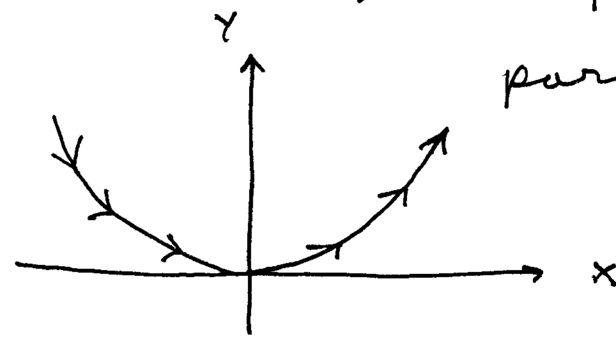
Section 11.1

1.) $\begin{cases} x = 3t \\ y = 9t^2 \end{cases}$ for $-\infty < t < \infty$;

$t = \frac{x}{3} \xrightarrow{\text{sub}} y = 9t^2 = 9\left(\frac{x}{3}\right)^2 = 9 \cdot \frac{x^2}{9} = x^2$

$\rightarrow \boxed{y = x^2}$ parabola

left to right



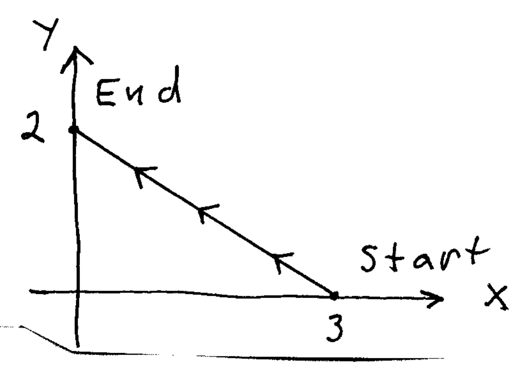
4.) $\begin{cases} x = 3 - 3t \\ y = 2t \end{cases}$ for $0 \le t \le 1$;

$x = 3 - 3t \rightarrow 3t = 3 - x \rightarrow t = 1 - \frac{1}{3}x \xrightarrow{\text{sub}}$

$y = 2t = 2\left(1 - \frac{1}{3}x\right) = 2 - \frac{2}{3}x \rightarrow$

$\boxed{y = 2 - \frac{2}{3}x}$

line

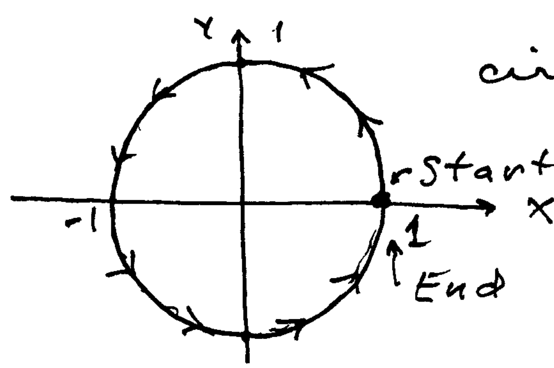


5.) $\begin{cases} x = \cos 2t \\ y = \sin 2t \end{cases}$ for $0 \le t \le \pi$;

$x^2 + y^2 = \cos^2 2t + \sin^2 2t = 1 \rightarrow$

$\boxed{x^2 + y^2 = 1}$ circle

once around, counter-clockwise

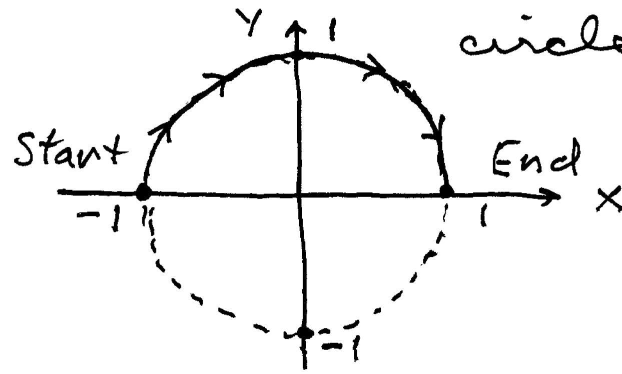


6.) $\begin{cases} x = \cos(\pi - t) \\ y = \sin(\pi - t) \end{cases}$ for $0 \le t \le \pi$;

