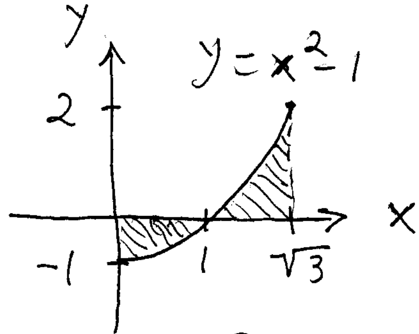
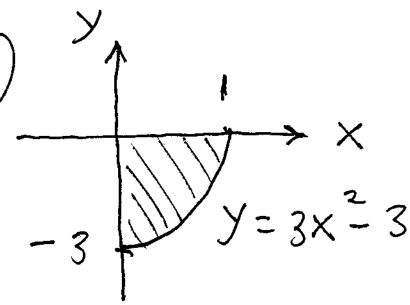
Section 5.3

55.)



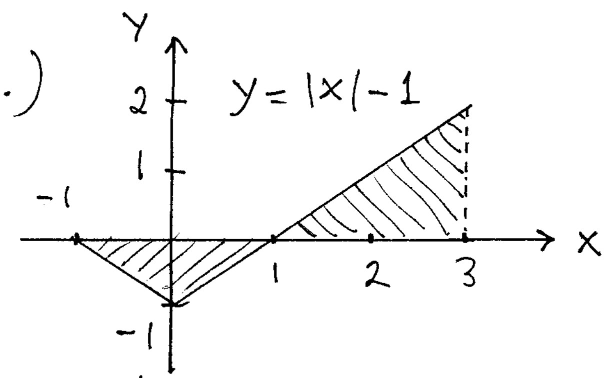
$$AVE = \frac{1}{\sqrt{3}-0}\int_0^{\sqrt{3}} (x^2-1)\,dx$$
$$= \frac{1}{\sqrt{3}}\left(\frac{x^3}{3}-x\right)\Big|_0^{\sqrt{3}}$$
$$= \frac{1}{\sqrt{3}}\left(\frac{(\sqrt{3})^3}{3}-\sqrt{3}\right)-\frac{1}{\sqrt{3}}\left(\frac{0^3}{3}-0\right)$$
$$= \frac{1}{\sqrt{3}}\left(\frac{\cancel{3}\sqrt{3}}{\cancel{3}}-\sqrt{3}\right) = 1-1 = 0$$

58.)



$$AVE = \frac{1}{1-0}\int_0^1 (3x^2-3)\,dx$$
$$= (x^3-3x)\Big|_0^1$$
$$= (1-3)-(0-0) = -2$$

61.)



a.) on $[-1,1]$:

$$AVE = \frac{1}{1-(-1)}\int_{-1}^1 (|x|-1)\,dx$$
$$= \frac{1}{2}\int_{-1}^0 ((-x)-1)\,dx$$
$$+ \frac{1}{2}\int_0^1 ((x)-1)\,dx = \frac{1}{2}\left(-\frac{x^2}{2}-x\right)\Big|_{-1}^0 + \frac{1}{2}\left(\frac{x^2}{2}-x\right)\Big|_0^1$$
$$= \frac{1}{2}(0-0)-\frac{1}{2}\left(\frac{-1}{2}+1\right) + \frac{1}{2}\left(\frac{1}{2}-1\right)-\frac{1}{2}(0-0)$$
$$= 0 - \frac{1}{4} + \frac{-1}{4} - 0 = -1/2$$

b.) on $[1,3]$:

$$AVE = \frac{1}{3-1}\int_1^3 (|x|-1)\,dx = \frac{1}{2}\int_1^3 (x-1)\,dx$$
$$= \frac{1}{2}\left(\frac{x^2}{2}-x\right)\Big|_1^3 = \frac{1}{2}\left(\frac{9}{2}-3\right)-\frac{1}{2}\left(\frac{1}{2}-1\right)$$

$= \frac{1}{2}\left(\frac{3}{2}\right) - \frac{1}{2}\left(\frac{-1}{2}\right) = \frac{3}{4} + \frac{1}{4} = 1$

c.) on $[-1, 3]$:

$AVE = \frac{1}{3-(-1)} \int_{-1}^{3} (|x|-1)\, dx$

$= \frac{1}{4}\int_{-1}^{0} ((-x)-1)\, dx + \frac{1}{4}\int_{0}^{3} ((x)-1)\, dx$

