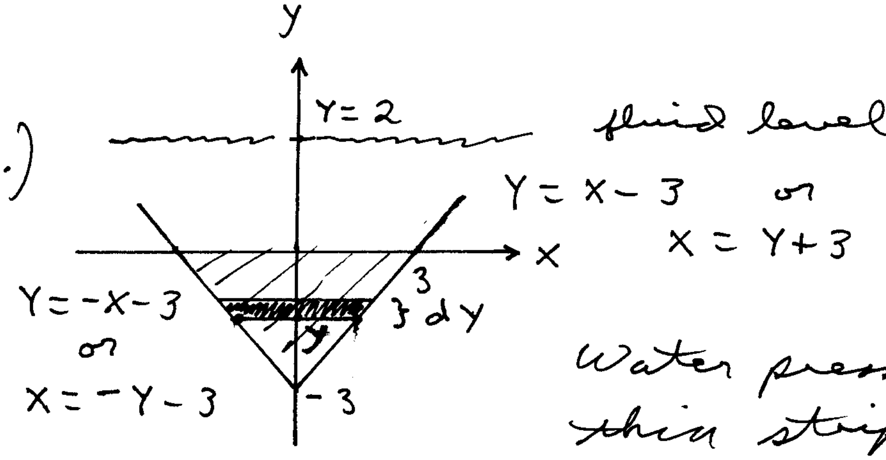
Section 6.5

34.)



Water pressure on thin strip is

$\approx$ (area) (depth) (density)

$= [((y+3)-(-y-3))\cdot dy]\cdot(2-y)\cdot(62.4)$

$= (62.4)(2y+6)(2-y)\,dy$

$= (124.8)(y+3)(2-y)\,dy$

$= (124.8)(-y^2-y+6)\,dy$; so total force is

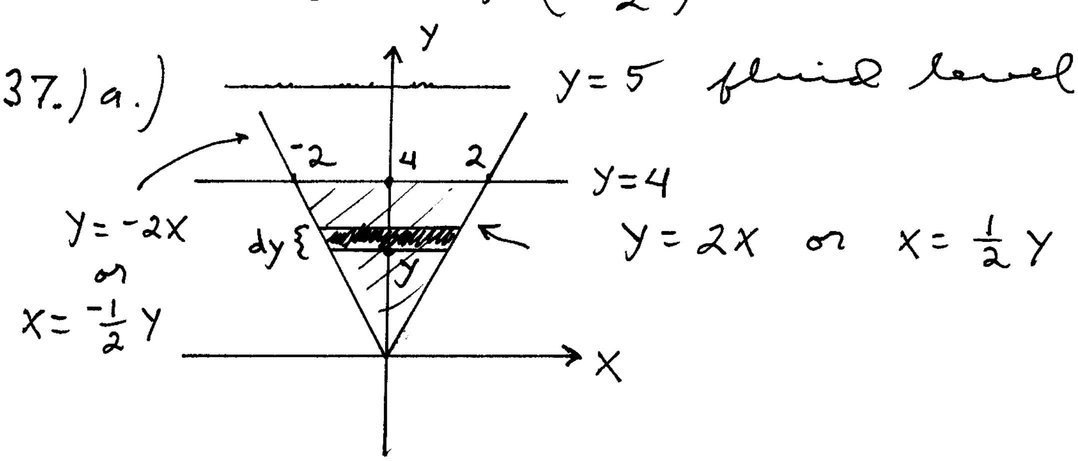
$$P = \int_{-3}^{0} (124.8)(-y^2-y+6)\,dy$$

$$= (124.8)\left(-\frac{y^3}{3}-\frac{y^2}{2}+6y\right)\Big|_{-3}^{0}$$

$$= 0-(124.8)\left(9-\frac{9}{2}-18\right)$$

$$= -(124.8)\left(-\frac{27}{2}\right) = 1684.8 \text{ lbs.}$$

37.) a.)



