

Section 7.1

$$\begin{aligned}
 2.) \quad & \int_{-1}^0 \frac{3 \, dx}{3x-2} \quad (\text{Let } u = 3x-2 \rightarrow \\
 & du = 3 \, dx; \quad x: -1 \rightarrow 0 \quad \text{so } u: -5 \rightarrow -2) \\
 & = \int_{-5}^{-2} \frac{1}{u} \, du = \ln|u| \Big|_{-5}^{-2} = \ln 2 - \ln 5 \\
 & = \ln(2/5)
 \end{aligned}$$

$$\begin{aligned}
 3.) \quad & \int \frac{2y}{y^2-25} \, dy \quad (\text{Let } u = y^2-25 \rightarrow \\
 & du = 2y \, dy) \\
 & = \int \frac{1}{u} \, du = \ln|u| + C = \ln|y^2-25| + C
 \end{aligned}$$

$$\begin{aligned}
 5.) \quad & \int \frac{3 \sec^2 t}{6+3 \tan t} \, dt \quad (\text{Let } u = 6+3 \tan t \rightarrow \\
 & du = 3 \cdot \sec^2 t \, dt) \\
 & = \int \frac{1}{u} \, du = \ln|u| + C = \ln|6+3 \tan t| + C
 \end{aligned}$$

$$\begin{aligned}
 6.) \quad & \int \frac{\sec y \tan y}{2+\sec y} \, dy \quad (\text{Let } u = 2+\sec y \rightarrow \\
 & du = \sec y \tan y \, dy) \\
 & = \int \frac{1}{u} \, du = \ln|u| + C = \ln|2+\sec y| + C
 \end{aligned}$$

$$\begin{aligned}
 7.) \quad & \int \frac{1}{2\sqrt{x}+2x} \, dx = \frac{1}{2} \int \frac{1}{\sqrt{x}(1+\sqrt{x})} \, dx \\
 & (\text{Let } u = 1+\sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}} \, dx \rightarrow 2 \, du = \frac{1}{\sqrt{x}} \, dx) \\
 & = \frac{1}{2} \int \frac{2}{u} \, du = \ln|u| + C = \ln|1+\sqrt{x}| + C
 \end{aligned}$$

$$\begin{aligned}
 8.) \quad & \int \frac{\sec x \, dx}{\sqrt{\ln(\sec x + \tan x)}} \quad (\text{Let } u = \ln(\sec x + \tan x)) \\
 & \rightarrow du = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} \, dx = \frac{\sec x (\cancel{\sec x + \tan x})}{\cancel{\sec x + \tan x}} \, dx \\
 & = \int \frac{1}{\sqrt{u}} \, du = \int u^{-1/2} \, du = \frac{u^{1/2}}{1/2} + C = \frac{1}{2} \ln(\sec x + \tan x)^{1/2} + C
 \end{aligned}$$

$$9.) \int_{\ln 2}^{\ln 3} e^x dx = e^x \Big|_{\ln 2}^{\ln 3} = e^{\ln 3} - e^{\ln 2} = 3 - 2 = 1$$

$$12.) \int \frac{\ln(\ln x)}{x \ln x} dx \quad (\text{let } u = \ln(\ln x) \rightarrow du = \frac{1}{\ln x} \cdot \frac{1}{x} dx)$$

$$= \int u du = \frac{1}{2} u^2 + c = \frac{1}{2} (\ln(\ln x))^2 + c$$

$$13.) \int_{\ln 4}^{\ln 9} e^{x/2} dx \quad (\text{let } u = \frac{x}{2} \rightarrow du = \frac{1}{2} dx$$

$$\rightarrow 2 du = dx$$

and $x: \ln 4 \rightarrow \ln 9$ so $u: \frac{1}{2} \ln 4 \rightarrow \frac{1}{2} \ln 9$)

$$= 2 \int_{\frac{1}{2} \ln 4}^{\frac{1}{2} \ln 9} e^u du = 2 e^u \Big|_{\frac{1}{2} \ln 4}^{\frac{1}{2} \ln 9}$$

