

Section 6.5

1.) $F(x) = kx$, where x is displacement;
 Work = $\int_a^b F(x) dx$ so $1800 \text{ J} = \int_0^3 kx dx \rightarrow$
 $1800 = k \frac{x^2}{2} \Big|_0^3 = \frac{9}{2} k \rightarrow k = 400 \text{ N/m.}$

2.) $F(x) = kx \rightarrow 800 \text{ lb.} = k \cdot (4 \text{ in.}) \rightarrow$

a.) $k = 200 \text{ lb./in.}$

b.) Work = $\int_0^2 F(x) dx = \int_0^2 200x dx$
 $= 100x^2 \Big|_0^2 = 400 \text{ in.-lbs.}$

c.) $1600 = kx \rightarrow 1600 = (200)x \rightarrow$
 $x = 8 \text{ in.}$

4.) $F(x) = kx \rightarrow 90 \text{ N} = k \cdot (1 \text{ m.}) \rightarrow k = 90$ so
 $F(x) = kx$; then Work = $\int_0^5 F(x) dx$
 $= \int_0^5 90x dx = 45x^2 \Big|_0^5 = 1125 \text{ N.}$

7.) work required
 to lift small piece
 of rope to top is

$\approx (\text{weight}) (\text{distance})$

$= (\text{density}) (\text{length}) (\text{distance})$

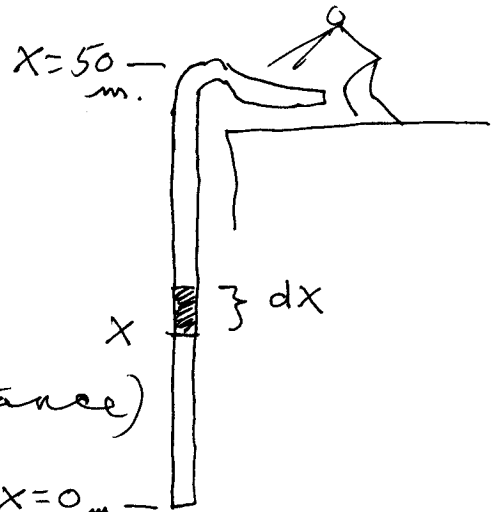
$= (0.624) (dx) (50 - x)$

$= (0.624) (50 - x) dx$; total work is

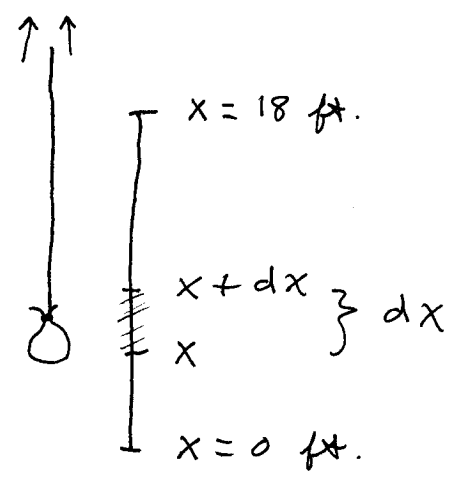
Work = $\int_0^{50} (0.624) (50 - x) dx$

$= (0.624) \left(50x - \frac{x^2}{2} \right) \Big|_0^{50} = (0.624) (2500 - 1250)$

$= 780 \text{ J.}$

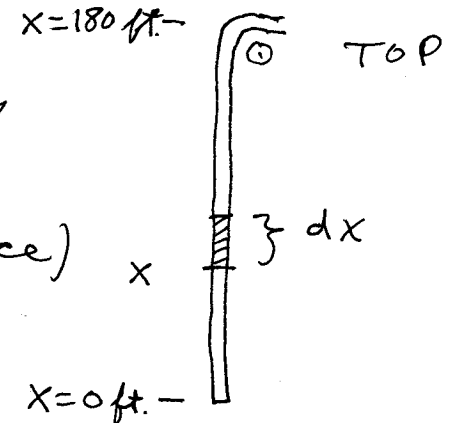


8.) Bag loses 4 lbs./ft.;
 work required to lift
 bag from x to $x+dx$ is
 \approx (weight) \cdot (distance)
 $= (144 - 4x) dx$;



