

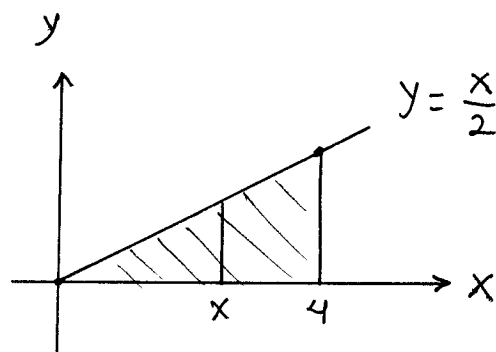
Section 6.4

$$9.) \text{ Area} = 2\pi \int_0^4 f(x) \cdot \sqrt{1 + (f'(x))^2} dx$$

$$= 2\pi \int_0^4 \left(\frac{x}{2}\right) \cdot \sqrt{1 + \left(\frac{1}{2}\right)^2} dx$$

$$= \pi \int_0^4 x \cdot \sqrt{\frac{5}{4}} dx$$

$$= \frac{\sqrt{5}}{2} \pi \cdot \frac{x^2}{2} \Big|_0^4 = 4\sqrt{5} \pi$$



$$13.) \text{ Area} = 2\pi \int_0^2 f(x) \cdot \sqrt{1 + (f'(x))^2} dx$$

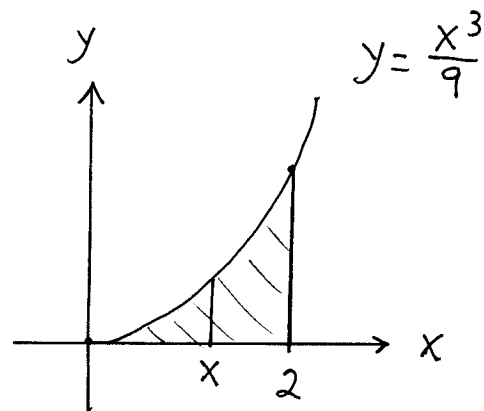
$$= 2\pi \int_0^2 \frac{x^3}{9} \cdot \sqrt{1 + \left(\frac{x^2}{3}\right)^2} dx$$

$$= \frac{2}{9} \pi \int_0^2 x^3 \cdot \sqrt{1 + \frac{1}{9}x^4} dx$$

$$= \frac{2}{9} \pi \cdot \frac{9}{4} \left(1 + \frac{1}{9}x^4\right)^{3/2} \Big|_0^2$$

$$= \frac{\pi}{3} \left(\left(1 + \frac{16}{9}\right)^{3/2} - 1^{3/2} \right) = \frac{\pi}{3} \left(\left(\frac{25}{9}\right)^{3/2} - 1 \right)$$

$$= \frac{\pi}{3} \left(\frac{125}{27} - 1 \right) = \frac{\pi}{3} \cdot \frac{98}{27} = \frac{98}{81} \pi$$

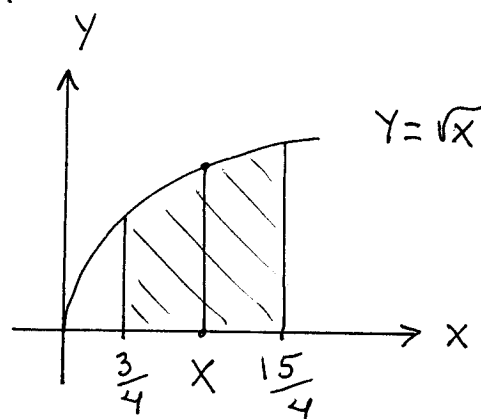


$$14.) \text{ Area} = 2\pi \int_{3/4}^{15/4} f(x) \cdot \sqrt{1 + (f'(x))^2} dx$$

$$= 2\pi \int_{3/4}^{15/4} \sqrt{x} \cdot \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2} dx$$

$$= 2\pi \int_{3/4}^{15/4} \sqrt{x} \cdot \sqrt{1 + \frac{1}{4x}} dx$$

$$= 2\pi \int_{3/4}^{15/4} \sqrt{x} \cdot \sqrt{\frac{4x+1}{4x}} dx = 2\pi \int_{3/4}^{15/4} \sqrt{x} \cdot \frac{\sqrt{4x+1}}{2\sqrt{x}} dx$$



$$= \pi \cdot \frac{1}{4} \left(\frac{4x+1}{3/2} \right)^{3/2} \Big|_{\frac{3}{4}}^{\frac{15}{4}} = \frac{\pi}{6} \left(16^{3/2} - 4^{3/2} \right)$$