$$= 2 e^{\frac{1}{2} \ln 9} - 2 e^{\frac{1}{2} \ln 4} = 2 e^{\ln 9^{1/2}} - 2 e^{\ln 4^{1/2}}$$

$$= 2(3) - 2(2) = 2$$

$$15.) \int \frac{e^{\sqrt{r}}}{\sqrt{r}} dr \quad (\text{let } u = \sqrt{r} \rightarrow du = \frac{1}{2\sqrt{r}} dr$$

$$\rightarrow 2 du = \frac{1}{\sqrt{r}} dr)$$

$$= 2 \int e^u du = 2 e^u + c = 2 e^{\sqrt{r}} + c$$

$$20.) \int \frac{e^{-1/x^2}}{x^3} dx \quad (\text{let } u = \frac{-1}{x^2} \rightarrow du = \frac{2}{x^3} dx \rightarrow$$

$$\frac{1}{2} du = \frac{1}{x^3} dx)$$

$$= \frac{1}{2} \int e^u du = \frac{1}{2} e^u + c = \frac{1}{2} e^{-1/x^2} + c$$

$$22.) \int e^{\csc(\pi+t)} \csc(\pi+t) \cot(\pi+t) dt$$

$$(\text{let } u = \csc(\pi+t) \rightarrow du = -\csc(\pi+t) \cot(\pi+t) dt$$

$$\rightarrow -du = \csc(\pi+t) \cot(\pi+t) dt)$$

$$24.) \int_0^{\sqrt{\ln \pi}} 2x e^{x^2} \cos(e^{x^2}) dx \quad (\text{let } u = e^{x^2} \rightarrow$$

$$\begin{aligned}
 du &= 2xe^{x^2} dx; \quad x: 0 \rightarrow \sqrt{\ln \pi} \text{ and} \\
 & \quad u: e^0 \rightarrow e^{\ln \pi} \text{ or } u: 1 \rightarrow \pi \\
 &= \int_1^\pi \cos u \, du = \sin u \Big|_1^\pi = \sin \pi - \sin 1 \\
 &= -\sin 1
 \end{aligned}$$

$$\begin{aligned}
 25.) \int \frac{e^n}{1+e^n} dr & \quad (\text{Let } u = 1+e^n \rightarrow du = e^n dr) \\
 &= \int \frac{1}{u} du = \ln|u| + c = \ln|1+e^n| + c
 \end{aligned}$$

$$\begin{aligned}
 26.) \int \frac{1}{1+e^x} dx &= \int \frac{1}{1+e^x} \cdot \frac{e^{-x}}{e^{-x}} dx \\
 &= \int \frac{e^{-x}}{e^{-x}+1} dx \quad (\text{Let } u = e^{-x}+1 \rightarrow \\
 & \quad du = -e^{-x} dx \rightarrow -du = e^{-x} dx) \\
 &= -\int \frac{1}{u} du = -\ln|u| + c = -\ln|e^{-x}+1| + c
 \end{aligned}$$

$$\begin{aligned}
 28.) \int_{-2}^0 5^{-\theta} d\theta & \quad (\text{Let } u = -\theta \rightarrow du = -d\theta \rightarrow \\
 & \quad -du = d\theta; \quad x: -2 \rightarrow 0 \text{ so } u: 2 \rightarrow 0) \\
 &= -\int_2^0 5^u du = -\frac{5^u}{\ln 5} \Big|_2^0 = \frac{-5^0}{\ln 5} - \frac{-5^2}{\ln 5} \\
 &= \frac{-1}{\ln 5} + \frac{25}{\ln 5} = \frac{24}{\ln 5}
 \end{aligned}$$