$x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1 \rightarrow$

$\boxed{x^2 + y^2 = 1}$ circle

½ of circle, clockwise



8.) $\begin{cases} x = 4 \sin t \\ y = 5 \cos t \end{cases}$ for $0 \le t \le 2\pi$;

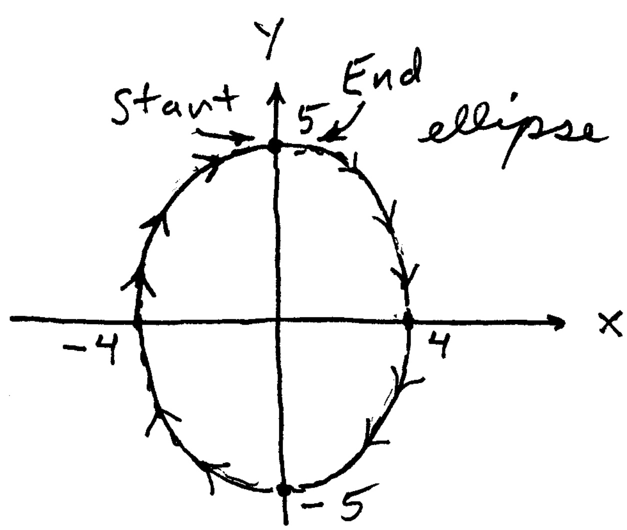
$\frac{x}{4} = \sin t$ and $\frac{y}{5} = \cos t \longrightarrow$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = \sin^2 t + \cos^2 t = 1 \longrightarrow$

$\boxed{\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1}$

ellipse

once around, clockwise



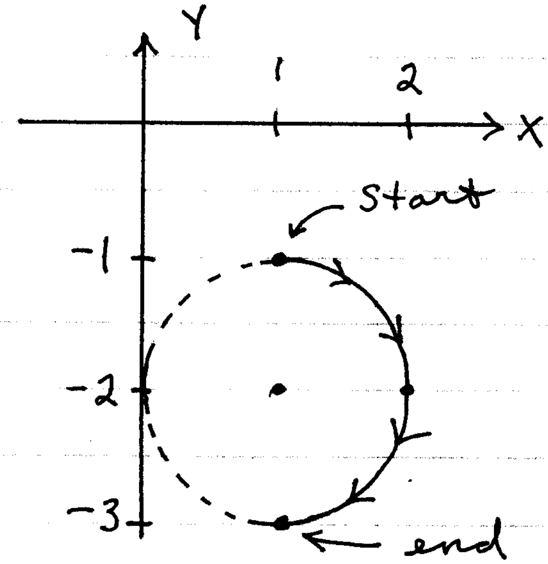
10.) $\begin{cases} X = 1 + \sin t \\ Y = \cos t - 2 \end{cases} \rightarrow \begin{cases} X - 1 = \sin t \\ Y + 2 = \cos t \end{cases}$ for $0 \le t \le \pi$

$\rightarrow (X-1)^2 + (Y+2)^2$

$= \sin^2 t + \cos^2 t = 1$

$\rightarrow \boxed{(X-1)^2 + (Y+2)^2 = 1}$

$\frac{1}{2}$ of circle, clockwise

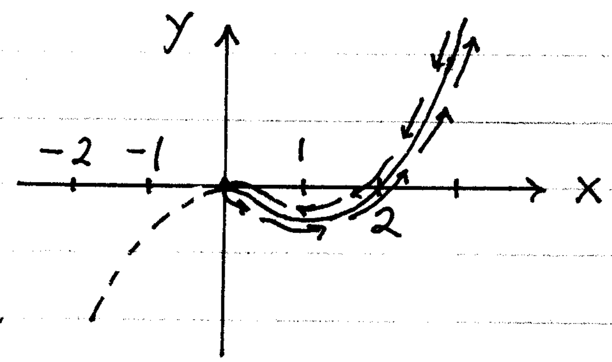


11.) $\begin{cases} X = t^2 \\ Y = t^6 - 2t^4 = (t^2)^3 - 2(t^2)^2 \end{cases}$ for $-\infty < t < \infty$

$\rightarrow \boxed{Y = X^3 - 2X^2}$

$= X^2(X-2)$

cubic polynomial, changes direction at (0, 0)



12.) $\begin{cases} X = \dfrac{t}{t-1} \\ Y = \dfrac{t-2}{t+1} \end{cases}$ for $-1 < t < 1$

