Section 11.2

1.) $\begin{cases} X = 2\cos t \\ Y = 2\sin t \end{cases}$

a.) $\dfrac{dY}{dx} = \dfrac{dY/dt}{dX/dt} = \dfrac{2\cos t}{-2\sin t} = \dfrac{-\cos t}{\sin t}$; if $t = \frac{\pi}{4}$

$\rightarrow X = \sqrt{2}$, $Y = \sqrt{2}$, slope $Y' = \dfrac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$;

tangent line is $Y - \sqrt{2} = -1(X - \sqrt{2}) \rightarrow$

$\boxed{Y = 2\sqrt{2} - X}$

b.) $\dfrac{d^2Y}{dx^2} = \dfrac{d}{dX}\left(\dfrac{dY}{dx}\right) = \dfrac{\frac{d}{dt}\left(\frac{dY}{dx}\right)}{\frac{dX}{dt}}$

$= \dfrac{\frac{d}{dt}\left(\frac{-\cos t}{\sin t}\right)}{\frac{-2\sin t}{1}} = \dfrac{\sin t \cdot (\sin t) - (-\cos t) \cdot \cos t}{\sin^2 t \cdot (-2\sin t)}$

$= \dfrac{\sin^2 t + \cos^2 t}{-2\sin^3 t} = \dfrac{-1}{2\sin^3 t}$; if $t = \frac{\pi}{4} \rightarrow$

$\dfrac{d^2Y}{dx^2} = \dfrac{-1}{2\left(\frac{\sqrt{2}}{2}\right)^3} = \dfrac{-1}{2\left(\frac{2\sqrt{2}}{8}\right)} = \dfrac{-1}{\frac{\sqrt{2}}{2}} = -\sqrt{2}$

5.) $\begin{cases} X = t \\ Y = \sqrt{t} \end{cases}$

a.) $\dfrac{dY}{dX} = \dfrac{\frac{dY}{dt}}{\frac{dX}{dt}} = \dfrac{\frac{1}{2\sqrt{t}}}{1} = \dfrac{1}{2\sqrt{t}}$; if $t = \frac{1}{4} \rightarrow$

$X = \frac{1}{4}$, $Y = \frac{1}{2}$, slope $Y' = 1$; tangent line is $Y - \frac{1}{2} = 1\left(x - \frac{1}{4}\right) \rightarrow \boxed{Y = X + \frac{1}{4}}$

b.) $\frac{d^2Y}{dX^2} = \frac{d}{dX}\left(\frac{dY}{dX}\right) = \frac{\frac{d}{dt}\left(\frac{dY}{dX}\right)}{\frac{dX}{dt}} = \frac{\frac{d}{dt}\left(\frac{1}{2}t^{-1/2}\right)}{1}$

$= \frac{-1}{4}t^{-3/2}$; if $t = \frac{1}{4} \rightarrow$

$\frac{d^2Y}{dX^2} = \frac{-1}{4}\left(\frac{1}{4}\right)^{-3/2} = \frac{-1}{4}\left(\frac{1}{2}\right)^{-3} = \frac{-1}{4}(8) = -2$

6.) $\begin{cases} X = \sec^2 t - 1 \\ Y = \tan t \end{cases}$

a.) $\frac{dY}{dX} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{\cancel{\sec^2} t}{2\cancel{\sec} t \cdot \cancel{\sec t} \tan t} = \frac{1}{2}\cot t$;

if $t = -\frac{\pi}{4} \rightarrow X = \left(\sec\left(-\frac{\pi}{4}\right)\right)^2 - 1 = 2 - 1 = 1$,
$Y = \tan(-\pi/4) = -1$, slope
$Y' = \frac{1}{2}\cot\left(-\frac{\pi}{4}\right) = \frac{1}{2}(-1) = \frac{-1}{2}$; tangent line

is $Y - (-1) = \frac{-1}{2}(X - 1) \rightarrow Y + 1 = \frac{-1}{2}X + \frac{1}{2} \rightarrow$

$\boxed{Y = \frac{-1}{2}X - \frac{1}{2}}$

b.) $\frac{d^2Y}{dX^2} = \frac{d}{dX}\left(\frac{dY}{dX}\right) = \frac{\frac{d}{dt}\left(\frac{dY}{dX}\right)}{\frac{dX}{dt}} = \frac{\frac{d}{dt}\left(\frac{1}{2}\cot t\right)}{2\sec^2 t \cdot \tan t}$

