3, -3 \\ \frac{1}{2}, & \text{if } x = 3 \\ 0, & \text{if } x = -3 . \end{cases}$

Determine if f is continuous at $x = 3$.

5.) Using limits, determine the value(s) of constants A and B so that the following function is continuous for all values of x :

$$f(x) = \begin{cases} Ax + B, & \text{if } x < 0 \\ 12, & \text{if } 0 \leq x \leq 2 \\ Bx^2 - A, & \text{if } x > 2 . \end{cases}$$

6.

(a) State the definition of the derivative.

(b) Use the definition of the derivative to differentiate the function

$$f(x) = \frac{x}{x+7}$$