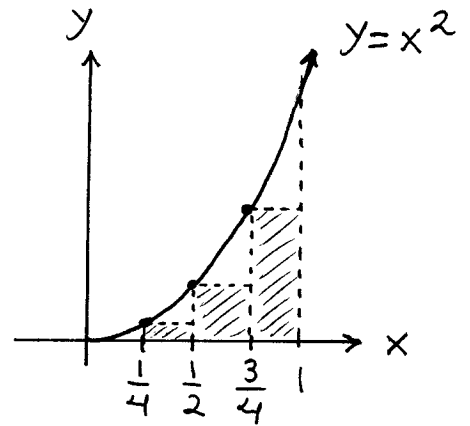


## Section 5.1

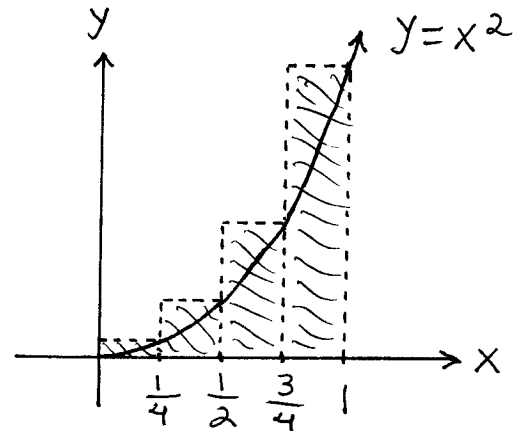
1.) b.) Lower Sum:

$$\begin{aligned} & (0)^2 \cdot \frac{1}{4} + \left(\frac{1}{4}\right)^2 \cdot \frac{1}{4} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} \\ &= \frac{1}{4} \left( \frac{1}{16} + \frac{1}{4} + \frac{9}{16} \right) \\ &= \frac{1}{4} \cdot \frac{14}{16} = \frac{7}{32} \end{aligned}$$



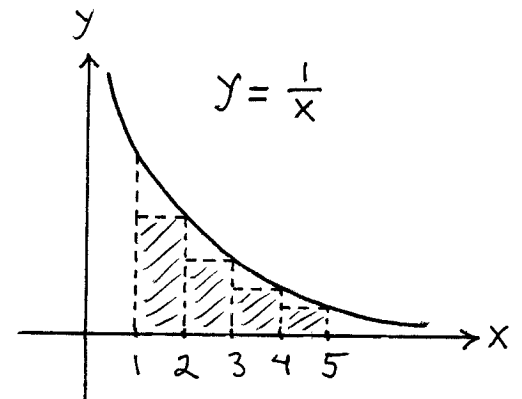
d.) Upper Sum:

$$\begin{aligned} & \left(\frac{1}{4}\right)^2 \cdot \frac{1}{4} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + (1)^2 \cdot \frac{1}{4} \\ &= \frac{1}{4} \left( \frac{1}{16} + \frac{1}{4} + \frac{9}{16} + 1 \right) \\ &= \frac{1}{4} \cdot \frac{30}{16} = \frac{15}{32} \end{aligned}$$



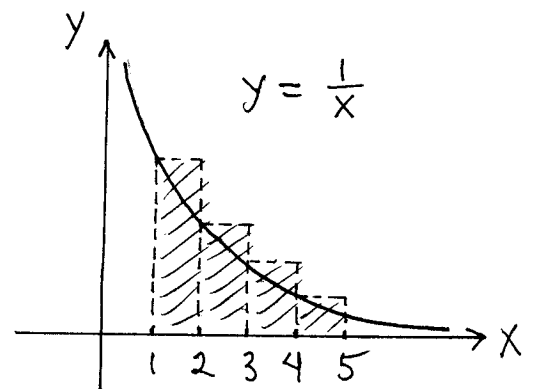
3.) b.) Lower Sum:

$$\begin{aligned} & \frac{1}{2} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{4} \cdot 1 + \frac{1}{5} \cdot 1 \\ &= \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60} \\ &= \frac{77}{60} \end{aligned}$$