$= \frac{\frac{1}{2} \cdot -\csc^2 t}{2\sec^2 t \cdot \tan t} = \frac{-1}{4} \cdot \frac{\frac{1}{\sin^2 t}}{\frac{1}{\cos^2 t} \cdot \frac{\sin t}{\cos t}}$

$$= \frac{-1}{4} \cdot \frac{\cos^3 t}{\sin^3 t} \quad ; \quad \text{if } t = -\frac{\pi}{4} \rightarrow$$

$$\frac{d^2 Y}{dx^2} = \frac{-1}{4} \cdot \frac{\left(\sqrt{2}/2\right)^3}{\left(-\sqrt{2}/2\right)^3} = -\frac{1}{4}(-1)^3 = \frac{1}{4}$$

9.) $\begin{cases} x = 2t^2 + 3 \\ y = t^4 \end{cases}$

a.) $\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{4t} = t^2$; if $t = -1 \rightarrow$

$x = 5$, $Y = 1$, slope $Y' = 1$; tangent line is $Y - 1 = 1(x - 5) \rightarrow \boxed{Y = x - 4}$

b.) $\frac{d^2 Y}{dx^2} = \frac{d}{dx}\left(\frac{dY}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dY}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(t^2)}{4t}$

$= \frac{2t}{4t} = \frac{1}{2}$; if $t = -1 \rightarrow \frac{d^2 Y}{dx^2} = \frac{1}{2}$

11.) $\begin{cases} x = t - \sin t \\ Y = 1 - \cos t \end{cases}$

a.) $\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dx}{dt}} = \frac{-(-\sin t)}{1 - \cos t} = \frac{\sin t}{1 - \cos t}$; if $t = \frac{\pi}{3} \rightarrow$

$x = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$, $Y = 1 - \frac{1}{2} = \frac{1}{2}$, slope $Y' = \frac{\sqrt{3}/2}{1 - 1/2}$

$= \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$; tangent line is

$$\boxed{Y - \frac{1}{2} = \sqrt{3}\left(x - \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right)}$$.

b.) $\dfrac{d^2Y}{dx^2} = \dfrac{d}{dx}(Y') = \dfrac{\frac{d}{dt}(Y')}{\frac{dx}{dt}}$

$$= \frac{\dfrac{(1-\cos t)\cos t - \sin t\cdot(\sin t)}{(1-\cos t)^2}}{\dfrac{1-\cos t}{1}}$$

$$= \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^2} \cdot \frac{1}{1-\cos t}$$

$$= \frac{\cos t - (\cos^2 t + \sin^2 t)}{(1-\cos t)^3} = \frac{\cos t - 1}{(1-\cos t)^3}$$

$$= \frac{-(1-\cos t)}{(1-\cos t)^3} = \frac{-1}{(1-\cos t)^2}\ ;\ \text{if } t = \frac{\pi}{3} \rightarrow$$

$$\frac{d^2Y}{dx^2} = \frac{-1}{(1-\frac{1}{2})^2} = \frac{-1}{1/4} = -4$$

13.) $\begin{cases} x = \dfrac{1}{t+1} \\ Y = \dfrac{t}{t-1} \end{cases}$

$$\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dx}{dt}} = \frac{\dfrac{(t-1)(1) - t(1)}{(t-1)^2}}{\dfrac{-1}{(t+1)^2}} = \frac{\cancel{-1}}{(t-1)^2} \cdot \frac{(t+1)^2}{\cancel{-1}}$$

$$= \frac{(t+1)^2}{(t-1)^2}\ ;\ \text{if } t = 2 \rightarrow x = \frac{1}{3},\ Y = 2,$$

slope $y' = 9$; tangent line is

$$y - 2 = 9\left(x - \frac{1}{3}\right) \rightarrow \boxed{y = 9x - 1}$$

b.) $$\frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{\frac{d}{dt}(y')}{\frac{dx}{dt}}$$

$$= \frac{\frac{(t-1)^2 \cdot 2(t+1) - (t+1)^2 \cdot 2(t-1)}{(t-1)^4}}{\frac{-1}{(t+1)^2}}$$

