

Section 5.3

$$10.) \text{ a.) } \int_1^9 -2 f(x) dx = -2 \int_1^9 f(x) dx = (-2)(-1) = 2$$

$$\text{ b.) } \int_7^9 (f(x) + h(x)) dx = \int_7^9 f(x) dx + \int_7^9 h(x) dx \\ = (5) + (4) = 9$$

$$\text{ c.) } \int_7^9 (2f(x) - 3h(x)) dx = 2 \int_7^9 f(x) dx - 3 \int_7^9 h(x) dx \\ = 2(5) - 3(4) = -2$$

$$\text{ d.) } \int_9^1 f(x) dx = - \int_1^9 f(x) dx = -(-1) = 1$$

$$\text{ e.) } \int_1^7 f(x) dx = \int_1^9 f(x) dx - \int_7^9 f(x) dx \\ = (-1) - (5) = -6$$

$$\text{ f.) } \int_9^7 [h(x) - f(x)] dx = - \int_7^9 [h(x) - f(x)] dx \\ = \int_7^9 (f(x) - h(x)) dx = \int_7^9 f(x) dx - \int_7^9 h(x) dx \\ = (5) - (4) = 1$$

$$11.) \text{ a.) } \int_1^2 f(u) du = \int_1^2 f(x) dx = 5$$

$$\text{ b.) } \int_1^2 \sqrt{3} \cdot f(z) dz = \sqrt{3} \int_1^2 f(z) dz \\ = \sqrt{3} \cdot (5) = 5\sqrt{3}$$

$$\text{ c.) } \int_2^1 f(t) dt = - \int_1^2 f(t) dt = -5$$

$$\text{ d.) } \int_1^2 -f(x) dx = - \int_1^2 f(x) dx = -5$$

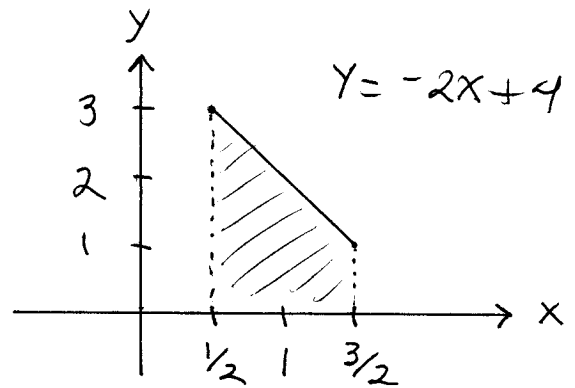
$$13.) \text{ a.) } \int_3^4 f(z) dz = \int_0^4 f(z) dz - \int_0^3 f(z) dz \\ = (7) - (3) = 4$$

$$b.) \int_4^3 f(t) dt = - \int_3^4 f(t) dt = -(4) = -4$$

$$16.) \int_{\frac{1}{2}}^{\frac{3}{2}} (-2x+4) dx$$

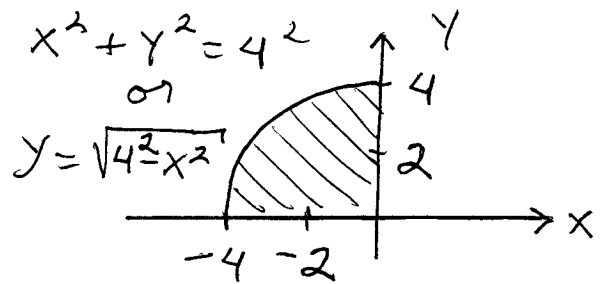
$$= \frac{1}{2} (3+1) \cdot \left(\frac{3}{2} - \frac{1}{2} \right)$$