; then $X(t-1) = t \rightarrow$

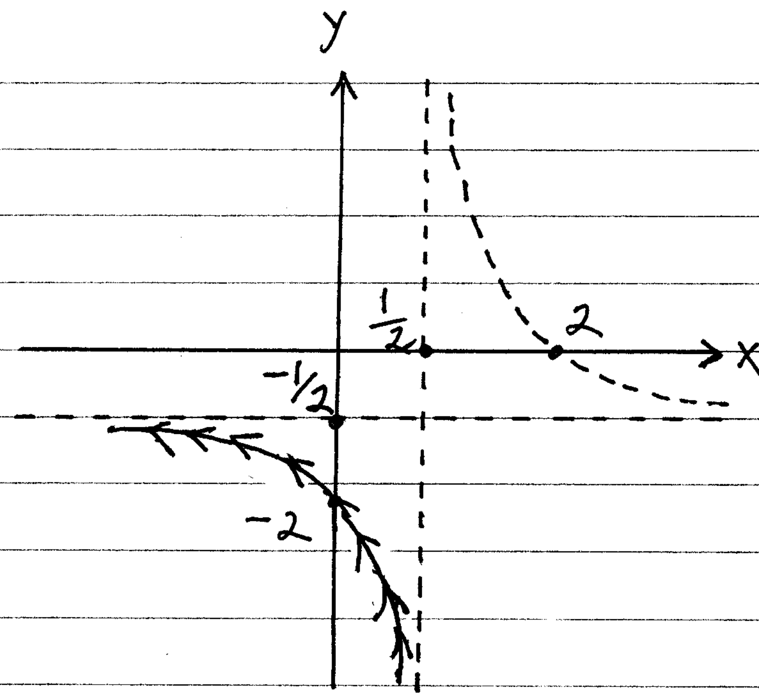
$Xt - X = t \rightarrow Xt - t = X \rightarrow t(X-1) = X$

$\rightarrow t = \dfrac{X}{X-1} \longrightarrow$ (SUB) $\longrightarrow Y = \dfrac{\left(\frac{X}{X-1}\right) - 2}{\left(\frac{X}{X-1}\right) + 1} \cdot \dfrac{X-1}{X-1}$

$$\rightarrow Y = \frac{x - 2(x-1)}{x + (x-1)}$$

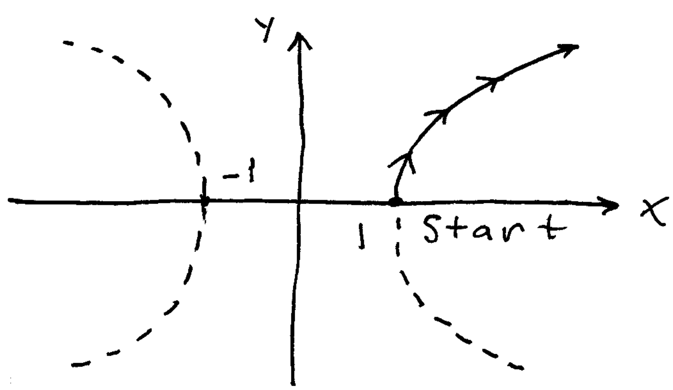
$$\rightarrow \boxed{Y = \frac{2-x}{2x-1}}$$

hyperbola, left half only



14.) $\begin{cases} x = \sqrt{t+1} \\ y = \sqrt{t} \end{cases}$ for $t \geq 0$

$\left.\begin{array}{l} x^2 = t+1 \\ Y^2 = t \end{array}\right\}$ $x^2 = Y^2 + 1$ $\longrightarrow$ $\boxed{x^2 - Y^2 = 1}$



(hyperbola) with $x \geq 0$, $Y \geq 0$

16.) $\begin{cases} x = -\sec t \\ Y = \tan t \end{cases}$ for $-\frac{\pi}{2} < t < \frac{\pi}{2}$