Water pressure on thin strip is

$\approx$ (area) (depth) (density)

$= [((\tfrac{1}{2}y)-(-\tfrac{1}{2}y))\cdot dy]\cdot(5-y)\cdot(62.4)$

$= (62.4)(5y-y^2)\,dy$;

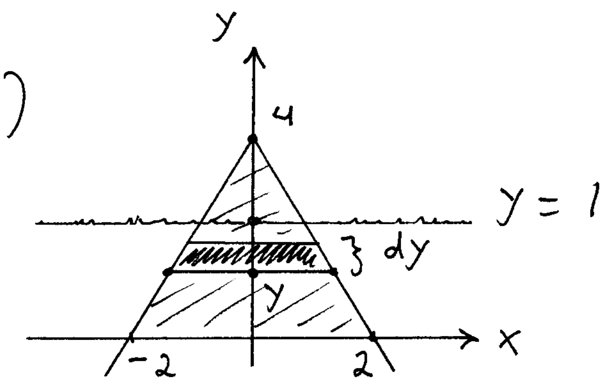
so total force is

$$P = \int_0^4 (62.4)(5y - y^2)\, dy$$

$$= (62.4)\left(\frac{5}{2}y^2 - \frac{y^3}{3}\right)\Big|_0^4$$

$$= (62.4)\left(40 - \frac{64}{3}\right) = 1164.8 \text{ lbs.}$$

38.)



$y = 2x + 4$ or $x = \frac{1}{2}(y - 4)$

$y = -2x + 4$ or $x = \frac{1}{2}(4 - y)$

water pressure on thin strip is

$$\approx (\text{area})(\text{depth})(\text{density})$$

$$= \left[\left(\frac{1}{2}(4-y) - \frac{1}{2}(y-4)\right) \cdot dy\right] \cdot (1 - y) \cdot (62.4)$$

$$= (62.4)(4-y)(1-y)\, dy$$

$$= (62.4)(y^2 - 5y + 4)\, dy \,;$$

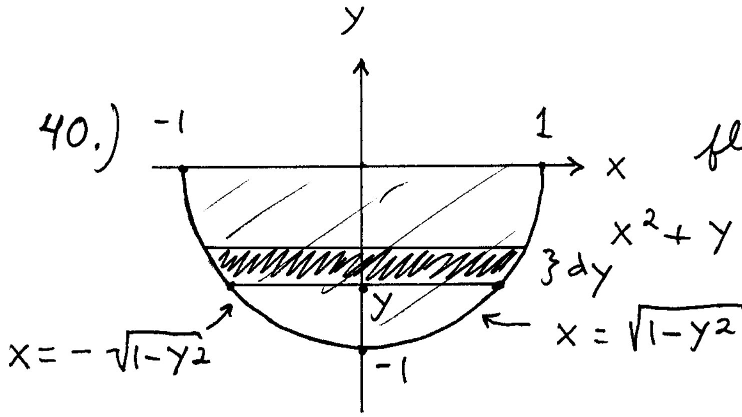
so total force is

$$P = \int_0^1 (62.4)(y^2 - 5y + 4)\, dy$$

$$= (62.4)\left(\frac{y^3}{3} - \frac{5}{2}y^2 + 4y\right)\Big|_0^1$$

$$= (62.4)\left(\frac{1}{3} - \frac{5}{2} + 4\right) = (62.4)\left(\frac{11}{6}\right) = 114.4 \text{ lbs.}$$

40.)



$x^2 + y^2 = 1$ or $x = \pm\sqrt{1-y^2}$

Water pressure on thin strip is

$\approx$ (area) (depth) (density)

$= \left[\left(\sqrt{1-y^2} - \left(-\sqrt{1-y^2}\right)\right) \cdot dy\right] \cdot (0-y)\,(62.4)$

$= -(62.4) \cdot y\left(2\sqrt{1-y^2}\right)$

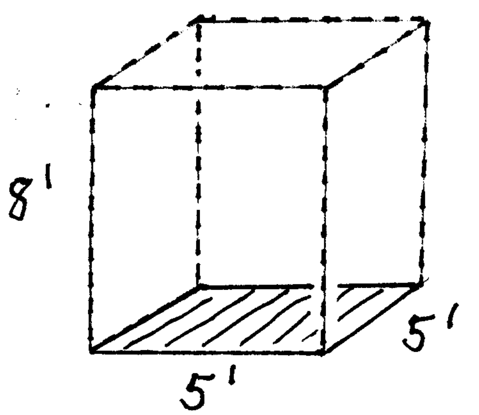
$= -(124.8) \cdot y\sqrt{1-y^2}$; so total force is

$P = \int_{-1}^{0} -(124.8) \cdot y\sqrt{1-y^2}\ dy$

$= -(124.8) \cdot \frac{-1}{\cancel{2}} \frac{(1-y^2)^{3/2}}{3/\cancel{2}} \Big|_{-1}^{0}$

$= (41.6)\left(1^{3/2} - 0^{3/2}\right) = 41.6$ lbs.

41.) a.)