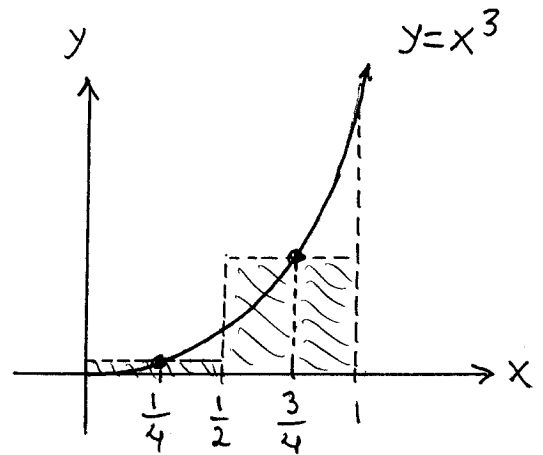
d.) Upper Sum:

$$\begin{aligned} & 1 \cdot 1 + \frac{1}{2} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{4} \cdot 1 \\ &= \frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12} \\ &= \frac{25}{12} \end{aligned}$$



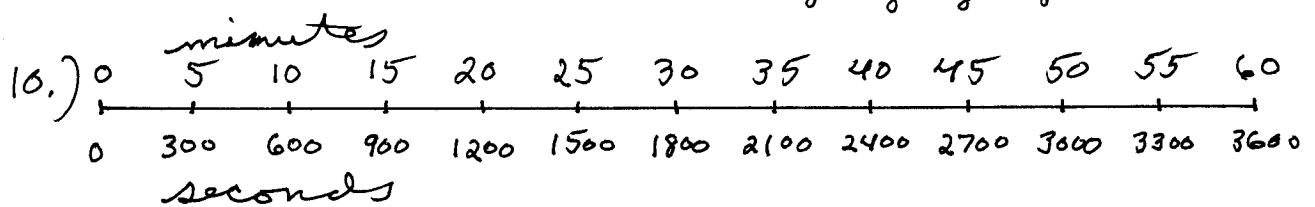
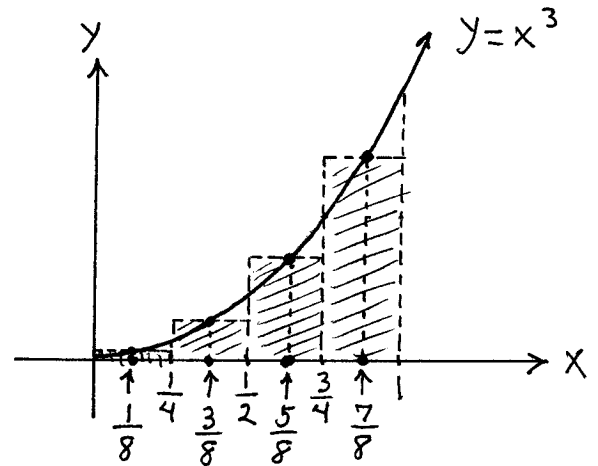
6.) a.) midpoint sum:

$$\begin{aligned} & \left(\frac{1}{4}\right)^3 \cdot \frac{1}{2} + \left(\frac{3}{4}\right)^3 \cdot \frac{1}{2} \\ &= \frac{1}{2} \left(\frac{1}{64} + \frac{27}{64}\right) \\ &= \frac{1}{2} \cdot \frac{28}{64} = \frac{14}{64} = \frac{7}{32} \end{aligned}$$



b.) midpoint sum:

$$\begin{aligned} & \left(\frac{1}{8}\right)^3 \cdot \frac{1}{4} + \left(\frac{3}{8}\right)^3 \cdot \frac{1}{4} + \left(\frac{5}{8}\right)^3 \cdot \frac{1}{4} + \left(\frac{7}{8}\right)^3 \cdot \frac{1}{4} \\ &= \frac{1}{4} \left(\frac{1}{512} + \frac{27}{512} + \frac{125}{512} + \frac{343}{512}\right) \\ &= \frac{1}{4} \cdot \frac{496}{512} = \frac{124}{512} = \frac{31}{128} \end{aligned}$$