$= \frac{1}{4}\left(-\frac{x^2}{2} - x\right)\Big|_{-1}^{0} + \frac{1}{4}\left(\frac{x^2}{2} - x\right)\Big|_{0}^{3}$

$= \frac{1}{4}(0-0) - \frac{1}{4}\left(-\frac{1}{2}+1\right) + \frac{1}{4}\left(\frac{9}{2} - 3\right) - \frac{1}{4}(0-0)$

$= 0 - \frac{1}{8} + \frac{3}{8} - 0 = \frac{1}{4}$

Section 5.4

3.) $\int_0^2 x(x-3)\,dx = \int_0^2 (x^2-3x)\,dx$

$= \left(\frac{1}{3}x^3 - \frac{3}{2}x^2\right)\Big|_0^2 = \left(\frac{8}{3}-6\right)-(0-0) = \frac{-10}{3}$

7.) $\int_0^1 (x^2+x^{1/2})\,dx = \left(\frac{1}{3}x^3 + \frac{2}{3}x^{3/2}\right)\Big|_0^1$

$= \left(\frac{1}{3}+\frac{2}{3}\right) - (0+0) = 1$

9.) $\int_0^{\pi/3} 2\sec^2 x\,dx = 2\tan x\,\Big|_0^{\pi/3}$

$= 2\tan\frac{\pi}{3} - 2\tan 0 = 2\cdot\frac{\sin\frac{\pi}{3}}{\cos\pi/3} - 2\,\frac{\sin 0}{\cos 0}$

$= 2\cdot\frac{\sqrt{3}/2}{1/2} - 2\cdot\frac{0}{1} = 2\sqrt{3}$

12.) $\int_0^{\frac{\pi}{3}} 4\sec u\tan u\,du = 4\sec u\,\Big|_0^{\pi/3}$

$= 4\sec\frac{\pi}{3} - 4\sec 0 = 4\cdot\frac{1}{\cos\frac{\pi}{3}} - 4\cdot\frac{1}{\cos 0}$

$= 4\cdot\frac{1}{1/2} - 4\cdot\frac{1}{1} = 8-4 = 4$

13.) $\int_{\frac{\pi}{2}}^{0} \frac{1+\cos 2t}{2}\,dt = \int_{\frac{\pi}{2}}^{0}\left(\frac{1}{2}+\frac{1}{2}\cos 2t\right)dt$

$= \left(\frac{1}{2}t + \frac{1}{2}\cdot\frac{1}{2}\sin 2t\right)\Big|_{\frac{\pi}{2}}^{0}$

$= \left(0+\frac{1}{4}\cancelto{0}{\sin 0}\right) - \left(\frac{\pi}{4}+\frac{1}{4}\cancelto{0}{\sin\pi}\right) = -\frac{\pi}{4}$

16.) $\int_0^{\frac{\pi}{6}} (\sec x+\tan x)^2\,dx = \int_0^{\frac{\pi}{6}}(\sec^2 x + 2\sec x\tan x + \tan^2 x)\,dx$

$\sec^2 x - 1$

$= \int_0^{\frac{\pi}{6}}(2\sec^2 x + 2\sec x\tan x - 1)\,dx$

$= (2\tan x + 2\sec x - x)\,\Big|_0^{\pi/6}$

$$= \left(2\tan\frac{\pi}{6} + 2\sec\frac{\pi}{6} - \frac{\pi}{6}\right)$$

$$- \left(2\cancel{\tan}\overset{0}{0} + 2\cancel{\sec}\overset{1}{0} - 0\right)$$

$$= 2\cdot\left(\frac{1}{\sqrt{3}}\right) + 2\left(\frac{2}{\sqrt{3}}\right) - \frac{\pi}{6} - 2$$

$$= \frac{6}{\sqrt{3}} - \frac{\pi}{6} - 2$$

20.) $\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4)\,dt = \int_{-\sqrt{3}}^{\sqrt{3}} (t^3+t^2+4t+4)\,dt$

$$= \left(\frac{1}{4}t^4 + \frac{1}{3}t^3 + 2t^2 + 4t\right)\Big|_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \left(\cancel{\tfrac{1}{4}\cdot 9} + \frac{1}{\cancel{3}}\cdot\cancel{3}\sqrt{3} + \cancel{2\cdot 3} + 4\sqrt{3}\right) - \left(\cancel{\tfrac{1}{4}\cdot 9} - \frac{1}{\cancel{3}}\cdot\cancel{3}\sqrt{3} + \cancel{2\cdot 3} - 4\sqrt{3}\right)$$