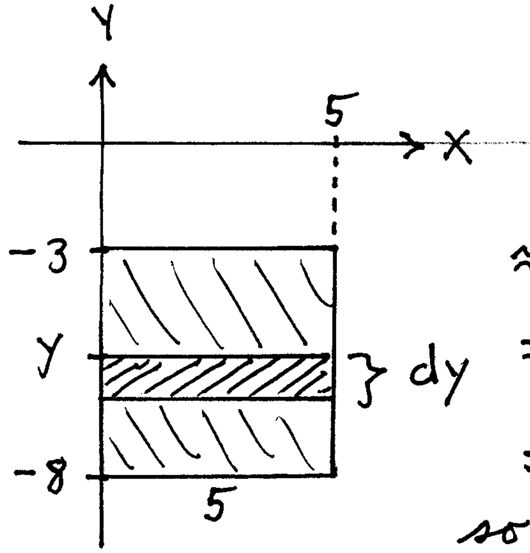
total force of H_2O is

$P = $ (area)(depth)(density)

$= (25)\,(8)\,(62.4)$

$= 12{,}480$ lbs.

b.)



Water pressure on thin strip is

$\approx$ (area)(depth)(density)

$= (5\,dy)(0-Y)(62.4)$

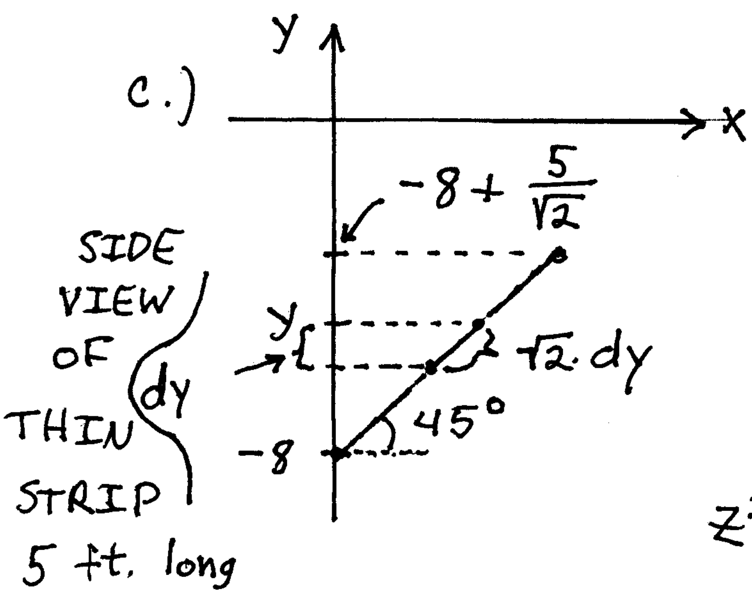
$= -312Y\,dY$;

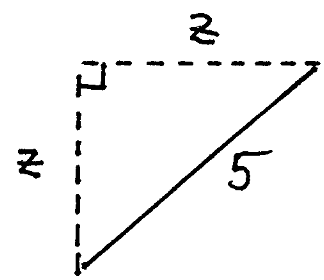
so total force is

$$P = \int_{-8}^{-3} -312Y\,dy = -156Y^2\Big|_{-8}^{-3}$$

$$= -156(-3)^2 - -156(-8)^2 = 8580 \text{ lbs.}$$

c.)

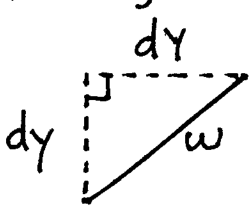




$z^2 + z^2 = 5^2 \rightarrow$

$2z^2 = 25 \rightarrow$

$z^2 = \frac{25}{2} \rightarrow \boxed{z = \frac{5}{\sqrt{2}}}$



$(dY)^2 + (dY)^2 = w^2 \rightarrow$

$2(dY)^2 = w^2 \rightarrow \boxed{w = \sqrt{2}\cdot dy}$;

water pressure on thin strip is

$\approx$ (area)(depth)(density)

$= (5\cdot\sqrt{2}\,dY)(0-Y)(62.4)$

$= -312\sqrt{2}\cdot y\,dy$;