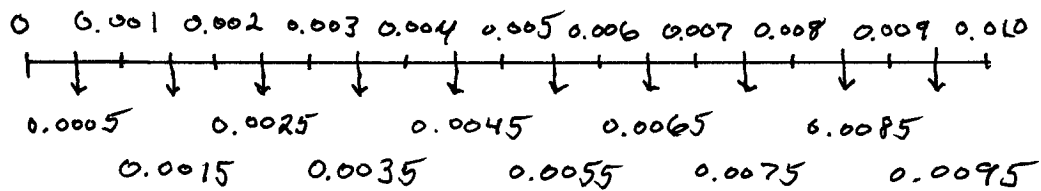


$$\begin{array}{ccccc} \text{Distance} & = & \text{Velocity} & \times & \text{Time} \\ (\text{m.}) & & (\text{m./sec.}) & & (\text{sec.}) \end{array}$$

a.) Distance  $\approx (1)(300) + (1.2)(300) + (1.7)(300) + (2.0)(300) + (1.8)(300) + (1.6)(300) + (1.4)(300) + (1.2)(300) + (1.0)(300) + (1.8)(300) + (1.5)(300) + (1.2)(300) = 5220 \text{ m.}$

b.) Distance  $\approx (1.2)(300) + (1.7)(300) + (2.0)(300) + (1.8)(300) + (1.6)(300) + (1.4)(300) + (1.2)(300) + (1.0)(300) + (1.8)(300) + (1.5)(300) + (1.2)(300) + (1.0)(300) = 4920 \text{ m.}$

12.) Use midpoints of subintervals to determine velocities, which can be estimated from the graph:



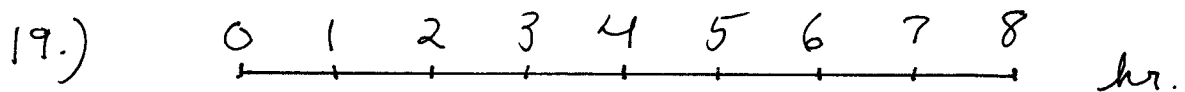
hrs.

$$a.) \text{ Distance} = \text{Velocity} \times \text{Time}$$

(mi.)                  (mph)                  (hr.)

$$\begin{aligned} \text{Distance} &\approx (20)(0.001) + (50)(0.001) \\ &+ (72)(0.001) + (90)(0.001) + (103)(0.001) \\ &+ (112)(0.001) + (120)(0.001) + (128)(0.001) \\ &+ (134)(0.001) + (140)(0.001) = 0.969 \text{ mi.} \end{aligned}$$

b.) at approx.  $t = 0.006$  hrs. the car reaches its halfway point of 0.4845 mi ( $\frac{1}{2}$  of area under graph); its speed at that time is approx. 118 mph.



$$a.) \text{ Leakage} = \text{Rate} \times \text{Time}$$

(gal.)                  (gal./hr.)                  (hr.)

i.) Lower Estimate (left-hand endpoints):

$$\begin{aligned} \text{Leakage} &\approx (50)(1) + (70)(1) + (97)(1) \\ &+ (136)(1) + (190)(1) = 543 \text{ gal.} \end{aligned}$$

ii.) Upper Estimate (right-hand endpoints) :

$$\text{Leakage} \approx (70)(1) + (97)(1) + (136)(1) + (190)(1) + (265)(1) = 758 \text{ gal.}$$

b.) i.) Lower Estimate :

$$\text{Leakage} \approx (50)(1) + (70)(1) + (97)(1) + (136)(1) + (190)(1) + (265)(1) + (369)(1) + (516)(1) = 1693 \text{ gal.}$$

ii.) Upper Estimate :

$$\text{Leakage} \approx (70)(1) + (97)(1) + (136)(1) + (190)(1) + (265)(1) + (369)(1) + (516)(1) + (720)(1) = 2363 \text{ gal.}$$

c.) after 8 hrs. leakage rate is constant at 720 gal. / hr. ;

i.) worst case (shortest time) :

$$25,000 - 2363 = 22,637 \text{ gal. ;}$$

$$\frac{22,637 \text{ gal.}}{720 \frac{\text{gal.}}{\text{hr.}}} \approx 31.44 \text{ more hours}$$

ii.) best case (longest time) :

$$25,000 - 1693 = 23,307 \text{ gal. ;}$$

$$\frac{23,307 \text{ gal.}}{720 \frac{\text{gal.}}{\text{hr.}}} \approx 32.37 \text{ more hours}$$