$$= \frac{2(t+1)\cancel{(t-1)} \cdot [(t-1) - (t+1)]}{(t-1)^{\cancel{4}\,3}} \cdot \frac{(t+1)^2}{-1}$$

$$= \frac{4(t+1)^3}{(t-1)^3} \; ; \text{ if } t = 2 \rightarrow$$

$$\frac{d^2y}{dx^2} = 4(27) = 108$$

14.) $$\begin{cases} x = t + e^t \\ y = 1 - e^t \end{cases}$$

a.) $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-e^t}{1+e^t} \; ; \text{ if } t = 0 \rightarrow x = 1, \; y = 0,$$

slope $y' = \frac{-1}{2}$; tangent line is

$$y - 0 = \frac{-1}{2}(x - 1) \rightarrow y = \frac{-1}{2}x + \frac{1}{2}$$

b.) $$\frac{d^2y}{dx^2} = \frac{d(y')}{dx} = \frac{d/dt\,(y')}{dx/dt}$$

$$= \frac{\dfrac{(1+e^t)(-e^t)-(-e^t)(e^t)}{(1+e^t)^2}}{\dfrac{1+e^t}{1}}$$

$$= \frac{-e^t-\cancel{e^{2t}}+\cancel{e^{2t}}}{(1+e^t)^2}\cdot\frac{1}{1+e^t} = \frac{-e^t}{(1+e^t)^3};$$

$$\text{if } t=0 \rightarrow \frac{d^2Y}{dX^2} = \frac{-1}{2^3} = \frac{-1}{8}$$

15.) $\begin{cases} X^3+2t^2=9 \\ 2Y^3-3t^2=4 \end{cases} \xrightarrow{D}$

$$3X^2\cdot\frac{dx}{dt}+4t=0 \rightarrow \frac{dx}{dt} = \frac{-4t}{3X^2};$$

$$6Y^2\cdot\frac{dY}{dt}-6t=0 \rightarrow \frac{dY}{dt} = \frac{6t}{6Y^2} = \frac{t}{Y^2};$$

if $t=2 \rightarrow X^3 = 9-8=1 \rightarrow X=1$ and

$2Y^3 = 4+12 \rightarrow Y^3=8 \rightarrow Y=2$; slope

$$\frac{dY}{dX} = \frac{dY/dt}{dX/dt} = \frac{2/(2)^2}{-8/3(1)^2} = \frac{2}{4}\cdot\frac{3}{-8} = -\frac{3}{16}$$

18.) $\begin{cases} X\sin t + 2X = t \\ t\sin t - 2t = Y \end{cases} \xrightarrow{D}$

$$X\cdot\cos t + \frac{dX}{dt}\cdot\sin t + 2\cdot\frac{dX}{dt} = 1 \longrightarrow$$

$$(2+\sin t)\frac{dX}{dt} = 1 - X\cos t \rightarrow \frac{dX}{dt} = \frac{1-X\cos t}{2+\sin t};$$

$t\cos t + (1)\sin t - 2 = \frac{dY}{dt}$; if $t=\pi \rightarrow$

$X\cdot(0)+2X=\pi \longrightarrow X=\pi/2$; $\pi\overset{0}{\cancel{\sin\pi}} - 2\pi = Y \rightarrow$

$Y = -2\pi$; slope

$$\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{\pi \cos\pi + \sin\pi - 2}{\frac{1 - \pi\cos\pi}{2 + \sin\pi}}$$

$$= \frac{-\pi - 2}{\frac{1+\pi}{2}} = (-\pi - 2)\cdot\frac{2}{1+\pi} = \frac{-2\pi - 4}{\pi + 1}$$

19.) $\begin{cases} X = t^3 + t \\ Y + 2t^3 = 2X + t^2 \end{cases} \xrightarrow{D} \frac{dX}{dt} = 3t^2 + 1$;

$$\frac{dY}{dt} + \cancel{6t^2} = 2\cdot\frac{dX}{dt} + 2t = 2(3t^2+1) + 2t = \cancel{6t^2} + 2t + 2 \rightarrow$$

$\frac{dY}{dt} = 2t + 2$; if $t = 1 \rightarrow X = 2$, $Y + 2 = 2(2) + 1 \rightarrow$
$Y = 3$; slope

$$\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dX}{dt}} = \frac{2(1) + 2}{3(1)^2 + 1} = \frac{4}{4} = 1$$