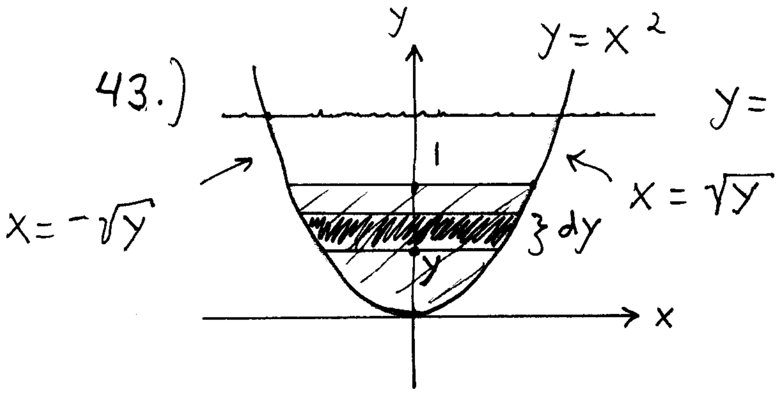
so total force is

$$P = \int_{-8}^{-8+5/\sqrt{2}} -312\sqrt{2} \cdot y \, dy$$

$$= -312\sqrt{2} \cdot \frac{1}{2} y^2 \Big|_{-8}^{-8+5/\sqrt{2}}$$

$$= \frac{-312}{\sqrt{2}}\left(-8+\frac{5}{\sqrt{2}}\right)^2 - \frac{-312}{\sqrt{2}}(-8)^2 \approx 9722.3 \text{ lbs.}$$

43.)



a.) Water pressure on thin strip is

$\approx$ (area)(depth)(density)

$$= \left[\left(\sqrt{y} - (-\sqrt{y})\right) \cdot dy\right] \cdot (2-y) \cdot (50)$$

$$= (2\sqrt{y} \cdot dy)(2-y)(50)$$

$$= 100\left(2\sqrt{y} - y^{3/2}\right)$$; so total force is

$$P = \int_0^1 100\left(2\sqrt{y} - y^{3/2}\right) dy$$

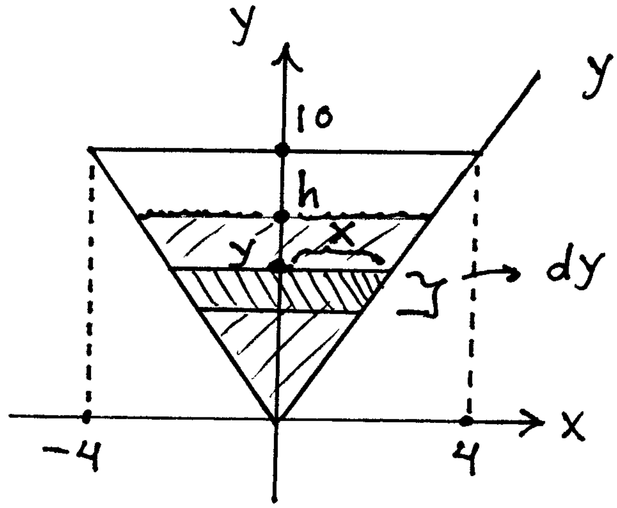
$$= 100\left(2 \cdot \frac{y^{3/2}}{3/2} - \frac{y^{5/2}}{5/2}\right)\Big|_0^1$$

$$= 100\left(\frac{4}{3}y^{3/2} - \frac{2}{5}y^{5/2}\right)\Big|_0^1$$

$$= 100\left(\frac{4}{3} - \frac{2}{5}\right)$$

$$= 100 \cdot \frac{14}{15} = 93\tfrac{1}{3} \text{ lbs.}$$

44.)



$y = \frac{5}{2}x$ or $x = \frac{2}{5}y$

water pressure on thin strip is

$\approx$ (area)(depth)(density)

$= \left(2\left(\frac{2}{5}y\right)dy\right)\cdot(h-y)(62.4)$

$= 49.92\,(hy - y^2)\,dy$; so total force is

$P = \int_0^h 49.92\,(hy - y^2)\,dy$

$= 49.92\left(\frac{h}{2}y^2 - \frac{1}{3}y^3\right)\Big|_0^h$

$= 49.92\left(\frac{1}{2}h^3 - \frac{1}{3}h^3\right)$

$= 49.92\left(\frac{1}{6}h^3\right)$

$= 8.32\,h^3 = 6667$ lbs. $\longrightarrow$

$h^3 \approx 801.32 \longrightarrow h \approx (801.32)^{1/3} \approx 9.29$ ft. ;

then total volume of H_2O is

Vol $= \frac{1}{2}$ (base)(height)(30)

$= \frac{1}{\cancel{2}}\left(\cancel{2}\left(\frac{2}{5}\right)(9.29)\right)(9.29)(30)$

≈ 1035.65 ft.^{3}