## Section 5.2

$$2.) \sum_{k=1}^3 \frac{k-1}{k} = \frac{0}{1} + \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$$

$$3.) \sum_{k=1}^4 \cos k\pi = \cos \pi + \cos 2\pi \\ + \cos 3\pi + \cos 4\pi \\ = (-1) + (1) + (-1) + (1) = 0$$

$$4.) \sum_{k=1}^5 \sin k\pi = \sin \pi + \sin 2\pi \\ + \sin 3\pi + \sin 4\pi \\ = (0) + (0) + (0) + (0) = 0$$

$$5.) \sum_{k=1}^3 (-1)^{k+1} \cdot \sin \frac{\pi}{k} = (1) \cdot \sin \pi \\ + (-1) \cdot \sin \frac{\pi}{2} + (1) \cdot \sin \frac{\pi}{3} \\ = (1)(0) + (-1)(1) + (1)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2} - 1$$

7.) all of them.

8.) a.) yes b.) yes c.) no

$$11.) 1 + 2 + 3 + 4 + 5 + 6 = \sum_{i=1}^6 i$$

$$12.) 1 + 4 + 9 + 16 = 1^2 + 2^2 + 3^2 + 4^2 \\ = \sum_{i=1}^4 i^2$$

$$16.) -\frac{1}{5} + \frac{2}{5} - \frac{3}{5} + \frac{4}{5} - \frac{5}{5} = \sum_{i=1}^5 (-1)^i \cdot \frac{i}{5}$$

$$17.) a.) \sum_{k=1}^n 3a_k = 3 \cdot \sum_{k=1}^n a_k = 3(-5) = -15$$

$$b.) \sum_{k=1}^n \frac{b_k}{6} = \frac{1}{6} \cdot \sum_{k=1}^n b_k = \frac{1}{6}(6) = 1$$

$$c.) \sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k \\ = (-5) + (6) = 1$$

$$d.) \sum_{k=1}^n (a_k - b_k) = \sum_{k=1}^n a_k - \sum_{k=1}^n b_k \\ = (-5) - (6) = -11$$

$$e.) \sum_{k=1}^n (b_k - 2a_k) = \sum_{k=1}^n b_k - 2 \cdot \sum_{k=1}^n a_k \\ = (6) - 2(-5) = 16$$

$$20.) a.) \sum_{k=1}^{13} k = 1 + 2 + 3 + \dots + 13 = \frac{13(14)}{2} = 91$$

$$b.) \sum_{k=1}^{13} k^2 = 1^2 + 2^2 + 3^2 + \dots + 13^2 \\ = \frac{13(14) \cdot (26+1)}{6} = 819$$

$$c.) \sum_{k=1}^{13} k^3 = 1^3 + 2^3 + 3^3 + \dots + 13^3 \\ = \left( \frac{13(14)}{2} \right)^2 = 8281$$

$$23.) \sum_{k=1}^6 (3 - k^2) = \sum_{k=1}^6 3 - \sum_{k=1}^6 k^2 \\ = 3(6) - \frac{6(7)(13)}{6} = 18 - 91 = -73$$

$$25.) \sum_{k=1}^5 k(3k+5) = \sum_{k=1}^5 (3k^2 + 5k)$$

$$= 3 \cdot \sum_{k=1}^5 k^2 + 5 \cdot \sum_{k=1}^5 k$$

$$= 3 \cdot \frac{5(6)(11)}{6} + 5 \cdot \frac{5(6)}{2} = 240$$

$$28.) \left( \sum_{k=1}^7 k \right)^2 - \sum_{k=1}^7 \frac{k^3}{4} = \left( \frac{7(8)}{2} \right)^2 - \frac{1}{4} \cdot \sum_{k=1}^7 k^3$$

