

Section 4.8

2.) a.) $6x \xrightarrow{AD} 3x^2$
 b.) $x^7 \xrightarrow{AD} \frac{1}{8}x^8$
 c.) $x^7 - 6x + 8 \xrightarrow{AD} \frac{1}{8}x^8 - 3x^2 + 8x$

5.) a.) $\frac{1}{x^2} = x^{-2} \xrightarrow{AD} \frac{x^{-1}}{-1}$
 b.) $\frac{5}{x^2} = 5 \cdot x^{-2} \xrightarrow{AD} 5 \cdot \frac{x^{-1}}{-1}$
 c.) $2 - \frac{5}{x^2} = 2 - 5 \cdot x^{-2} \xrightarrow{AD} 2x - 5 \cdot \frac{x^{-1}}{-1}$

8.) a.) $\frac{4}{3}x^{1/3} \xrightarrow{AD} \frac{4}{3} \cdot \frac{x^{4/3}}{4/3} = x^{4/3}$
 b.) $\frac{1}{3x^{1/3}} = \frac{1}{3} \cdot x^{-1/3} \xrightarrow{AD} \frac{1}{3} \cdot \frac{x^{2/3}}{2/3} = \frac{1}{2}x^{2/3}$
 c.) $x^{1/3} + x^{-1/3} \xrightarrow{AD} \frac{x^{4/3}}{4/3} + \frac{x^{2/3}}{2/3}$

11.) a.) $\frac{1}{x} \xrightarrow{AD} \ln|x|$
 b.) $\frac{7}{x} = 7 \cdot \frac{1}{x} \xrightarrow{AD} 7 \ln|x|$
 c.) $1 - \frac{5}{x} \xrightarrow{AD} x - 5 \ln|x|$

15.) a.) $\sec^2 x \xrightarrow{AD} \tan x$
 b.) $\frac{2}{3} \cdot \sec^2 \frac{x}{3} \xrightarrow{AD} \frac{2}{3} \cdot 3 \cdot \tan \frac{x}{3}$
 c.) $-\sec^2 \frac{3x}{2} \xrightarrow{AD} -1 \cdot \frac{2}{3} \tan \frac{3x}{2}$

20.) a.) $e^{-2x} \xrightarrow{AD} \frac{-1}{2} \cdot e^{-2x}$
 b.) $e^{\frac{4x}{3}} \xrightarrow{AD} \frac{3}{4} e^{\frac{4x}{3}}$

$$c.) e^{-\frac{x}{5}} \xrightarrow{AD} -5 \cdot e^{-\frac{x}{5}}$$

$$21.) a.) 3^x \xrightarrow{AD} \frac{1}{\ln 3} \cdot 3^x$$

$$b.) 2^{-x} \xrightarrow{AD} -1 \cdot \frac{1}{\ln 2} \cdot 2^{-x}$$

$$c.) \left(\frac{5}{3}\right)^x \xrightarrow{AD} \frac{1}{\ln\left(\frac{5}{3}\right)} \cdot \left(\frac{5}{3}\right)^x$$

$$23.) a.) \frac{2}{\sqrt{1-x^2}} \xrightarrow{AD} 2 \cdot \arcsin x$$

$$b.) \frac{1}{2} \cdot \frac{1}{x^2+1} \xrightarrow{AD} \frac{1}{2} \cdot \arctan x$$

$$c.) \frac{1}{1+4x^2} = \frac{1}{1+(2x)^2} \xrightarrow{AD} \frac{1}{2} \cdot \arctan(2x)$$

$$26.) \int (5-6x) dx = 5x - 3x^2 + C$$

$$34.) \int x^{-5/4} dx = \frac{x^{-1/4}}{-1/4} + C$$

$$36.) \int \left(\frac{1}{2} \cdot x^{1/2} + 2 \cdot x^{-1/2}\right) dx = \frac{1}{2} \cdot \frac{x^{3/2}}{3/2} + 2 \cdot \frac{x^{1/2}}{1/2} + C$$

$$39.) \int 2x(1-x^{-3}) dx = \int (2x - 2x^{-2}) dx \\ = x^2 - 2 \cdot \frac{x^{-1}}{-1} + C$$

$$46.) \int 3 \cos 5\theta d\theta = 3 \cdot \frac{1}{5} \sin 5\theta + C$$

$$50.) \int \frac{2}{5} \sec \theta \tan \theta d\theta = \frac{2}{5} \cdot \sec \theta + C$$

$$51.) \int (e^{3x} + 5e^{-x}) dx = \frac{1}{3} e^{3x} + 5 \cdot \frac{e^{-x}}{-1} + C$$