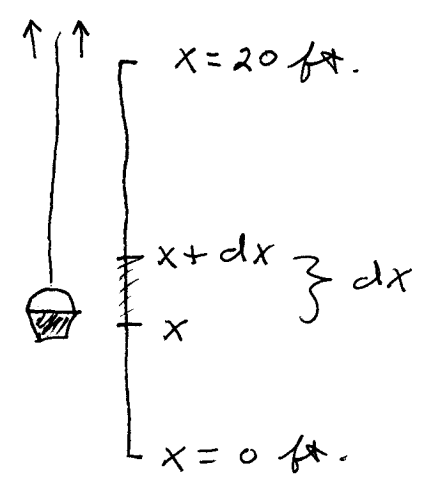
so total work is
 $Work = \int_0^{18} (144 - 4x) dx$
 $= (144x - 2x^2) \Big|_0^{18} = 2592 - 648 = 1944 \text{ ft.-lbs.}$

9.) work to move small
 piece of cable to top is
 \approx (weight) (distance)
 $=$ (density) (length) (distance)
 $= (4.5) (dx) (180 - x)$
 $= (4.5) (180 - x) dx$;



total work to lift cable is
 $Work = \int_0^{180} (4.5) (180 - x) dx$
 $= (4.5) (180x - \frac{x^2}{2}) \Big|_0^{180} = 4.5 (16,200)$
 $= 72,900 \text{ ft.-lbs.}$

11.) bucket weighs 5 lbs.;
 starts with 16 lbs. of
 H_2O in it, but leaks
 at rate of $\frac{16 \text{ lbs.}}{20 \text{ ft.}} = \frac{4 \text{ lb.}}{5 \text{ ft.}}$;

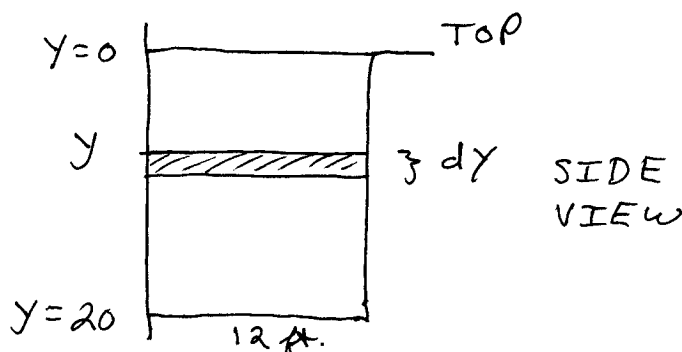
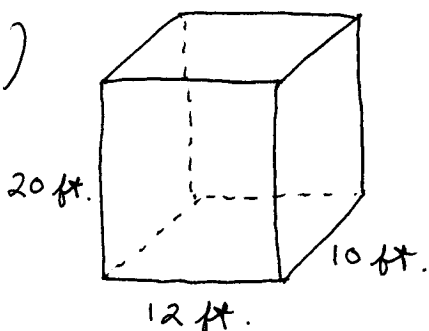


work to move bucket from

x to $x + dx$ is
 $\approx (\text{weight}) (\text{distance})$
 $= (21 - \frac{4}{5}x) (dx)$; total work
 to lift bucket is

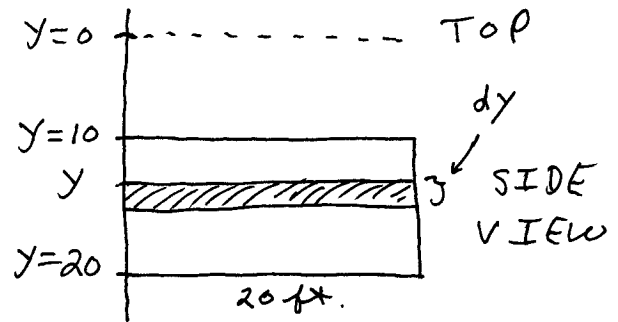
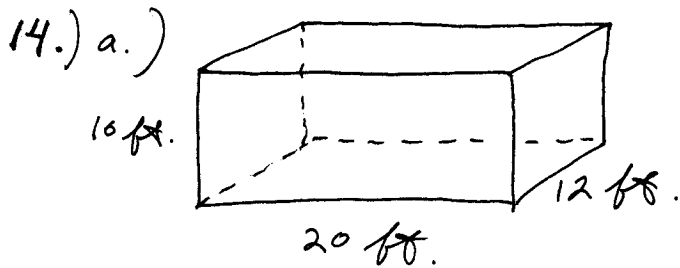
$$\begin{aligned}
 \text{Work} &= \int_0^{20} (21 - \frac{4}{5}x) dx \\
 &= (21x - \frac{4}{5} \cdot \frac{x^2}{2}) \Big|_0^{20} = 420 - 160 = 260 \text{ ft.-lbs.}
 \end{aligned}$$