$$= 784 - \frac{1}{4} \cdot \left( \frac{7(8)}{2} \right)^2 = 588$$

$$29.) \text{ a.) } \sum_{k=1}^7 3 = 7(3) = 21$$

$$\text{ b.) } \sum_{k=1}^{500} 7 = 500(7) = 3500$$

$$\text{ c.) } \sum_{k=3}^{264} 10 = \sum_{k=1}^{264} 10 - \sum_{k=1}^2 10$$

$$= 264(10) - 2(10) = 2620$$

$$30.) \text{ a.) } \sum_{k=9}^{36} k = \sum_{k=1}^{36} k - \sum_{k=1}^8 k$$

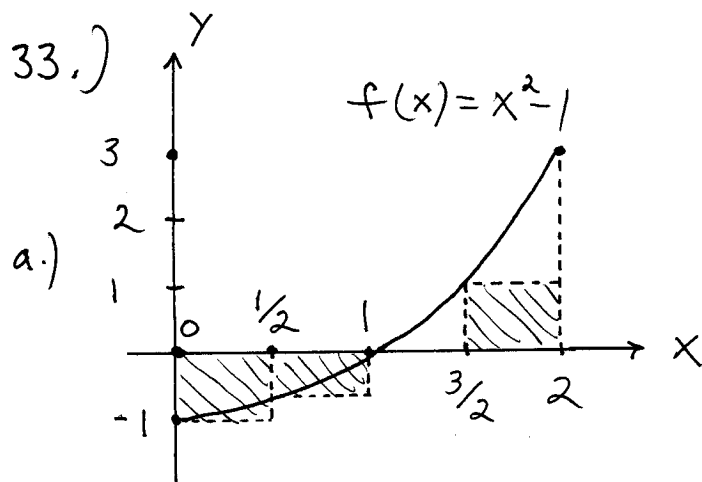
$$= \frac{1}{2}(36)(36+1) - \frac{1}{2}(8)(8+1) = 630$$

$$\text{ b.) } \sum_{k=3}^{17} k^2 = \sum_{k=1}^{17} k^2 - \sum_{k=1}^2 k^2$$

$$= \frac{1}{6}(17)(17+1)(34+1) - (1^2 + 2^2)$$

$$= 1780$$

$$\begin{aligned}
 \text{c.) } \sum_{k=18}^{71} k(k-1) &= \sum_{k=18}^{71} (k^2 - k) \\
 &= \sum_{k=18}^{71} k^2 - \sum_{k=18}^{71} k \\
 &= \sum_{k=1}^{71} k^2 - \sum_{k=1}^{17} k^2 - \left\{ \sum_{k=1}^{71} k - \sum_{k=1}^{17} k \right\} \\
 &= \frac{1}{6} (71)(71+1)(142+1) - \frac{1}{6} (17)(18)(34+1) \\
 &\quad - \left\{ \frac{1}{2} (71)(71+1) - \frac{1}{2} (17)(17+1) \right\} = 117,648
 \end{aligned}$$



$$\Delta x_i = \frac{1}{2} \text{ for } i=1, 2, 3, 4;$$

$$c_1 = 0, c_2 = \frac{1}{2}, c_3 = 1, c_4 = \frac{3}{2};$$

$$\begin{aligned}
 \sum_{i=1}^4 f(c_i) \cdot \Delta x_i &= f(c_1) \cdot \Delta x_1 + f(c_2) \cdot \Delta x_2 \\
 &\quad + f(c_3) \cdot \Delta x_3 + f(c_4) \cdot \Delta x_4
 \end{aligned}$$

$$= f(0) \cdot \frac{1}{2} + f\left(\frac{1}{2}\right) \cdot \frac{1}{2} + f(1) \cdot \frac{1}{2} + f\left(\frac{3}{2}\right) \cdot \frac{1}{2}$$

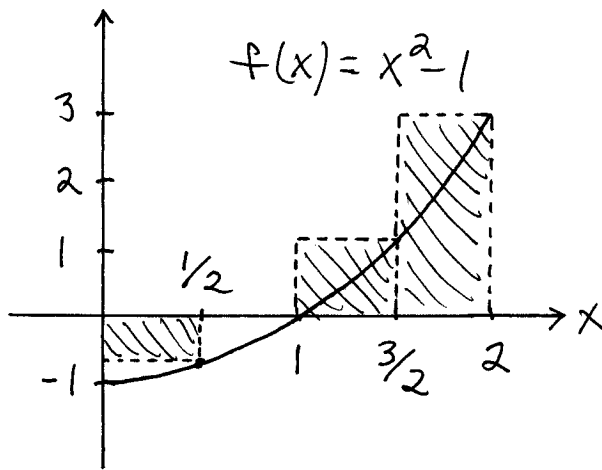
$$= (-1) \cdot \frac{1}{2} + \left(-\frac{3}{4}\right) \cdot \frac{1}{2} + (0) \cdot \frac{1}{2} + \left(\frac{5}{4}\right) \cdot \frac{1}{2}$$