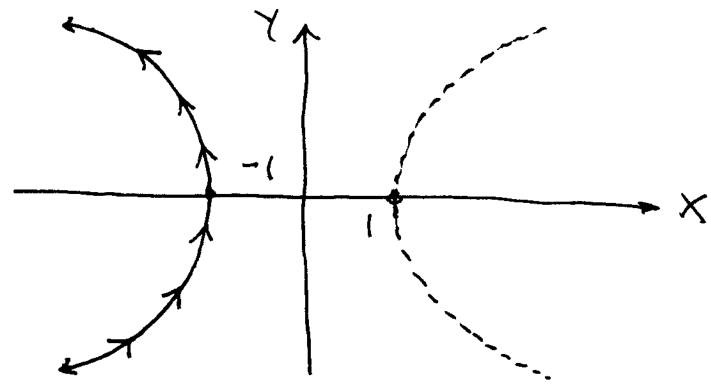
$\left.\begin{array}{l} x^2 = \sec^2 t \\ Y^2 = \tan^2 t \end{array}\right\}$ $\begin{array}{l} 1 + \tan^2 t = \sec^2 t \rightarrow \\ 1 + Y^2 = x^2 \longrightarrow \end{array}$

$\boxed{x^2 - y^2 = 1}$

(hyperbola)

$x < 0$

$-\infty < y < \infty$



19.) a.) $\begin{cases} x = a \sin(t + \frac{\pi}{2}) \\ y = a \cos(t + \frac{\pi}{2}) \end{cases}$ for $0 \le t \le 2\pi$

b.) $\begin{cases} x = a \cos t \\ y = a \sin t \end{cases}$ for $0 \le t \le 2\pi$

c.) $\begin{cases} x = a \sin(t + \frac{\pi}{2}) \\ y = a \cos(t + \frac{\pi}{2}) \end{cases}$ for $0 \le t \le 4\pi$

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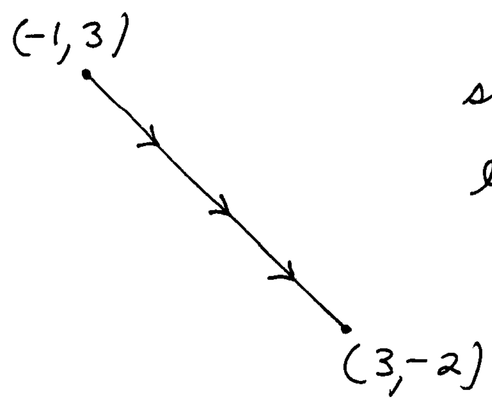
d.) $\begin{cases} x = a \cos t \\ y = a \sin t \end{cases}$ for $0 \le t \le 4\pi$

21.)



slope $= \dfrac{3-(-2)}{-1-3} = \dfrac{-5}{4}$;

line $y - 3 = \frac{-5}{4}(x - (-1)) \rightarrow$

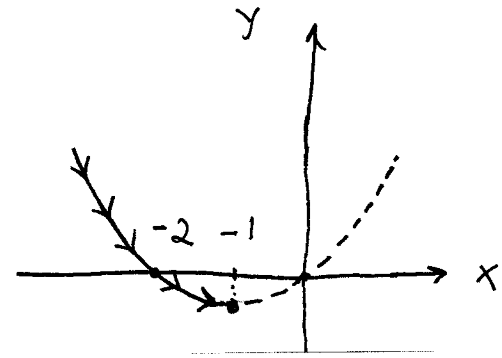
$y - 3 = \frac{-5}{4}x - \frac{5}{4} \rightarrow$

$y = \frac{-5}{4}x + \frac{7}{4}$

$\begin{cases} x = t \\ y = \frac{-5}{4}t + \frac{7}{4} \end{cases}$ for $-1 \le t \le 3$

24.) $Y = x^2 + 2x = x(x+2)$

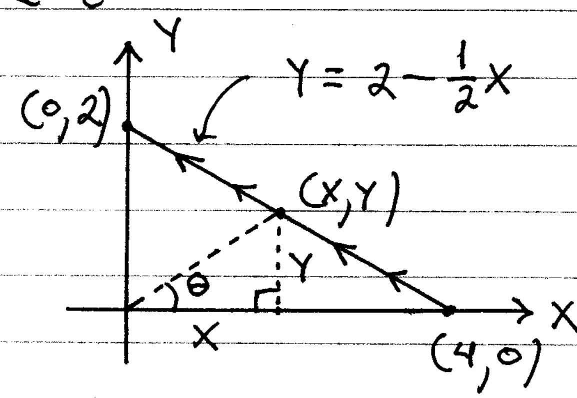
$$\begin{cases} x = t \\ y = t^2 + 2t \end{cases} \quad \text{for } t \le -1$$