$$= (2) \cdot (1) = 2$$



$$18.) \int_{-4}^0 \sqrt{4^2 - x^2} dx$$

$$= \frac{1}{4} \pi (4)^2 = 4\pi$$

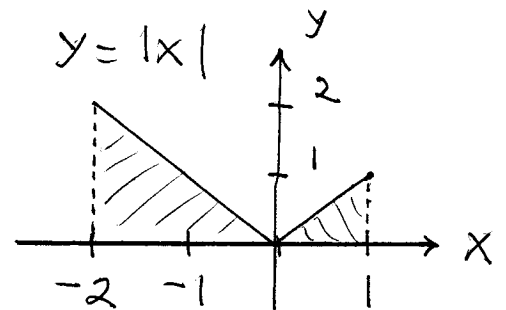


$$19.) \int_{-2}^1 |x| dx$$

$$= \int_{-2}^0 |x| dx + \int_0^1 |x| dx$$

$$= \frac{1}{2} (2)(2) + \frac{1}{2} (1)(1)$$

$$= 2 + \frac{1}{2} = \frac{5}{2}$$

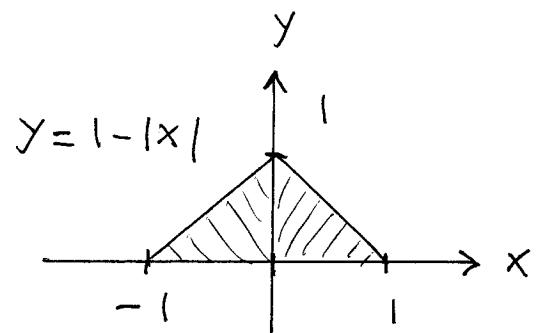


$$20.) \int_{-1}^1 (1 - |x|) dx$$

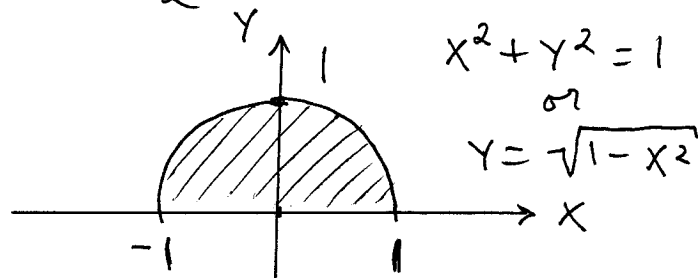
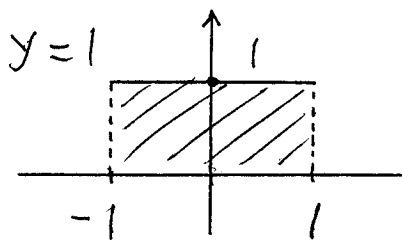
$$= \int_{-1}^0 (1 - |x|) dx$$

$$+ \int_0^1 (1 - |x|) dx$$

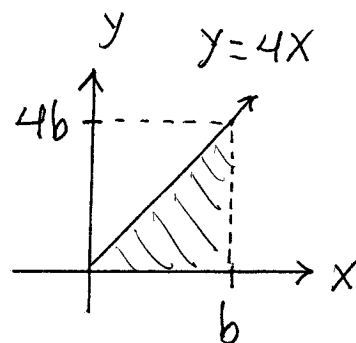
$$= \frac{1}{2} (1)(1) + \frac{1}{2} (1)(1) = 1$$



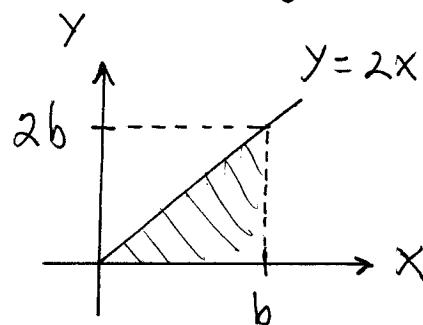
$$\begin{aligned}
 22.) \int_{-1}^1 (1 + \sqrt{1-x^2}) dx \\
 &= \int_{-1}^1 1 dx + \int_{-1}^1 \sqrt{1-x^2} dx \\
 &= (2)(1) + \frac{1}{2}\pi(1)^2 = 2 + \frac{\pi}{2}
 \end{aligned}$$



$$\begin{aligned}
 24.) \int_0^b 4x dx \\
 &= \frac{1}{2}(b)(4b) = 2b^2
 \end{aligned}$$



$$\begin{aligned}
 53.) \text{Area} &= \int_0^b 2x dx \\
 &= \frac{1}{2}(b)(2b) = b^2
 \end{aligned}$$



$$73.) \frac{1}{2} = \frac{1}{1+1^2} \leq \frac{1}{1+x^2} \leq \frac{1}{1+0^2} = 1 \rightarrow$$

$$\frac{1}{2}(1-0) \leq \int_0^1 \frac{1}{1+x^2} dx \leq 1(1-0) \rightarrow$$

$$\frac{1}{2} \leq \int_0^1 \frac{1}{1+x^2} dx \leq 1$$

$$75.) -1 \leq \sin x \leq 1 \rightarrow -1 \leq \sin^2 x \leq 1 \rightarrow$$

$$-1(1-0) \leq \int_0^1 \sin x^2 dx \leq 1(1-0) \rightarrow$$

$$-1 \leq \int_0^1 \sin x^2 dx \leq 1$$

$$\text{so } \int_0^1 \sin x^2 dx \neq 2.$$

76.) on interval $[0, 1]$:

$$\sqrt{8} = \sqrt{0+8} \leq \sqrt{x+8} \leq \sqrt{1+8} = \sqrt{9} = 3 \rightarrow$$