$$60.) \int \frac{1}{2}(1 - \cos 6t) dt = \frac{1}{2} \left(t - \frac{1}{6} \sin 6t \right) + C$$

$$62.) \int \left(\frac{2}{\sqrt{1-y^2}} - y^{-1/4} \right) dy = 2 \cdot \arcsin y - \frac{y^{3/4}}{3/4} + C$$

$$66.) \int (2 + \tan^2 \theta) d\theta = \int (2 + (\sec^2 \theta - 1)) d\theta \\ = \int (1 + \sec^2 \theta) d\theta = \theta + \tan \theta + C$$

$$69.) \int \cos \theta (\tan \theta + \sec \theta) d\theta \\ = \int \cos \theta \left(\frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \right) d\theta \\ = \int (\sin \theta + 1) d\theta = -\cos \theta + \theta + C$$

$$78.) D(xe^x - e^x + C) = x \cdot e^x + 1 \cdot e^x - e^x + 0 = xe^x$$

$$79.) D\left(\frac{1}{a} \cdot \arctan\left(\frac{x}{a}\right) + C\right) = \frac{1}{a} \cdot \frac{1}{1 + \left(\frac{x}{a}\right)^2} \cdot \frac{1}{a} + 0 \\ = \frac{1}{a^2 + x^2}$$

$$80.) D\left(\arcsin\left(\frac{x}{a}\right)\right) = \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a} = \frac{1}{a} \cdot \frac{1}{\sqrt{\frac{a^2 - x^2}{a^2}}} \\ = \frac{1}{a} \cdot \frac{1}{\sqrt{\frac{1}{a^2}(a^2 - x^2)}} = \frac{1}{a} \cdot \frac{1}{\frac{1}{a} \sqrt{a^2 - x^2}} = \frac{1}{\sqrt{a^2 - x^2}}$$

$$83.) b.) \int x \sin x dx = -x \cos x + C ; \text{ FALSE :} \\ D(-x \cos x + C) = -x \cdot (-\sin x) + (-1) \cdot \cos x + 0 \\ = x \sin x - \cos x$$

$$c.) \int x \sin x dx = -x \cos x + \sin x + C ; \text{ TRUE :}$$

$$D(-x \cos x + \sin x + C) = -x \cdot (-\sin x) + (-1) \cdot \cos x + \sin x + 0 \\ = x \sin x$$

86.) b.) $\int \sqrt{2x+1} dx = \sqrt{x^2+x} + C$; FALSE:

$$D(\sqrt{x^2+x} + C) = \frac{1}{2}(x^2+x)^{-1/2} \cdot (2x+1) + 0 \\ = \frac{2x+1}{2\sqrt{x^2+x}}$$

c.) $\int \sqrt{2x+1} dx = \frac{1}{3}(\sqrt{2x+1})^3 + C$; TRUE:

$$D\left(\frac{1}{3}(\sqrt{2x+1})^3 + C\right) = \frac{1}{3} \cdot 3(\sqrt{2x+1})^2 \cdot \frac{1}{2}(2x+1)^{-1/2} \cdot 2 \\ = \frac{2x+1}{\sqrt{2x+1}} = \sqrt{2x+1}$$

90.) $\frac{dy}{dx} = -x \xrightarrow{AD} y = -\frac{x^2}{2} + C$ and $x=-1, y=1 \rightarrow$

$$1 = -\frac{(-1)^2}{2} + C \rightarrow 1 = -\frac{1}{2} + C \rightarrow C = \frac{3}{2} \rightarrow$$

$$y = -\frac{x^2}{2} + \frac{3}{2} \rightarrow \text{b.)}$$

96.) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2} \xrightarrow{AD} y = \frac{1}{2} \cdot \frac{x^{1/2}}{1/2} + C$

$$\rightarrow y = \sqrt{x} + C \text{ and } x=4, y=0 \rightarrow$$

$$0 = \sqrt{4} + C = 2 + C \rightarrow C = -2 \rightarrow y = \sqrt{x} - 2$$

98.) $\frac{ds}{dt} = \cos t + \sin t \xrightarrow{AD} s = \sin t - \cos t + C$

$$\text{and } x=\pi, s=1 \rightarrow 1 = \sin \pi - \cos \pi + C \rightarrow$$

$$1 = 0 - (-1) + C \rightarrow 1 = 1 + C \rightarrow C = 0 \rightarrow$$

$$s = \sin t - \cos t$$

$$106.) \frac{d^2y}{dx^2} = 0 \xrightarrow{AD} \frac{dy}{dx} = c \text{ and } x=0, y'=2 \rightarrow$$