20.) $\begin{cases} t = \ln(X - t) \\ Y = te^t \end{cases} \xrightarrow{D}$

$$1 = \frac{1}{X - t}\cdot\left(\frac{dX}{dt} - 1\right) \rightarrow X - t = \frac{dX}{dt} - 1 \rightarrow$$

$\frac{dX}{dt} = X - t + 1$; $\frac{dY}{dt} = te^t + (1)e^t$; if $t = 0 \rightarrow$

$0 = \ln(X - 0) \rightarrow X = 1$, $Y = 0e^0 = 0$; slope

$$\frac{dY}{dx} = \frac{dY/dt}{dX/dt} = \frac{0e^0 + e^0}{1 - 0 + 1} = \frac{1}{2}$$

25.) $\text{Arc} = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^{\pi} \sqrt{(-\sin t)^2 + (1+\cos t)^2}\, dt$

$= \int_0^{\pi} \sqrt{\underline{\sin^2 t + \cos^2 t} + 2\cos t + 1}\, dt$

$= \int_0^{\pi} \sqrt{2 + 2\cos t}\, dt$

$= \int_0^{\pi} \sqrt{2(1+\cos t)\frac{(1-\cos t)}{(1-\cos t)}}\, dt$

$= \int_0^{\pi} \sqrt{2} \cdot \sqrt{\frac{1-\cos^2 t}{1-\cos t}}\, dt$

$= \sqrt{2} \int_0^{\pi} \frac{\sqrt{\sin^2 t}}{\sqrt{1-\cos t}}\, dt$

$= \sqrt{2} \int_0^{\pi} \frac{|\sin t|}{\sqrt{1-\cos t}}\, dt$

$= \sqrt{2} \int_0^{\pi} \frac{\sin t}{\sqrt{1-\cos t}}\, dt$

(Let $u = 1-\cos t \rightarrow du = \sin t\, dt$;
$t: 0 \rightarrow \pi$ so $u: 0 \rightarrow 2$)

$= \sqrt{2} \int_0^{2} \frac{1}{\sqrt{u}}\, du = \sqrt{2} \cdot \int_0^{2} u^{-1/2}\, du$

$= \sqrt{2} \cdot \left.\frac{u^{1/2}}{1/2}\right|_0^2 = 2\sqrt{2} \cdot \sqrt{2} = 4$

26.) $\text{Arc} = \int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$= \int_0^{\sqrt{3}} \sqrt{(3t^2)^2 + \left(\tfrac{3}{\cancel{2}}\cdot \cancel{2}t\right)^2}\, dt$$

$$= \int_0^{\sqrt{3}} \sqrt{9t^4 + 9t^2}\, dt = \int_0^{\sqrt{3}} \sqrt{9t^2(t^2+1)}\, dt$$

$$= 3\int_0^{\sqrt{3}} t\sqrt{t^2+1}\, dt = \cancel{3}\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}(t^2+1)^{3/2}\Big|_0^{\sqrt{3}}$$

$$= 4^{3/2} - 1^{3/2} = 8 - 1 = 7$$

27.) $\text{Arc} = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$= \int_0^4 \sqrt{(t)^2 + \left(\tfrac{\cancel{3}}{\cancel{2}}\cdot\tfrac{1}{\cancel{3}}(2t+1)^{1/2}\cdot\cancel{2}\right)^2}\, dt$$

$$= \int_0^4 \sqrt{t^2 + 2t + 1}\, dt$$

$$= \int_0^4 \sqrt{(t+1)^2}\, dt = \int_0^4 (t+1)\, dt$$

$$= \left(\tfrac{1}{2}t^2 + t\right)\Big|_0^4 = 8 + 4 = 12$$

29.) $\text{Arc} = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\left(\cancel{-8\sin t} + 8t\cos t + \cancel{8\sin t}\right)^2 + \left(\cancel{8\cos t} - (8t\cdot{}^-\sin t + \cancel{8\cos t})\right)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{(8t\cos t)^2 + (8t\sin t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{64t^2\cos^2 t + 64t^2\sin^2 t}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{64t^2(\underbrace{\cos^2 t + \sin^2 t}_{1})}\, dt$$

$$= \int_0^{\frac{\pi}{2}} 8t\,dt = 4t^2 \Big|_0^{\frac{\pi}{2}} = 4\left(\frac{\pi}{2}\right)^2 = \pi^2$$