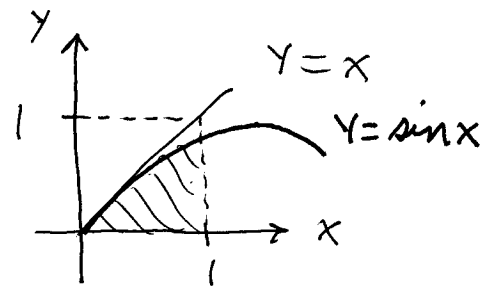
$$2\sqrt{2} \leq \sqrt{x+8} \leq 3 \rightarrow$$

$$2\sqrt{2}(1-0) \leq \int_0^1 \sqrt{x+8} dx \leq 3 \cdot (1-0) \rightarrow$$

$$2\sqrt{2} \leq \int_0^1 \sqrt{x+8} dx \leq 3.$$

$$79.) \sin x \leq x \rightarrow \int_0^1 \sin x dx \leq \int_0^1 x dx$$

$$\rightarrow \int_0^1 \sin x dx \leq \frac{1}{2}(1)(1) = \frac{1}{2}$$



Worksheet 1

1.) a.) $\begin{array}{ccccccccccc} -1 & x_1 & x_2 & x_3 & \dots & x_i & \dots & 2 = x_n \\ | & | & | & | & & | & & | & & | \\ \hline & & \underbrace{\hspace{2cm}} & & & & & & & \end{array}$

$$\Delta x_i = \frac{2 - (-1)}{n} = \frac{3}{n} \text{ for } i=1, 2, 3, \dots$$

and $x_i = -1 + \frac{3}{n}i$; $f(x) = 5$; then

$$\int_{-1}^2 5 \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n 5 \cdot \frac{3}{n} = \lim_{n \rightarrow \infty} \frac{15}{n} \cdot \left(\sum_{i=1}^n 1 \right)$$

$$= \frac{15}{n} \cdot n = 15$$

b.) $\begin{array}{ccccccccccc} 0 & x_1 & x_2 & x_3 & \dots & x_i & \dots & 2 = x_n \\ | & | & | & | & & | & & | & & | \\ \hline & & \underbrace{\hspace{2cm}} & & & & & & & \end{array}$

$$\Delta x_i = \frac{2-0}{n} = \frac{2}{n} \text{ for } i=1, 2, 3, \dots$$

and $x_i = 0 + \frac{2}{n}i = \frac{2}{n}i$ for $i=1, 2, 3, \dots$;
 $f(x) = x+3$; then

$$\int_0^2 (x+3) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{2}{n}i\right) \cdot \frac{2}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{2}{n}i + 3\right) \frac{2}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{4}{n^2}i + \frac{6}{n}\right) = \lim_{n \rightarrow \infty} \left[\frac{4}{n^2} \left(\sum_{i=1}^n i\right) + \frac{6}{n} \cdot \left(\sum_{i=1}^n 1\right) \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{4}{n^2} \cdot \frac{n(n+1)}{2} + \frac{6}{n} \cdot n \right]$$

$$= \lim_{n \rightarrow \infty} \left[2 \cdot \left(1 + \frac{1}{n}\right) + 6 \right] = 2(1) + 6 = 8$$

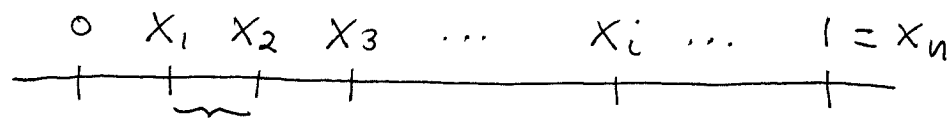
c.) $\begin{array}{ccccccccccc} -3 & x_1 & x_2 & x_3 & \dots & x_i & \dots & 0 = x_n \\ | & | & | & | & & | & & | & & | \\ \hline & & \underbrace{\hspace{2cm}} & & & & & & & \end{array}$