$$2 = c \rightarrow \frac{dy}{dx} = 2 \xrightarrow{AD} y = 2x + c \text{ and}$$

$$x=0, y=0 \rightarrow 0 = 2(0) + c \rightarrow c = 0 \rightarrow$$

$$y = 2x$$

$$109.) y''' = 6 \xrightarrow{AD} y'' = 6x + c \text{ and } x=0, y'' = -8 \rightarrow$$

$$-8 = 6(0) + c \rightarrow c = -8 \rightarrow y'' = 6x - 8 \xrightarrow{AD}$$

$$y' = 3x^2 - 8x + c \text{ and } x=0, y' = 0 \rightarrow$$

$$0 = 3(0)^2 - 8(0) + c \rightarrow c = 0 \rightarrow y' = 3x^2 - 8x \xrightarrow{AD}$$

$$y = x^3 - 4x^2 + c \text{ and } x=0, y = 5 \rightarrow$$

$$5 = (0)^3 - 4(0)^2 + c \rightarrow c = 5 \rightarrow y = x^3 - 4x^2 + 5$$

$$117.) y' = \sin x - \cos x \xrightarrow{AD} y = -\cos x - \sin x + c$$

$$\text{and } x = -\pi, y = -1 \rightarrow -1 = -\cos(-\pi) - \sin(-\pi) + c$$

$$\rightarrow -1 = -(-1) - (0) + c \rightarrow c = -2 \rightarrow$$

$$y = -\cos x - \sin x - 2$$

120.) Let $S = S(t)$ be height (m.) at time t seconds; assume $S'' = 20 \text{ m./sec.}^2$

$$\rightarrow S' = 20t + c \text{ and } t=0, S' = 0 \text{ m./sec.} \rightarrow$$

$$0 = 20(0) + c \rightarrow c = 0 \rightarrow \underline{S' = 20t}, \rightarrow$$

$$S = 10t^2 + c \text{ and } t=0, S = 0 \text{ m.} \rightarrow$$

$$0 = 10(0)^2 + c \rightarrow c = 0 \rightarrow \underline{S = 10t^2};$$

$$\text{if } t = 1 \text{ min.} = 60 \text{ sec.} \rightarrow$$

$$\text{height } S = 10(60)^2 = 36,000 \text{ m. and}$$

$$\text{velocity } S' = 20(60) = 1200 \text{ m./sec.}$$

122.) assume s : distance, s' : velocity,
 s'' : acceleration, t : time (sec.)

$$t = 0 \text{ sec.}$$

$$t = T \text{ sec.}$$

$$s = 0 \text{ ft.}$$

$$s = 45 \text{ ft.}$$

$$s' = 44 \text{ ft./sec.}$$

$$s' = 0 \text{ ft./sec.}$$

If $\boxed{s'' = A}$ ft./sec.² \xrightarrow{AD}

$$s' = At + C \text{ and } t = 0, s' = 44 \rightarrow$$

$$44 = A(0) + C \rightarrow C = 44 \rightarrow$$

$$\boxed{s' = At + 44} \text{ and } t = T, s' = 0 \rightarrow$$

$$0 = AT + 44 \rightarrow \boxed{T = \frac{-44}{A}}; \text{ then}$$

$$s = A \frac{t^2}{2} + 44t + C \text{ and } t = 0, s = 0 \rightarrow$$

$$0 = A(0) + 44(0) + C \rightarrow C = 0 \rightarrow$$

$$s = \frac{A}{2} t^2 + 44t \text{ and } t = T, s = 45 \rightarrow$$

$$\boxed{45 = \frac{A}{2} T^2 + 44T}; \text{ SUB then}$$

$$45 = \frac{A}{2} \left(\frac{-44}{A}\right)^2 + 44 \left(\frac{-44}{A}\right) = \frac{968}{A} - \frac{1936}{A} = \frac{-968}{A} \rightarrow$$

$$45 = \frac{-968}{A} \rightarrow A = \frac{-968}{45} \text{ ft./sec.}^2 \approx \underline{\underline{-21.51 \text{ ft./sec.}^2}}$$

$$\text{and } T \approx 2.045 \text{ sec.}$$

125.)

$$s'' = a \rightarrow$$

$$s' = at + c \text{ and } t=0, s' = v_0 \rightarrow$$

$$v_0 = a(0) + c \rightarrow c = v_0 \rightarrow$$

$$s' = at + v_0 \rightarrow$$

$$s = \frac{a}{2}t^2 + v_0t + c \text{ and } t=0, s = s_0 \rightarrow$$

$$s_0 = \frac{a}{2}(0)^2 + v_0(0) + c \rightarrow c = s_0 \rightarrow$$

$$s = \frac{a}{2}t^2 + v_0t + s_0 .$$