

Math 21B  
Vogler  
Discrete and Continuous Compounding of Interest

EXAMPLE : Initially  $P$  dollars is deposited into a savings account where interest  $i$  is compounded for each of  $m$  consecutive interest earning periods. Assume that interest earned remains in the account and that no withdrawals are made and no additional money is deposited in the account. What is the amount  $A$  of money in the account after  $m$  periods ?

Period	Amount of money
1	$P + i \cdot P = P(1 + i)$
2	$P(1 + i) + i \cdot P(1 + i) = (1 + i) \cdot P(1 + i) = P(1 + i)^2$
3	$P(1 + i)^2 + i \cdot P(1 + i)^2 = (1 + i) \cdot P(1 + i)^2 = P(1 + i)^3$
4	$P(1 + i)^3 + i \cdot P(1 + i)^3 = (1 + i) \cdot P(1 + i)^3 = P(1 + i)^4$
$\vdots$	$\vdots$
$m$	$P(1 + i)^m$

So after  $m$  periods the total amount of money in the account is  $A = P(1 + i)^m$ . If the annual interest rate is  $r$  and interest is compounded  $n$  times per year for  $t$  years, then  $i = r/n$ ,  $m = nt$ , and

$$A = P(1 + r/n)^{nt} \quad (\text{Discrete Interest Formula}) .$$

If we let  $n$  go to infinity then (See class notes.)

$$A = Pe^{rt} \quad (\text{Continuous Interest Formula}) .$$