$$= 2\sqrt{3} + 8\sqrt{3} = 10\sqrt{3}$$

22.) $\int_{-3}^{-1} \frac{y^5 - 2y}{y^3}\,dy = \int_{-3}^{-1}\left(\frac{y^5}{y^3} - \frac{2y}{y^3}\right)dy$

$$= \int_{-3}^{-1}(y^2 - 2y^{-2})\,dy = \left(\frac{1}{3}y^3 - 2\cdot\frac{y^{-1}}{-1}\right)\Big|_{-3}^{-1}$$

$$= \left(\frac{1}{3}y^3 + \frac{2}{y}\right)\Big|_{-3}^{-1} = \left(\frac{-1}{3} - 2\right) - \left(-9 + \frac{-2}{3}\right)$$

$$= 7 + \tfrac{1}{3} = \tfrac{22}{3}$$

26.) $\int_0^{\frac{\pi}{3}} (\cos x + \sec x)^2\,dx = \int_0^{\frac{\pi}{3}}(\cos^2 x + 2 + \sec^2 x)\,dx$

$$= \int_0^{\frac{\pi}{3}}\left(\frac{1}{2}(1+\cos 2x) + 2 + \sec^2 x\right)dx$$

$$= \int_0^{\frac{\pi}{3}}\left(\frac{5}{2} + \frac{1}{2}\cos 2x + \sec^2 x\right)dx$$

$$= \left(\frac{5}{2}x + \frac{1}{4}\sin 2x + \tan x\right)\Big|_0^{\pi/3}$$

$$= \left(\frac{5}{2}\cdot\frac{\pi}{3} + \frac{1}{4}\overset{\sqrt{3}/2}{\sin \tfrac{2}{3}\pi} + \overset{\sqrt{3}}{\tan \pi/3}\right) - \left(0 + \frac{1}{4}\overset{0}{\sin 0} + \overset{0}{\tan 0}\right)$$

$$= \frac{5}{6}\pi + \frac{\sqrt{3}}{8} + \sqrt{3} = \frac{5}{6}\pi + \frac{9}{8}\sqrt{3}$$

29.) $\int_0^{\ln 2} e^{3x}\,dx = \frac{1}{3}e^{3x}\Big|_0^{\ln 2}$

$$= \frac{1}{3}e^{3\ln 2} - \frac{1}{3}e^{0} = \frac{1}{3}e^{\ln 2^3} - \frac{1}{3}\cdot 1$$

$$= \frac{1}{3}\cdot 8 - \frac{1}{3} = \frac{7}{3}$$

32.) $\int_0^{1/2} \frac{1}{1+4x^2}\,dx = \int_0^{1/2} \frac{1}{2}\cdot\frac{2}{1+(2x)^2}\,dx$

$$= \frac{1}{2}\arctan(2x)\Big|_0^{1/2} = \frac{1}{2}\arctan 1 - \frac{1}{2}\arctan 0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{2}(0) = \frac{\pi}{8}$$

34.) $\int_{-1}^{0} \pi^{x-1}\,dx = \int_{-1}^{0} \frac{1}{\ln\pi}\left(\pi^{x-1}\cdot\ln\pi\right)dx$

$$= \frac{1}{\ln\pi}\cdot\pi^{x-1}\Big|_{-1}^{0} = \frac{1}{\ln\pi}\left(\pi^{-1} - \pi^{-2}\right)$$

35.) $\int_0^1 x e^{x^2}\,dx = \int_0^1 \frac{1}{2}\left(2x\cdot e^{x^2}\right)dx$

$$= \frac{1}{2}e^{x^2}\Big|_0^1 = \frac{1}{2}\left(e^1 - e^0\right) = \frac{1}{2}(e-1)$$

36.) $\int_1^2 \frac{\ln x}{x}\,dx = \int_1^2 \frac{1}{x}\cdot\ln x\,dx$

$$= \frac{1}{2}(\ln x)^2\Big|_1^2 = \frac{1}{2}(\ln 2)^2 - \frac{1}{2}(\overset{0}{\ln 1})^2$$

$$= \frac{1}{2}(\ln 2)^2$$

39.) a.) $\frac{d}{dx}\left(\int_0^{\sqrt{x}} \cos t\, dt\right) = \frac{d}{dx}\left(\sin t \Big|_0^{\sqrt{x}}\right)$

