Knowing if a function is even or odd can sometimes lead to a relatively easy solution to a definite integral.

**DEFINITIONS:** Function $f$ is *even* if $f(x) = f(-x)$. Function $f$ is *odd* if $f(x) = -f(-x)$.

**EXAMPLE:**

- $f$ is even
- $f$ is odd

**REMARKS:**

I. If $f$ is even then $\int_{-a}^{0} f(x) \, dx = \int_{0}^{a} f(x) \, dx$ so that $\int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx$.

II. If $f$ is odd then $\int_{-a}^{0} f(x) \, dx = -\int_{0}^{a} f(x) \, dx$ so that $\int_{-a}^{a} f(x) \, dx = 0$.

**PROBLEM:** Show that $f(x) = x \sqrt{x^2 + \cos x}$ is an odd function, then evaluate the definite integral $\int_{-5}^{5} x \sqrt{x^2 + \cos x} \, dx$. 