27.) $$\begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases} \quad \text{for } 0 \le t \le \pi$$

28.) $$\begin{cases} x = 3\cos t \\ y = (3\cos t)^2 = 9\cos^2 t \end{cases} \quad \text{for } t \ge 0$$

31.) $\tan\theta = \dfrac{Y}{X}$



$= \dfrac{2 - \frac{1}{2}X}{X} = \dfrac{2}{X} - \dfrac{1}{2} \rightarrow$

$\dfrac{2}{X} = \dfrac{1}{2} + \tan\theta \longrightarrow$

$\dfrac{X}{2} = \dfrac{1}{\frac{1}{2} + \tan\theta} \rightarrow X = \dfrac{2}{\frac{1}{2} + \tan\theta} \cdot \dfrac{2}{2} = \dfrac{4}{1 + 2\tan\theta}$;

$Y = 2 - \frac{1}{2}X = 2 - \dfrac{1}{2} \cdot \dfrac{4}{1+2\tan\theta} = \dfrac{2}{1} - \dfrac{2}{1+2\tan\theta}$

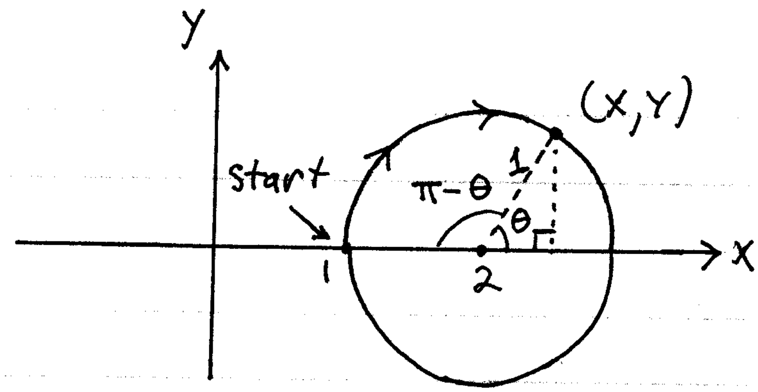
$= \dfrac{2 + 4\tan\theta - 2}{1 + 2\tan\theta} = \dfrac{4\tan\theta}{1 + 2\tan\theta}$ so

$$\begin{cases} X = \dfrac{4}{1+2\tan\theta} \\ Y = \dfrac{4\tan\theta}{1+2\tan\theta} \end{cases} \quad \text{for } 0 \le \theta \le \frac{\pi}{2}$$

33.)

$$\begin{cases} x = 2 + \cos(\pi - \theta) \\ y = \sin(\pi - \theta) \end{cases}$$

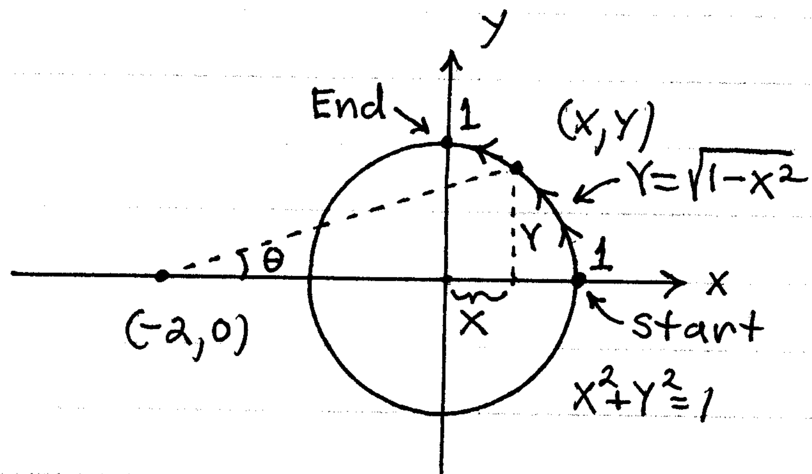
for $0 \le \theta \le 2\pi$



$(x-2)^2 + y^2 = 1$

34.) $\tan\theta = \dfrac{y}{x+2}$

$= \dfrac{\sqrt{1-x^2}}{x+2} \longrightarrow$



$x^2 + y^2 = 1$

$\tan\theta = \dfrac{\sqrt{1-x^2}}{x+2} \longrightarrow$

$\underbrace{\tan^2\theta}_{A} = \left(\dfrac{\sqrt{1-x^2}}{x+2}\right)^2 = \dfrac{1-x^2}{x^2+4x+4} \longrightarrow$