13.) a.)



work to move thin slice of H_2O to top is
 $\approx (\text{weight}) (\text{distance})$
 $= (\text{density}) (\text{volume}) (\text{distance})$
 $= (62.4) \cdot ((10)(12)(dy)) \cdot (y)$
 $= 7488 y dy$; so total work to
 empty the tank is

$$\begin{aligned}
 \text{Work} &= \int_0^{20} 7488 y dy = 3744 y^2 \Big|_0^{20} \\
 &= 1,497,600 \text{ ft.-lbs.}
 \end{aligned}$$



work to move thin slice of H_2O to top is
 \approx (weight) (distance)

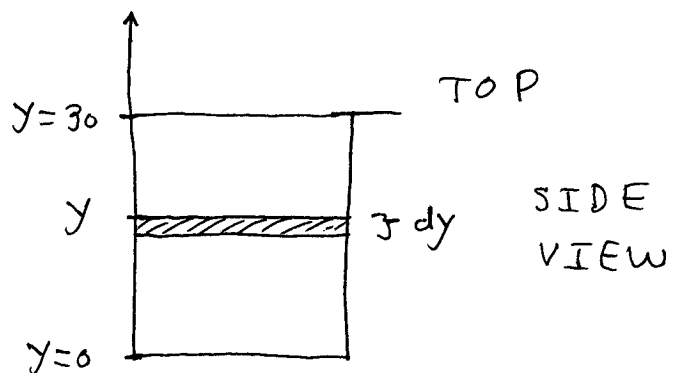
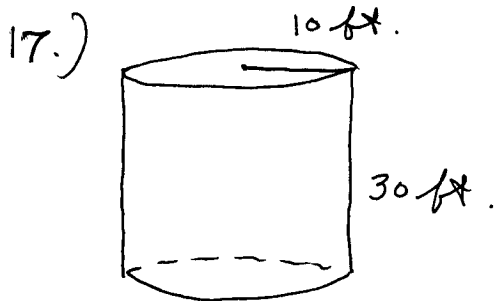
$$= (\text{density})(\text{volume})(\text{distance})$$

$$= (62.4) ((20)(12)(dy)) \cdot (y); \text{ so total}$$

work to move H_2O to top is

$$\text{Work} = \int_{10}^{20} 14,976 y \, dy = 7488 y^2 \Big|_{10}^{20}$$

$$= 7488 (400 - 100) = 2,246,400 \text{ ft.-lbs.}$$



work to move slice of kerosene to top is

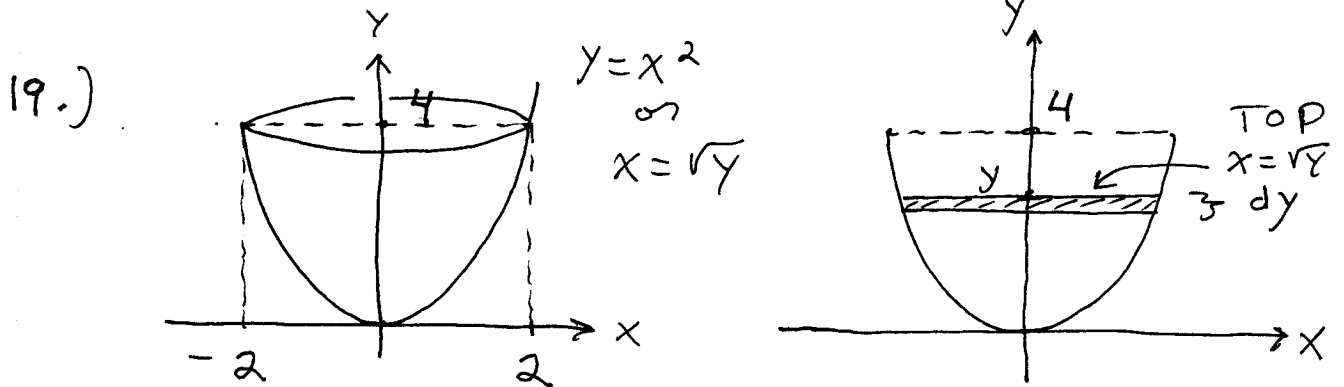
$$\approx (\text{weight})(\text{distance})$$

