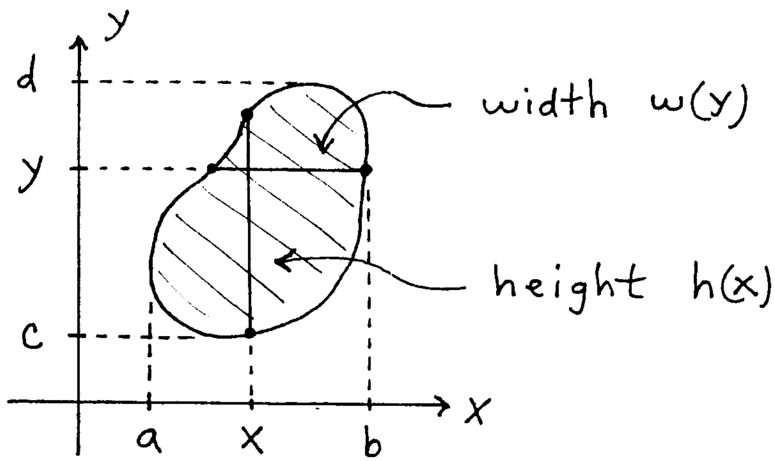
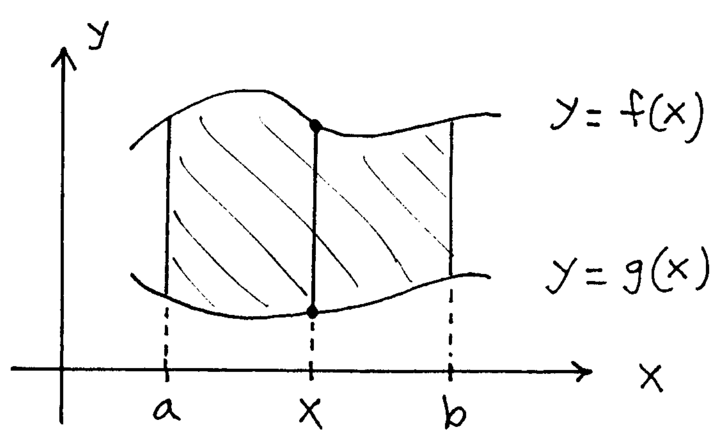
Math 21B
Vogler
Centroid— The Balance Point $(\bar{x}, \bar{y})$ of a Flat Plate of Uniform (Constant) Density

Consider a flat region R whose height at $x, a \le x \le b$, is given to be $h(x)$ and whose width at $y, c \le y \le d$, is given to be $w(y)$. Assume the density at point (x, y) is $\delta(x, y) = k$, a constant. The *standard formulas* for the coordinates of the centroid $(\bar{x}, \bar{y})$ of region R are

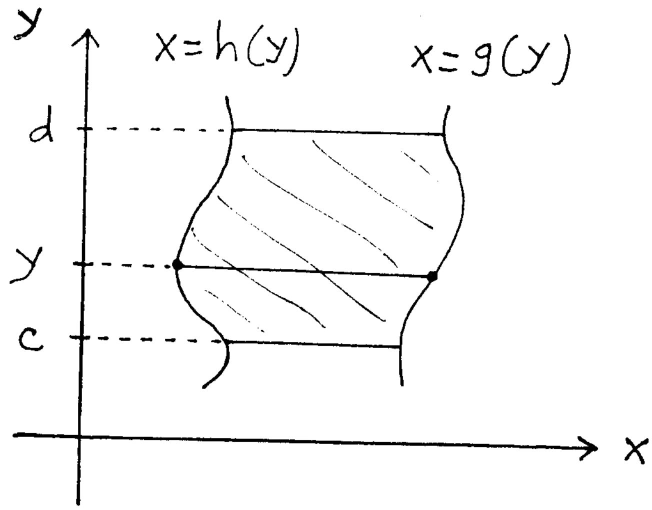


$$\bar{x} = \frac{\int_a^b x h(x)\, dx}{\int_a^b h(x)\, dx} \quad \text{and} \quad \bar{y} = \frac{\int_c^d y w(y)\, dy}{\int_c^d w(y)\, dy}$$

Following are two sets of *alternate formulas* and the corresponding regions.



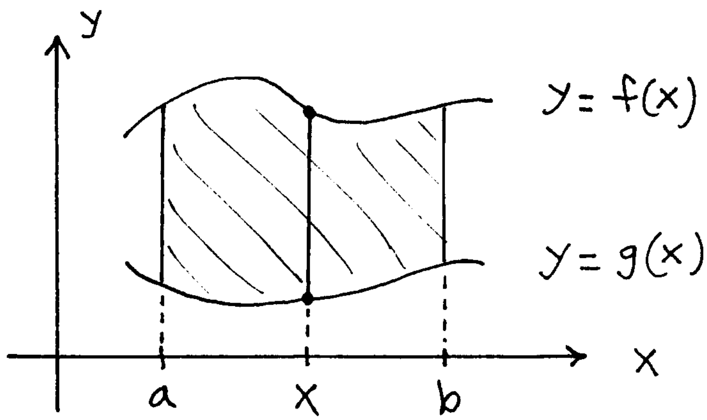
$$\bar{x} = \frac{\int_a^b x(f(x) - (g(x))\, dx}{\int_a^b (f(x) - g(x))\, dx} \quad \text{and} \quad \bar{y} = \frac{\int_a^b (1/2)((f(x))^2 - (g(x))^2)\, dx}{\int_a^b (f(x) - g(x))\, dx}$$



$$\bar{x} = \frac{\int_c^d (1/2)((g(y))^2 - (h(y))^2)\,dy}{\int_c^d (g(y) - h(y))\,dy} \quad \text{and} \quad \bar{y} = \frac{\int_c^d y(g(y) - h(y))\,dy}{\int_c^d (g(y) - h(y))\,dy}$$

Math 21B
Vogler
Center of Mass— The Balance Point $(\bar{x}, \bar{y})$ of a Flat Plate of Variable Density

Consider a flat region R bounded above by the graph of $y = f(x)$ and below by the graph of $y = g(x)$ for $a \leq x \leq b$,. Assume the density at point (x, y) is $\delta(x, y) = k(x)$, a function of x only (not y.). The *standard formulas* for the coordinates of the centroid $(\bar{x}, \bar{y})$ of region R are



$$\bar{x} = \frac{\int_a^b x(f(x) - (g(x))\delta(x,y)\,dx}{\int_a^b (f(x) - g(x))\delta(x,y)\,dx} \quad \text{and} \quad \bar{y} = \frac{\int_a^b (1/2)((f(x))^2 - (g(x))^2)\delta(x,y)\,dx}{\int_a^b (f(x) - g(x))\delta(x,y)\,dx}$$

REMARK : The integral $\displaystyle\int_a^b (f(x) - g(x))\delta(x,y)\,dx$ represents the TOTAL MASS of the plate with variable density.