$$= \frac{\pi}{6} (64 - 8) = \frac{56}{6} \pi = \frac{28}{3} \pi$$

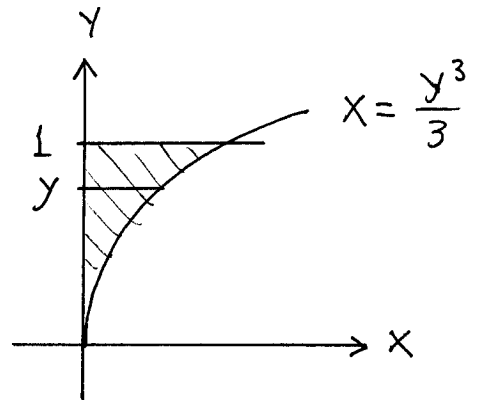
17.) Area = $2\pi \int_0^1 g(y) \cdot \sqrt{1+(g'(y))^2} dy$

$$= 2\pi \int_0^1 \left(\frac{y^3}{3} \right) \cdot \sqrt{1+(y^2)^2} dy$$

$$= \frac{2}{3} \pi \int_0^1 y^3 \cdot \sqrt{1+y^4} dy$$

$$= \frac{2}{3} \pi \cdot \frac{1}{4} \frac{(1+y^4)^{3/2}}{3/2} \Big|_0^1$$

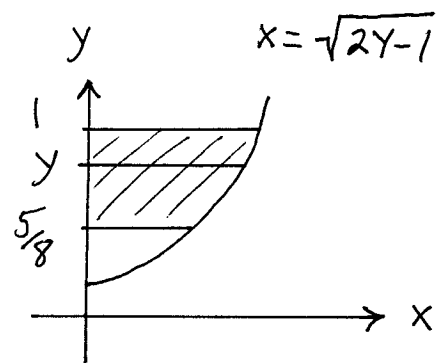
$$= \frac{\pi}{9} \cdot (2^{3/2} - 1^{3/2}) = \frac{\pi}{9} (2^{3/2} - 1)$$



20.) Area = $2\pi \int_{5/8}^1 g(y) \cdot \sqrt{1+(g'(y))^2} dy$

$$= 2\pi \int_{5/8}^1 \sqrt{2y-1} \cdot \sqrt{1+\left(\frac{1}{2}(2y-1)^{-1/2} \cdot 2\right)^2} dy$$

$$= 2\pi \int_{5/8}^1 \sqrt{2y-1} \cdot \sqrt{1+\frac{1}{2y-1}} dy$$



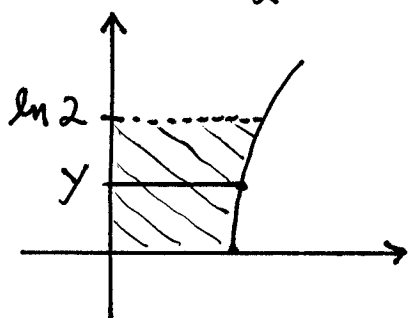
$$= 2\pi \int_{5/8}^1 \sqrt{2y-1} \cdot \sqrt{\frac{2y}{2y-1}} dy = 2\sqrt{2}\pi \int_{5/8}^1 \sqrt{y} dy$$

$$= 2\sqrt{2}\pi \frac{y^{3/2}}{3/2} \Big|_{5/8}^1 = \frac{4}{3}\sqrt{2}\pi \left(1^{3/2} - \left(\frac{5}{8}\right)^{3/2} \right)$$

$$= \frac{4}{3}\sqrt{2}\pi \left(1 - \left(\frac{5}{8}\right)^{3/2} \right)$$

$$x = \frac{1}{2}(e^y + e^{-y})$$

21.) Area = $2\pi \int_0^{\ln 2} g(y) \sqrt{1 + (g'(y))^2} dy$



$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^y + e^{-y}) \sqrt{1 + \left(\frac{1}{2}(e^y - e^{-y})\right)^2} dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^y + e^{-y}) \sqrt{1 + \frac{1}{4}(e^{2y} - 2 + e^{-2y})} dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^y + e^{-y}) \sqrt{\frac{1}{4}e^{2y} + \frac{1}{2} + \frac{1}{4}e^{-2y}} dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^y + e^{-y}) \sqrt{\left(\frac{1}{2}e^y\right)^2 + \frac{1}{2} + \left(\frac{1}{2}e^{-y}\right)^2} dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^y + e^{-y}) \sqrt{\left(\frac{1}{2}e^y + \frac{1}{2}e^{-y}\right)^2} dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^y + e^{-y}) \left(\frac{1}{2}e^y + \frac{1}{2}e^{-y}\right) dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2} \cdot \frac{1}{2} \cdot (e^y + e^{-y})^2 dy$$

$$= \frac{\pi}{2} \int_0^{\ln 2} (e^{2y} + 2 + e^{-2y}) dy$$

$$= \frac{\pi}{2} \left(\frac{1}{2}e^{2y} + 2y - \frac{1}{2}e^{-2y} \right) \Big|_0^{\ln 2}$$