$$= -\frac{4}{8} + -\frac{3}{8} + 0 + \frac{5}{8}$$

$$= -\frac{2}{8} = -\frac{1}{4}$$



b.)

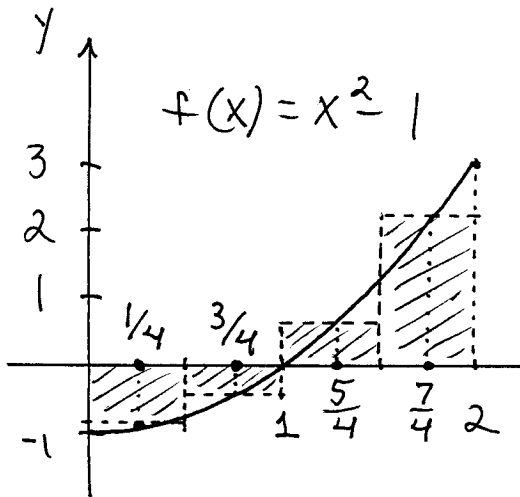


$$\Delta x_i = \frac{1}{2} \text{ for } i=1,2,3,4;$$

$$c_1 = \frac{1}{2}, c_2 = 1, c_3 = \frac{3}{2}, c_4 = 2;$$

$$\begin{aligned} \sum_{i=1}^4 f(c_i) \cdot \Delta x_i &= f(c_1) \cdot \Delta x_1 + f(c_2) \cdot \Delta x_2 \\ &\quad + f(c_3) \cdot \Delta x_3 + f(c_4) \cdot \Delta x_4 \\ &= f\left(\frac{1}{2}\right) \cdot \frac{1}{2} + f(1) \cdot \frac{1}{2} + f\left(\frac{3}{2}\right) \cdot \frac{1}{2} + f(2) \cdot \frac{1}{2} \\ &= \left(-\frac{3}{4}\right) \cdot \frac{1}{2} + (0) \cdot \frac{1}{2} + \left(\frac{5}{4}\right) \cdot \frac{1}{2} + (3) \cdot \frac{1}{2} \\ &= -\frac{3}{8} + 0 + \frac{5}{8} + \frac{12}{8} = \frac{14}{8} = \frac{7}{4} \end{aligned}$$

c.)



$$\Delta x_i = \frac{1}{4} \text{ for } i=1,2,3,4;$$

$$c_1 = \frac{1}{4}, c_2 = \frac{3}{4}, c_3 = \frac{5}{4}, c_4 = \frac{7}{4};$$

$$\begin{aligned} \sum_{i=1}^4 f(c_i) \cdot \Delta x_i &= f(c_1) \cdot \Delta x_1 + f(c_2) \cdot \Delta x_2 \\ &\quad + f(c_3) \cdot \Delta x_3 + f(c_4) \cdot \Delta x_4 \end{aligned}$$

$$\begin{aligned} &= f\left(\frac{1}{4}\right) \cdot \frac{1}{2} + f\left(\frac{3}{4}\right) \cdot \frac{1}{2} + f\left(\frac{5}{4}\right) \cdot \frac{1}{2} + f\left(\frac{7}{4}\right) \cdot \frac{1}{2} \\ &= \left(-\frac{15}{16}\right) \cdot \frac{1}{2} + \left(-\frac{7}{16}\right) \cdot \frac{1}{2} + \left(\frac{9}{16}\right) \cdot \frac{1}{2} + \left(\frac{33}{16}\right) \cdot \frac{1}{2} \\ &= \frac{-15}{32} + \frac{-7}{32} + \frac{9}{32} + \frac{33}{32} \\ &= \frac{20}{32} = \frac{5}{8} \end{aligned}$$