31.) $\begin{cases} x = \cos t \\ y = 2 + \sin t \end{cases}$, $0 \le t \le 2\pi$ (about x-axis)

$$\text{Area} = 2\pi \int_0^{2\pi} y \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= 2\pi \int_0^{2\pi} (2 + \sin t) \cdot \sqrt{(-\sin t)^2 + (\cos t)^2}\,dt$$

$$= 2\pi \int_0^{2\pi} (2 + \sin t) \cdot \sqrt{\underbrace{\sin^2 t + \cos^2 t}_{1}}\,dt$$

$$= 2\pi \int_0^{2\pi} (2 + \sin t)\,dt$$

$$= 2\pi\,(2t - \cos t)\Big|_0^{2\pi}$$

$$= 2\pi\left((4\pi - \cancel{\cos 2\pi}^{\,1}) - (0 - \cancel{\cos 0}^{\,1})\right) = 8\pi^2$$

32.) $\begin{cases} x = \frac{2}{3} t^{3/2} \\ y = 2\sqrt{t} \end{cases}$, $0 \le t \le \sqrt{3}$ (about y-axis)

$$\underline{\text{Area} = 2\pi \int_0^{\sqrt{3}} x \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt}$$

$$= 2\pi \int_0^{\sqrt{3}} x \cdot \sqrt{\left(\frac{2}{3}\cdot\frac{3}{2} t^{1/2}\right)^2 + \left(\cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{t}}\right)^2}\,dt$$

$$= 2\pi \int_0^{\sqrt{3}} \frac{2}{3} t^{3/2} \cdot \sqrt{t + \frac{1}{t}}\,dt$$

$$= \frac{4}{3}\pi \int_0^{\sqrt{3}} t^{3/2} \cdot \sqrt{\frac{t^2+1}{t}}\,dt$$

$$= \frac{4}{3}\pi \int_0^{\sqrt{3}} t^{3/2} \cdot \frac{\sqrt{t^2+1}}{\sqrt{t}}\,dt$$

$$= \frac{4}{3}\pi \int_0^{\sqrt{3}} t \cdot \sqrt{t^2+1}\,dt = \frac{4}{3}\pi \cdot \frac{1}{\cancel{2}}\,\frac{(t^2+1)^{3/2}}{3/\cancel{2}}\Big|_0^{\sqrt{3}}$$

$$= \frac{4}{9}\pi\left(4^{3/2} - 1^{3/2}\right) = \frac{4}{9}\pi(8-1) = \frac{28}{9}\pi$$

34.) $\begin{cases} X = \ln(\sec t + \tan t) - \sin t \\ Y = \cos t \, , \quad 0 \le t \le \frac{\pi}{3} \end{cases}$ (about x-axis)

$$\frac{dX}{dt} = \frac{\sec t \tan t + \sec^2 t}{\sec t + \tan t} - \cos t$$

$$= \frac{\sec t \cancel{(\tan t + \sec t)}}{\cancel{\sec t + \tan t}} - \cos t = \sec t - \cos t \, ;$$

$$\frac{dY}{dt} = -\sin t \; ; \quad \text{then}$$

$$\text{Area} = \int_0^{\frac{\pi}{3}} \sqrt{\left(\frac{dX}{dt}\right)^2 + \left(\frac{dY}{dt}\right)^2} \, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{(\sec t - \cos t)^2 + (-\sin t)^2} \, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 t - 2 + \underbrace{\cos^2 t + \sin^2 t}_{1}} \, dt$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 t - 1} \, dt = \int_0^{\frac{\pi}{3}} \sqrt{\tan^2 t} \, dt$$

$$= \int_0^{\frac{\pi}{3}} |\tan t| \, dt = \int_0^{\frac{\pi}{3}} \tan t \, dt$$

$$= \ln|\sec t| \Big|_0^{\frac{\pi}{3}} = \ln\left|\sec \frac{\pi}{3}\right| - \ln|\sec 0|$$

$$= \ln 2 - \underbrace{\ln 1}_{0} = \ln 2$$

38.) $\begin{cases} X = e^t \cos t \\ Y = e^t \sin t \, , \quad 0 \le t \le \pi \end{cases}$