$$\begin{aligned}
 32.) \int_0^{\frac{\pi}{4}} \left(\frac{1}{3}\right)^{\tan t} \cdot \sec^2 t \, dt & \quad (\text{Let } u = \tan t \rightarrow \\
 & \quad du = \sec^2 t \, dt; \quad t: 0 \rightarrow \frac{\pi}{4} \text{ so } u: 0 \rightarrow 1) \\
 &= \int_0^1 \left(\frac{1}{3}\right)^u du = \frac{\left(\frac{1}{3}\right)^u}{\ln(\frac{1}{3})} \Big|_0^1 = \frac{\frac{1}{3}}{\ln(\frac{1}{3})} - \frac{1}{\ln(\frac{1}{3})} \\
 &= \frac{-2/3}{\ln(\frac{1}{3})}
 \end{aligned}$$

$$\begin{aligned}
 33.) \int_2^4 x^{2x} (1 + \ln x) dx & \quad (\text{Let } u = x^{2x} \rightarrow \\
 \ln u = \ln x^{2x} & = 2x \cdot \ln x \xrightarrow{D} \\
 \frac{1}{u} \cdot \frac{du}{dx} & = 2x \cdot \frac{1}{x} + 2 \ln x \rightarrow du = u(2 + 2 \ln x) dx \\
 \rightarrow du & = 2 \cdot x^{2x} (1 + \ln x) dx \rightarrow \frac{1}{2} du = x^{2x} (1 + \ln x) dx ; \\
 x: 2 \rightarrow 4 & \text{ so } u: 2^4 \rightarrow 4^8 = 2^{16} \\
 = \int_{2^4}^{2^{16}} \frac{1}{2} du & = \frac{1}{2} u \Big|_{2^4}^{2^{16}} = \frac{1}{2} (65,536 - 16) \\
 & = 32,760
 \end{aligned}$$

$$\begin{aligned}
 36.) \int_1^e x^{(\ln 2) - 1} dx & = \frac{x^{\ln 2}}{\ln 2} \Big|_1^e = \frac{e^{\ln 2}}{\ln 2} - \frac{1^{\ln 2}}{\ln 2} \\
 & = \frac{2}{\ln 2} - \frac{1}{\ln 2} = \frac{1}{\ln 2}
 \end{aligned}$$

$$\begin{aligned}
 39.) \int_1^4 \frac{\ln 2 \cdot \log_2 x}{x} dx & = \ln 2 \int_1^4 \frac{\frac{\ln x}{\ln 2}}{x} dx \\
 & = \int_1^4 \frac{\ln x}{x} dx \quad (\text{Let } u = \ln x \rightarrow du = \frac{1}{x} dx; \\
 & \quad x: 1 \rightarrow 4 \text{ so } u: 0 \rightarrow \ln 4) \\
 & = \int_0^{\ln 4} u du = \frac{u^2}{2} \Big|_0^{\ln 4} = \frac{1}{2} (\ln 4)^2
 \end{aligned}$$

$$\begin{aligned}
 42.) \int_{\frac{1}{10}}^{10} \frac{\log_2(10x)}{x} dx & = \int_{\frac{1}{10}}^{10} \frac{\frac{\ln(10x)}{\ln 2}}{x} dx \\
 & = \frac{1}{\ln 2} \int_{\frac{1}{10}}^{10} \frac{\ln(10x)}{x} dx \quad (\text{Let } u = \ln(10x) \rightarrow \\
 du = \frac{1}{10x} \cdot 10 dx; & \quad x: \frac{1}{10} \rightarrow 10 \text{ so } u: 0 \rightarrow \ln 100) \\
 & = \frac{1}{\ln 2} \int_0^{\ln 100} u du = \frac{1}{\ln 2} \cdot \frac{u^2}{2} \Big|_0^{\ln 100}
 \end{aligned}$$

$$= \frac{1}{\ln 10^2} \cdot (\ln 100)^2 = \ln 100$$

$$49.) \frac{d^2y}{dx^2} = 2e^{-x} \rightarrow \frac{dy}{dx} = -2e^{-x} + c$$

$$\text{and } y'(0) = 0 \rightarrow 0 = -2 \cdot e^0 + c \rightarrow c = 2 \rightarrow$$