$$= \frac{\pi}{2} \cdot \frac{1}{2} (e^{2\ln 2} + 4\ln 2 - e^{-2\ln 2}) \Big|_0^{\ln 2}$$

$$= \frac{\pi}{4} (e^{2\ln 2} + 4\ln 2 - e^{-2\ln 2})$$

$$= \frac{\pi}{4} (e^{\ln 4} + 4\ln 2 - e^{\ln \frac{1}{4}})$$

$$= \frac{\pi}{4} \left(4 + 4 \ln 2 - \frac{1}{4} \right) = \frac{\pi}{4} \left(\frac{15}{4} + 4 \ln 2 \right)$$

$$25.) \text{ Area} = 2\pi \int_{-a}^a f(x) \cdot \sqrt{1 + (f'(x))^2} dx$$

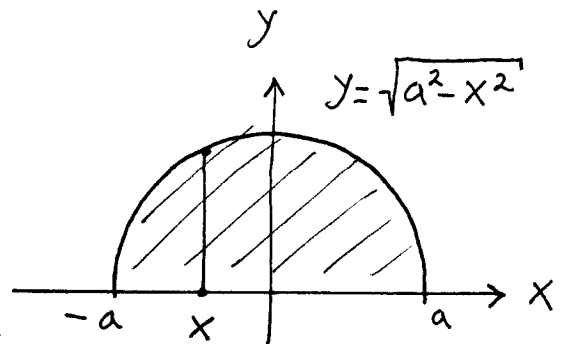
$$= 2\pi \int_{-a}^a \sqrt{a^2 - x^2} \cdot \sqrt{1 + \left(\frac{1}{2}(a^2 - x^2)^{-1/2} \cdot (-2x) \right)^2} dx$$

$$= 2\pi \int_{-a}^a \sqrt{a^2 - x^2} \sqrt{1 + \frac{x^2}{a^2 - x^2}} dx$$

$$= 2\pi \int_{-a}^a \sqrt{a^2 - x^2} \cdot \sqrt{\frac{a^2}{a^2 - x^2}} dx$$

$$= 2\pi \int_{-a}^a \sqrt{a^2 - x^2} \cdot \frac{\sqrt{a^2}}{\sqrt{a^2 - x^2}} dx = 2\pi a \int_{-a}^a 1 dx$$

$$= 2\pi a \cdot x \Big|_{-a}^a = 2\pi a (a - (-a)) = 4\pi a^2$$



$$32.) \text{ Area} = 2\pi \int_{-1}^1 f(x) \sqrt{1 + (f'(x))^2} dx$$

$$= 2 \cdot 2\pi \int_0^1 (1 - x^{2/3})^{3/2} \sqrt{1 + \left(\frac{2}{3}(1 - x^{2/3})^{1/2} \cdot \frac{-2}{3} x^{-1/3} \right)^2} dx$$

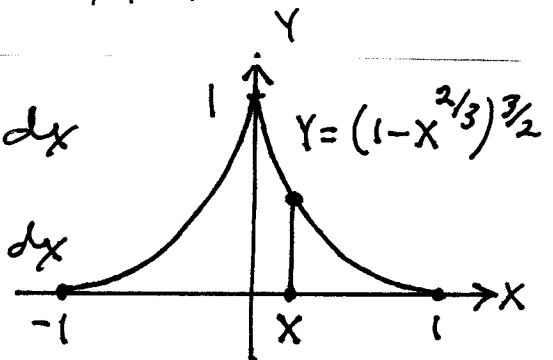
$$= 4\pi \int_0^1 (1 - x^{2/3})^{3/2} \sqrt{1 + \frac{1 - x^{2/3}}{x^{2/3}}} dx$$

$$= 4\pi \int_0^1 (1 - x^{2/3})^{3/2} \sqrt{\frac{x^{2/3} + 1 - x^{2/3}}{x^{2/3}}} dx$$

$$= 4\pi \int_0^1 (1 - x^{2/3})^{3/2} \sqrt{\frac{1}{x^{2/3}}} dx$$

$$= 4\pi \int_0^1 \frac{(1 - x^{2/3})^{3/2}}{x^{2/3}} dx$$

$$\left(\text{let } u = 1 - x^{2/3} \xrightarrow{D}$$



$$\begin{aligned} du &= -\frac{2}{3} x^{-1/3} dx \rightarrow -\frac{3}{2} du = \frac{1}{x^{1/3}} dx \\ &= 4\pi \cdot -\frac{3}{2} \int_{x=0}^{x=1} u^{3/2} du = -6\pi \cdot \frac{2}{5} u^{5/2} \Big|_{x=0}^{x=1} \\ &= -\frac{12}{5} \pi (1 - x^{2/3})^{5/2} \Big|_{x=0}^{x=1} \\ &= -\frac{12}{5} \pi (0) - -\frac{12}{5} \pi (1) = \frac{12}{5} \pi \end{aligned}$$