$$\frac{dX}{dt} = e^t \cdot -\sin t + e^t \cos t = e^t(\cos t - \sin t) \, ;$$

$$\frac{dY}{dt} = e^t\cos t + e^t\sin t = e^t(\cos t + \sin t);$$

$$\bar{X} = \frac{\int_a^b X\cdot Y\,dx}{\int_a^b Y\,dx} = \frac{\int_a^b X\cdot Y\,\frac{dx}{dt}\,dt}{\int_a^b Y\cdot\frac{dx}{dt}\,dt}$$

$$= \frac{\int_0^\pi (e^t\cos t)(e^t\sin t)e^t(\cos t - \sin t)\,dt}{\int_0^\pi (e^t\sin t)\,e^t(\cos t - \sin t)\,dt}$$

$$\bar{Y} = \frac{\int_a^b Y\cdot X\,dY}{\int_a^b Y\,dy} = \frac{\int_a^b Y\cdot X\cdot\frac{dY}{dt}\,dt}{\int_a^b Y\cdot\frac{dY}{dt}\,dt}$$

$$= \frac{\int_0^\pi (e^t\sin t)(e^t\cos t)e^t(\cos t + \sin t)\,dt}{\int_0^\pi (e^t\sin t)\,e^t(\cos t + \sin t)\,dt}$$

45.) $\begin{cases} x = \sin t \\ y = \sin 2t \end{cases}$

a.) $\frac{dY}{dx} = 0 \rightarrow \frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dx}{dt}} = \frac{2\cos 2t}{\cos t} = 0 \rightarrow$

$2\cos 2t = 0 \rightarrow \quad \cos 2t = 0 \rightarrow$
$2t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \rightarrow \boxed{t = \frac{\pi}{4}}, \frac{3\pi}{4}, \ldots$;

$t = \frac{\pi}{4} \rightarrow x = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}, \; Y = \sin\frac{\pi}{2} = 1$ so tangent is horizontal at $\underline{\left(\frac{\sqrt{2}}{2}, 1\right)}$

b.) $X = 0 \rightarrow \sin t = 0 \rightarrow t = \boxed{0}, \boxed{\pi}, 2\pi, \ldots$;
$Y = 0 \rightarrow \sin 2t = 0 \rightarrow 2t = 0, \pi, 2\pi, \ldots \rightarrow$
$t = \boxed{0}, \frac{\pi}{2}, \boxed{\pi}, \ldots$; then

$(x, Y) = (0, 0) \rightarrow \boxed{t = 0, \pi}$;

$\frac{dY}{dx} = \frac{\frac{dY}{dt}}{\frac{dx}{dt}} = \frac{2\cos 2t}{\cos t}$;

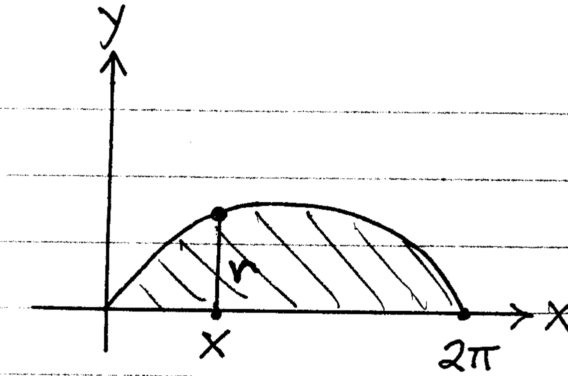
if $t = 0 \rightarrow x = 0, Y = 0$, and slope $Y' = \frac{2(1)}{1} = 2$;
so tangent line is $Y - 0 = 2(x - 0) \rightarrow \boxed{Y = 2X}$;

if $t = \pi \rightarrow x = 0, Y = 0$ and slope
$Y' = \frac{2(1)}{-1} = -2$; so tangent line is

$Y - 0 = -2(X - 0) \rightarrow \boxed{Y = -2X}$

48.) $\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$

for $0 \le t \le 2\pi$



$$\text{Vol} = \pi \int_a^b (\text{radius})^2 \, dx$$

$$= \pi \int_a^b y^2 \cdot \frac{dx}{dt} \cdot dt$$

$$= \pi \int_0^{2\pi} (1-\cos t)^2 \cdot (1-\cos t) \, dt$$

$$= \pi \int_0^{2\pi} (1-\cos t)^3 \, dt$$