$$= (\text{density})(\text{volume})(\text{distance})$$

$$= (51.2) (\pi (10)^2 \cdot dy) (30 - y)$$

$$= 5120 \pi (30 - y) \, dy; \text{ so work to empty tank is}$$

$$\begin{aligned}
 \text{Work} &= \int_0^{30} 5120\pi (30-y) dy \\
 &= 5120\pi \left(30y - \frac{y^2}{2} \right) \Big|_0^{30} \\
 &= 5120\pi (900 - 450) = 2,304,000\pi \text{ ft.-lbs.}
 \end{aligned}$$



work to move circular slice of H_2O to top is

$$\begin{aligned}
 &\approx (\text{weight})(\text{distance}) \\
 &= (\text{density})(\text{volume})(\text{distance}) \\
 &= (73) (\pi(\sqrt{y})^2 \cdot dy) (4-y) \\
 &= 73 \cdot \pi (4y - y^2) dy ;
 \end{aligned}$$

so total work to empty pipe is

$$\begin{aligned}
 \text{Work} &= \int_0^4 73 \pi (4y - y^2) dy \\
 &= 73 \pi \left(2y^2 - \frac{y^3}{3} \right) \Big|_0^4 \\
 &= 73 \pi \left(32 - \frac{64}{3} \right) \\
 &= (73) \frac{32}{3} \pi \approx 2446.25 \text{ ft.-lbs.}
 \end{aligned}$$

24.) $K.E. = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

$$(2 \text{ oz}) \left(\frac{1 \text{ lb.}}{16 \text{ oz.}} \right) = \frac{1}{8} \text{ lb.} ; F = mg \rightarrow$$

$$\frac{1}{8} = m \cdot 32 \rightarrow m = \frac{1}{256} \text{ slugs}; v_1 = 0 \text{ ft./sec.},$$

$$v_2 = 160 \text{ ft./sec.}; \text{ then}$$

$$\text{K.E.} = \frac{1}{2} \left(\frac{1}{256} \right) (160)^2 - \frac{1}{2} \left(\frac{1}{256} \right) (0)^2 = 50 \text{ ft.-lbs.}$$

$$25.) \quad F = mg \rightarrow 0.3125 = m \cdot (32) \rightarrow$$

$$m = \frac{0.3125}{32} \text{ slugs}; v_1 = 0 \text{ ft./sec.},$$

$$v_2 = \left(90 \frac{\text{mi}}{\text{hr.}} \right) \left(\frac{1 \text{ hr.}}{3600 \text{ sec.}} \right) \left(\frac{5280 \text{ ft.}}{1 \text{ mi.}} \right) = 132 \frac{\text{ft.}}{\text{sec.}}; \text{ then}$$

$$\text{K.E.} = \frac{1}{2} \left(\frac{0.3125}{32} \right) (132)^2 \approx 85.1 \text{ ft.-lbs.}$$

$$26.) \quad (1.6 \text{ oz}) \left(\frac{1 \text{ lb.}}{16 \text{ oz.}} \right) = \frac{1}{10} \text{ lb.}; F = mg \rightarrow$$

$$\frac{1}{10} = m(32) \rightarrow m = \frac{1}{320} \text{ slugs}; v_1 = 0 \text{ ft./sec.},$$

$$v_2 = 280 \frac{\text{ft.}}{\text{sec.}}, \text{ then}$$

$$\text{K.E.} = \frac{1}{2} \left(\frac{1}{320} \right) (280)^2 = 122.5 \text{ ft.-lbs.}$$