$$\frac{dy}{dx} = -2e^{-x} + 2 \rightarrow y = 2e^{-x} + 2x + c$$

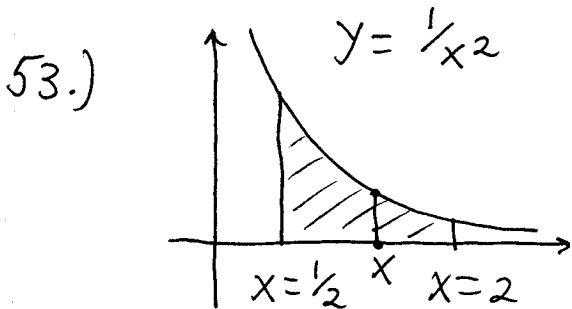
$$\text{and } y(0) = 1 \rightarrow 1 = 2 \cdot e^0 + 2(0) + c \rightarrow$$

$$c = -1 \rightarrow y = 2e^{-x} + 2x - 1$$

$$51.) \frac{dy}{dx} = 1 + \frac{1}{x} \rightarrow y = x + \ln|x| + c$$

$$\text{and } y(1) = 3 \rightarrow 3 = 1 + \ln 1 + c \rightarrow$$

$$c = 2 \rightarrow y = x + \ln|x| + 2$$



SHELL METHOD:

$$\text{Vol} = 2\pi \int_{\frac{1}{2}}^2 (x) \left(\frac{1}{x^2}\right) dx$$

radius \uparrow height \uparrow

$$= 2\pi \int_{\frac{1}{2}}^2 \frac{1}{x} dx = 2\pi \ln|x| \Big|_{\frac{1}{2}}^2$$

$$= 2\pi (\ln 2 - \ln \frac{1}{2})$$

$$= 2\pi (\ln 2 - (\ln 1 - \ln 2))$$

$$= 2\pi (2 \ln 2)$$

$$= 4\pi \ln 2$$

$$55.) y = \frac{x^2}{8} - \ln x \quad \text{for } 4 \leq x \leq 8 \rightarrow$$

$$\text{Arc} = \int_4^8 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_4^8 \sqrt{1 + \left(\frac{1}{4}x - \frac{1}{x}\right)^2} dx$$

$$= \int_4^8 \sqrt{1 + \frac{1}{16}x^2 - \frac{1}{2} + \frac{1}{x^2}} dx$$

$$= \int_4^8 \sqrt{\frac{1}{16}x^2 + \frac{1}{2} + \frac{1}{x^2}} dx$$

$$= \int_4^8 \sqrt{\left(\frac{1}{4}x + \frac{1}{x}\right)^2} dx$$

$$= \int_4^8 \left(\frac{1}{4}x + \frac{1}{x}\right) dx$$

$$= \left(\frac{1}{4} \frac{x^2}{2} + \ln|x|\right) \Big|_4^8$$

$$= \frac{1}{8}(64) + \ln 8 - \left(\frac{1}{8}(16) + \ln 4\right)$$

$$= 8 - 2 + \ln 8 - \ln 4$$

$$= 6 + \ln(8/4) = 6 + \ln 2$$

$$70.) b.) \text{ Show } \log_b x = \frac{\ln a}{\ln b} \log_a x :$$

$$\text{Let } y = \log_b x \rightarrow b^y = x \rightarrow \log_a b^y = \log_a x$$

$$\rightarrow y \cdot \log_a b = \log_a x \rightarrow y = \frac{1}{\log_a b} \cdot \log_a x$$

$$\rightarrow y = \frac{\frac{1}{\log_a b} \cdot \log_a x}{\frac{\ln b}{\ln a}} \quad (\text{By Rules From Class})$$

$$\rightarrow y = \frac{\ln a}{\ln b} \cdot \log_a x, \text{ i.e., } \log_b x = \frac{\ln a}{\ln b} \cdot \log_a x$$