$= \frac{d}{dx}\left(\sin\sqrt{x} - \cancel{\sin 0}^{\,0}\right) = \cos\sqrt{x} \cdot \frac{1}{2}x^{-1/2}$

b.) $\frac{d}{dx}\left(\int_0^{\sqrt{x}} \cos t\, dt\right) = \cos\sqrt{x} \cdot D(\sqrt{x})$

$= \cos\sqrt{x} \cdot \frac{1}{2}x^{-1/2}$

43.) a.) $\frac{d}{dx}\left(\int_0^{x^3} e^{-t}\, dt\right) = \frac{d}{dx}\left(-e^{-t}\Big|_0^{x^3}\right)$

$= \frac{d}{dx}\left(-e^{-x^3} - -e^0\right) = \frac{d}{dx}\left(-e^{-x^3} + 1\right)$

$= -e^{-x^3} \cdot -3x^2 = 3x^2 e^{-x^3}$

b.) $\frac{d}{dx}\left(\int_0^{x^3} e^{-t}\, dt\right) = e^{-x^3} \cdot D(x^3) = e^{-x^3} \cdot 3x^2$

45.) $Y = \int_0^x \sqrt{1+t^2}\, dt \xrightarrow{D} Y' = \sqrt{1+x^2}$

50.) $Y = \left(\int_0^x (t^3+1)^{10}\, dt\right)^3 \xrightarrow{D}$

$Y' = 3\left(\int_0^x (t^3+1)^{10}\, dt\right)^2 \cdot D\left(\int_0^x (t^3+1)^{10}\, dt\right)$

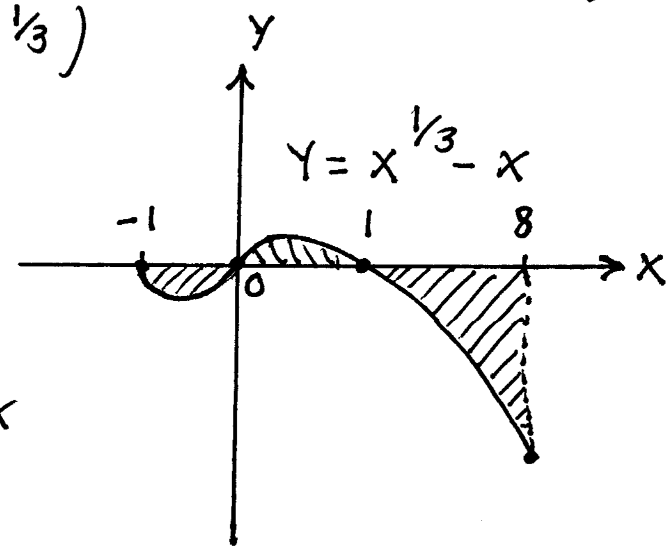
$= 3\left(\int_0^x (t^3+1)^{10}\, dt\right)^2 \cdot (x^3+1)^{10}$

52.) $D\left(\int_{\tan x}^0 \frac{1}{1+t^2}\, dt\right) = D\left(-\int_0^{\tan x} \frac{1}{1+t^2}\, dt\right)$

$= -\frac{1}{1+\tan^2 x} \cdot D(\tan x) = \frac{-1}{1+\tan^2 x} \cdot \sec^2 x$

60.) $Y = x^{1/3} - x = x^{1/3}(1 - x^{2/3}) = x^{1/3}(1 - (x^{1/3})^2)$

$= x^{1/3}(1 - x^{1/3})(1 + x^{1/3})$



$$\text{Area} = \int_{-1}^{0} [0 - (x^{1/3} - x)]\, dx + \int_{0}^{1} (x^{1/3} - x)\, dx + \int_{1}^{8} [0 - (x^{1/3} - x)]\, dx$$

$$= \left(-\tfrac{3}{4}x^{4/3} + \tfrac{1}{2}x^2\right)\Big|_{-1}^{0} + \left(\tfrac{3}{4}x^{4/3} - \tfrac{1}{2}x^2\right)\Big|_{0}^{1} + \left(-\tfrac{3}{4}x^{4/3} + \tfrac{1}{2}x^2\right)\Big|_{1}^{8}$$

$$= (0+0) - \left(-\tfrac{3}{4} + \tfrac{1}{2}\right) + \left(\tfrac{3}{4} - \tfrac{1}{2}\right) - (0-0) + \left(-\tfrac{3}{4}(8)^{4/3} + \tfrac{1}{2}(8)^2\right) - \left(-3/4 + 1/2\right)$$

$$= \tfrac{3}{4} - \tfrac{1}{2} + \tfrac{3}{4} - \tfrac{1}{2} - \tfrac{3}{4}(16) + 32 + \tfrac{3}{4} - \tfrac{1}{2}$$

