Math 21B
Vogler
Properties of the Definite Integral

I.) Properties of the Definite Integral

a.) $\int_a^a f(x)\,dx = 0$ b.) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

c.) $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$ d.) $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

e.) If $f(x) \geq 0$ then $\int_a^b f(x)\,dx \geq 0$ (if $a < b$)

f.) If $f(x) \geq g(x)$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$ (if $a < b$)

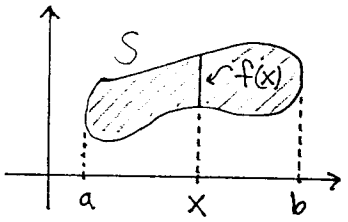
g.) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

h.) If $m \leq f(x) \leq M$ then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

II.) Applications of the Definite Integral

a.) Area of region : If $f(x)$ is the height of region S at x . then total area of S from a to b is

$$\text{AREA} = \int_a^b f(x)\,dx$$



b.) Mass of string : If $f(x)$ is the density (mass/length units) of string at x , then total mass of string from a to b is

$$\text{MASS} = \int_a^b f(x)\,dx$$

c.) Distance traveled : If $f(t)$ is the speed of an object at time t , then total distance traveled from time a to time b is

$$\text{DISTANCE} = \int_a^b f(t)\,dt$$

d.) Volume of solid : If $A(x)$ is the cross-sectional area of a solid S at x , then total volume of S from a to b is

$$\text{VOLUME} = \int_a^b A(x)\,dx$$