$A(x^2+4x+4) = 1 - x^2 \longrightarrow Ax^2 + 4Ax + 4A + x^2 - 1 = 0 \longrightarrow$

$(A+1)x^2 + (4A)x + (4A-1) = 0 \longrightarrow$

$$x = \frac{-4A \pm \sqrt{(4A)^2 - 4(A+1)(4A-1)}}{2(A+1)}$$

$$= \frac{-4A \pm \sqrt{16A^2 - 4(4A^2 + 3A - 1)}}{2(A+1)}$$

$$= \frac{-4A \pm \sqrt{\cancel{16A^2} - \cancel{16A^2} - 12A + 4}}{2(A+1)}$$

$$= \frac{-4A \pm \sqrt{4(1-3A)}}{2(A+1)} = \frac{\cancel{2}(-2A) + \cancel{2}\sqrt{1-3A}}{\cancel{2}(A+1)}$$

x is ≥ 0

$$\rightarrow \quad X = \frac{-2A + \sqrt{1-3A}}{A+1} \; ; \quad \text{then}$$

$$Y = \sqrt{1-X^2} = \sqrt{1 - \left(\frac{\sqrt{1-3A} - 2A}{A+1}\right)^2}$$

$$= \sqrt{\frac{(A+1)^2}{(A+1)^2} - \frac{1-3A-4A\sqrt{1-3A}+4A^2}{(A+1)^2}}$$

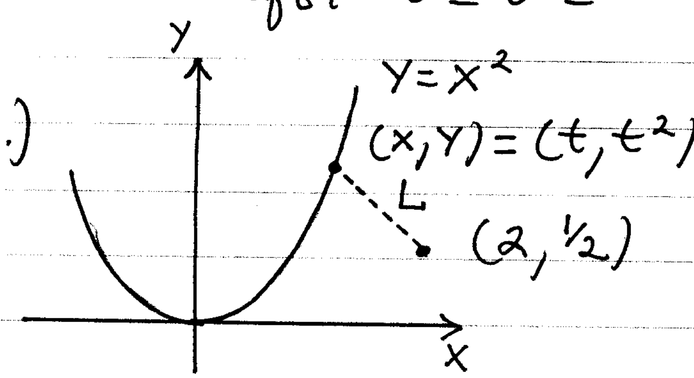
$$= \sqrt{\frac{A^2+2A+1-1+3A+4A\sqrt{1-3A}-4A^2}{(A+1)}}$$

$$= \frac{\sqrt{5A-3A^2+4A\sqrt{1-3A}}}{A+1} \; ; \quad \text{thus,}$$

$$\begin{cases} X = \dfrac{-2\tan^2\theta + \sqrt{1-3\tan^2\theta}}{\tan^2\theta + 1} \\[2ex] Y = \dfrac{\sqrt{5\tan^2\theta - 3\tan^4\theta + 4\tan^2\theta\sqrt{1-3\tan^2\theta}}}{\tan^2\theta + 1} \end{cases}$$

for $0 \le \theta \le \arctan\left(\frac{1}{2}\right)$

39.)



Minimize distance

$$L = \sqrt{(t-2)^2 + \left(t^2 - \frac{1}{2}\right)^2}$$

$$\xrightarrow{D} L' = \frac{1}{2}\left((t-2)^2 + \left(t^2-\frac{1}{2}\right)^2\right)^{-1/2}\left[2(t-2) + 2\left(t^2-\frac{1}{2}\right)\cdot 2t\right]$$

$$= \frac{\cancel{\frac{1}{2}} \cdot \cancel{2}\left[(t-2)+\left(t^2-\frac{1}{2}\right)2t\right]}{\sqrt{(t-2)^2+\left(t^2-\frac{1}{2}\right)^2}} = 0 \rightarrow$$

$$\cancel{t}-2+2t^3-\cancel{t} = 0 \rightarrow 2t^3 = 2 \rightarrow t^3 = 1 \rightarrow t = 1:$$

$-$ 0 $+$ L'

$t = 1$

point $\begin{cases} X = t = 1 \\ Y = t^2 = 1 \end{cases}$ and min. distance

$$L = \sqrt{(1-2)^2+\left(1-\frac{1}{2}\right)^2} = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$