$$= 3\left(\tfrac{3}{4}\right) - 3\left(\tfrac{1}{2}\right) - 12 + 32$$

$$= \tfrac{9}{4} - \tfrac{6}{4} + \tfrac{80}{4}$$

$$= \tfrac{83}{4}$$

61.) Area of shaded region is area of rectangle — area under the curve, i.e.,

$$\text{Area} = 2\pi - \int_0^{\pi} (1+\cos x)\,dx$$

$$= 2\pi - (x+\sin x)\Big|_0^{\pi} = 2\pi - [(\pi + \cancel{\sin \pi}^{\,0}) - (0 + \cancel{\sin 0}^{\,0})]$$

$$= \pi$$

75.) $T = 85 - 3(25-t)^{1/2}$

a.) $t=0 \rightarrow T = 85 - 3(5) = 70\ ^\circ F$

$t=16 \rightarrow T = 85 - 3(3) = 76\ ^\circ F$

$t=25 \rightarrow T = 85 - 3(0) = 85\ ^\circ F$

b.) $\text{AVE} = \frac{1}{25-0}\int_0^{25}\{85 - 3(25-t)^{1/2}\}\,dt$

$$= \frac{1}{25}\left\{85t - {}^{-}3\cdot\frac{2}{3}(25-t)^{3/2}\right\}\Big|_0^{25}$$

$$= \frac{1}{25}\left\{85t + 2(25-t)^{3/2}\right\}\Big|_0^{25}$$

$$= \frac{1}{25}(85\cdot(25)) - \frac{1}{25}(2(125)) = 75\ ^\circ F$$

77.) $\int_1^x f(t)\,dt = x^2 - 2x + 1 \xrightarrow{D} f(x) = 2x - 2$

78.) $\int_0^x f(t)\,dt = x\cdot\cos\pi x \xrightarrow{D}$

$f(x) = x\cdot{}^{-}\pi\sin\pi x + (1)\cdot\cos\pi x \rightarrow$

$f(4) = -4\pi \underbrace{\sin 4\pi}_{0} + \underbrace{\cos 4\pi}_{1} = 1$

80.) $g(x) = 3 + \int_1^{x^2} \sec(t-1)\,dt$ and

$g(-1) = 3 + \int_1^{(-1)^2} \sec(t-1)\,dt = 3 + \underbrace{\cancel{\int_1^1 \sec(t-1)\,dt}}_{0}$

$\rightarrow g(-1) = 3$; $\xrightarrow{D} g'(x) = \sec(x^2-1) \cdot 2x$

and $g'(-1) = -2\cancel{\sec}\overset{\circ}{0} \longrightarrow g'(-1) = -2$ so

linearization is

$L(x) = g(-1) + g'(-1)\,(x-(-1)) \longrightarrow$

$L(x) = 3 + {}^{-}2\,(x+1) \longrightarrow L(x) = 1 - 2x$.

83.) position: $s = \int_0^t f(x)\,dx$ m. $\xrightarrow{D}$

velocity: $s' = f(t)$ m./sec.

a.) $s'(5) = f(5) = 2$ m./sec.

acceleration: $s'' = f'(t)$

b.) $s''(5) = f'(5) < 0$

c.) $s(3) = \int_0^3 f(x)\,dx$ = area of $\triangle$

$= \frac{1}{2}(3)(3) = \frac{9}{2}$

d.) s is largest when "net area"

$s = \int_0^t f(x)\,dx$ is largest, i.e.,

s is largest when $t = 6$ sec.

e.) acceleration $s'' = f'(x) = 0$

when $x = 4$ sec., $x = 7$ sec.

f.) move toward origin : $s' < 0 \rightarrow$
$s' = f(t) < 0 \rightarrow 6 < t < 9$ seconds ;

move away from origin : $s' > 0 \rightarrow$
$s' = f(t) > 0 \rightarrow 0 < t < 6$ seconds

g.) $s(9) = \int_0^9 f(x)\,dx > 0$ (since region above x-axis is larger than region below x-axis.

84.) $$\lim_{x\to\infty} \frac{\int_1^x \frac{1}{\sqrt{t}}\,dt}{\sqrt{x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{\frac{1}{2}x^{-1/2}}$$

L'Hopital

$$= \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{1}{\cancel{\sqrt{x}}} \cdot \frac{2\cancel{\sqrt{x}}}{1} = 2$$