$$\Delta x_i = \frac{0 - (-3)}{n} = \frac{3}{n} \text{ for } i=1, 2, 3, \dots$$

and $x_i = -3 + \frac{3}{n}i$ for $i=1, 2, 3, \dots, j$

$f(x) = x^2 + 2x$; then

$$\begin{aligned} \int_{-3}^0 (x^2 + 2x) dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n (x_i^2 + 2x_i) \cdot \frac{3}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(-3 + \frac{3}{n}i\right)^2 + 2\left(-3 + \frac{3}{n}i\right) \right] \cdot \frac{3}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[9 - \frac{18}{n}i + \frac{9}{n^2}i^2 - 6 + \frac{6}{n}i \right] \cdot \frac{3}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[3 - \frac{12}{n}i + \frac{9}{n^2}i^2 \right] \cdot \frac{3}{n} \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{9}{n} - \frac{36}{n^2} \cdot i + \frac{27}{n^3} \cdot i^2 \right] \\ &= \lim_{n \rightarrow \infty} \left\{ \frac{9}{n} \cdot \left(\sum_{i=1}^n 1\right) - \frac{36}{n^2} \cdot \left(\sum_{i=1}^n i\right) + \frac{27}{n^3} \cdot \left(\sum_{i=1}^n i^2\right) \right\} \\ &= \lim_{n \rightarrow \infty} \left\{ \frac{9}{n} \cdot (n) - \frac{36}{n^2} \cdot \frac{n(n+1)}{2} + \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right\} \\ &= \lim_{n \rightarrow \infty} \left\{ 9 - 18 \cdot \left(1 + \frac{1}{n}\right) + \frac{9}{2} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \right\} \\ &= 9 - 18(1) + \frac{9}{2}(1)(2) = 0 \end{aligned}$$

d.) 

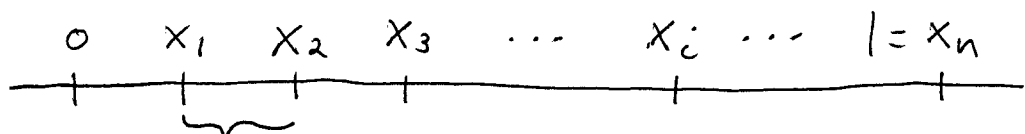
$$\Delta x_i = \frac{1-0}{n} = \frac{1}{n} \text{ for } i=1, 2, 3, \dots$$

and $x_i = 0 + \frac{1}{n}i = \frac{i}{n}$ for $i=1, 2, 3, \dots, j$

$f(x) = x^3$; then

$$\int_0^1 x^3 dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i$$

$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{i}{n}\right) \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i}{n}\right)^3 \cdot \frac{1}{n} \\
&= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^3}{n^3} \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{1}{n^4} \left(\sum_{i=1}^n i^3 \right) \\
&= \lim_{n \rightarrow \infty} \frac{1}{n^4} \cdot \left(\frac{n(n+1)}{2} \right)^2 = \lim_{n \rightarrow \infty} \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4} \\
&= \lim_{n \rightarrow \infty} \frac{1}{4} \cdot \left(\frac{n+1}{n} \right)^2 = \lim_{n \rightarrow \infty} \frac{1}{4} \left(1 + \frac{1}{n} \right)^2 = \frac{1}{4} (1)^2 = \frac{1}{4}
\end{aligned}$$

e.) 

$$\Delta x_i = \frac{1-0}{n} = \frac{1}{n} \text{ for } i=1, 2, 3, \dots$$

and $x_i = 0 + \frac{1}{n} i = \frac{i}{n}$ for $i=1, 2, 3, \dots$;

$f(x) = 2^x$; then

$$\int_0^1 2^x dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{i}{n}\right) \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n 2^{i/n} \cdot \frac{1}{n}$$

$$= \lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \cdot (2^{1/n} + 2^{2/n} + 2^{3/n} + \dots + 2^{n/n}) \right\}$$

$$= \lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \left((2^{1/n}) + (2^{1/n})^2 + (2^{1/n})^3 + \dots + (2^{1/n})^n \right) \right\}$$

$$= \lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \cdot 2^{1/n} \cdot (1 + (2^{1/n}) + (2^{1/n})^2 + (2^{1/n})^3 + \dots + (2^{1/n})^{n-1}) \right\}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \cdot 2^{1/n} \cdot \frac{1 - (2^{1/n})^{(n-1)+1}}{1 - 2^{1/n}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \cdot 2^{1/n} \cdot \frac{1 - 2}{1 - 2^{1/n}}$$

$$= \lim_{n \rightarrow \infty} \frac{-1}{n} \cdot \frac{2^{1/n}}{1 - 2^{1/n}} \cdot \frac{2^{-1/n}}{2^{-1/n}}$$

$$= \lim_{n \rightarrow \infty} \frac{-1}{n} \cdot \frac{2^0}{2^{-1/n} - 2^0}$$

$$= \lim_{n \rightarrow \infty} \frac{-\frac{1}{n}}{2^{-1/n} - 1}$$

"0/0"

$$\lim_{n \rightarrow \infty} \frac{\cancel{1} \cdot 2}{2^{-1/n} \cdot \cancel{\frac{1}{n^2}} \cdot \ln 2}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{2^{-1/n} \cdot \ln 2}$$

$$= \frac{1}{2^0 \cdot \ln 2}$$

$$= \frac{1}{\ln 2}$$