

Math 21B
Vogler
Discussion Sheet 9

1.) Use any method to determine the following indefinite integrals (antiderivatives).

$$\begin{array}{lll} \text{a.) } \int \frac{x+2}{x^2+4x+5} dx & \text{b.) } \int \frac{x+1}{x^2+4x+5} dx & \text{c.) } \int \frac{x+4}{x^2+4x+3} dx \\ \text{f.) } \int \frac{x}{x^2+4x+13} dx & \text{g.) } \int \frac{1}{25x^2+9} dx & \end{array}$$

2.) Use partial fractions to integrate the following.

$$\begin{array}{ll} \text{a.) } \int \frac{x^2}{x^2-1} dx & \text{b.) } \int \frac{x+3}{(x-1)^2(x+2)} dx \\ \text{c.) } \int \frac{7-x^2}{(x^2+4)(x+4)^2} dx & \text{d.) } \int \frac{1}{x^3+1} dx \end{array}$$

3.) Write the partial fractions decomposition for each. DO NOT SOLVE FOR THE UNKNOWN CONSTANTS !

$$\begin{array}{lll} \text{a.) } \frac{x^2+7x-5}{(7x^2+3)^2 x^2 (x+3)^3} & \text{b.) } \frac{1}{x^4+x^2+1} & \text{c.) (challenging) } \frac{1}{x^4+1} \end{array}$$

4.) Integrate $\int \frac{1}{x(x^2+1)^2} dx$ using

a.) trig substitution. b.) partial fractions.

5.) Assume that $f''(x) = x^2 e^{3x}$. Find a number M so that $\max_{0 \leq x \leq 2} |f''(x)| \leq M$.

6.) Assume that $f''(x) = \frac{7}{2x+3}$. Find a number M so that $\max_{-1 \leq x \leq 1} |f''(x)| \leq M$.

7.) Assume that $f^{(4)}(x) = \frac{x-3}{5-x}$. Find a number M so that $\max_{-2 \leq x \leq 3} |f^{(4)}(x)| \leq M$.

8.) Consider the definite integral $\int_0^4 \ln(x^2+1) dx$.

a.) Find T_4 , the Trapezoidal Estimate using $n = 4$.

b.) What should n be in order that the Trapezoidal Estimate T_n estimate the exact value of this definite integral with absolute error at most 0.0001?

9.) Consider the definite integral $\int_0^2 \frac{x+1}{x+3} dx$.

a.) Find S_4 , the Simpson Estimate using $n = 4$.

b.) What should n be in order that the Simpson Estimate S_n estimate the exact value of this definite integral with absolute error at most 0.0001?

10.) Consider the region R lying below the graph of $y = \frac{1}{x}$ and above the x-axis on the interval $[1, \infty)$.

a.) Determine if R has finite or infinite area.

b.) Form a solid by revolving R about the x-axis. Determine if the resulting volume is finite or infinite.

c.) Form a solid by revolving R about the y-axis. Determine if the resulting volume is finite or infinite.

11.) Compute the following improper integrals.

a.) $\int_1^{\infty} \frac{1}{x(x+4)} dx$ b.) $\int_{-\infty}^0 e^{3x} dx$ c.) $\int_{-1}^{\infty} \frac{1}{\sqrt{x+1}} dx$

d.) $\int_{-\infty}^{\sqrt{3}} \frac{1}{x^2+9} dx$ e.) $\int_1^{e^2+1} \frac{7}{x-1} dx$ f.) $\int_0^e x^2 \ln x dx$

g.) $\int_2^{\infty} \frac{1}{x(\ln x)^2} dx$ h.) $\int_0^5 \frac{1}{\sqrt{25-x^2}} dx$ i.) $\int_1^{\infty} \frac{24}{2x^2+5x+2} dx$

j.) $\int_0^5 \frac{8x}{x^2-9} dx$ k.) $\int_{-\infty}^{\infty} x^2 e^{x^3} dx$ l.) $\int_0^{\pi/2} \csc x \cot x dx$

m.) $\int_0^{\infty} x e^{-5x} dx$ n.) $\int_0^{\infty} \frac{1}{x^2} dx$ o.) $\int_0^1 \ln x dx$ p.) $\int_0^{\infty} \frac{e^{-1/x}}{x^2} dx$

12.) Use the Comparison Test to show that the following improper integral converges, i.e., is finite :

$$\int_1^{\infty} \frac{1}{\sqrt{x^3+16}} dx$$

13.) Use the Comparison Test to show that the following improper integral diverges, i.e., is infinite :

$$\int_1^{\infty} \frac{x+4}{\sqrt{x^3+16}} dx$$

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“ What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises.”– G. Lichtenberg

“ I hear and I forget. I see and I remember. I do and I understand.”– Chinese Proverb