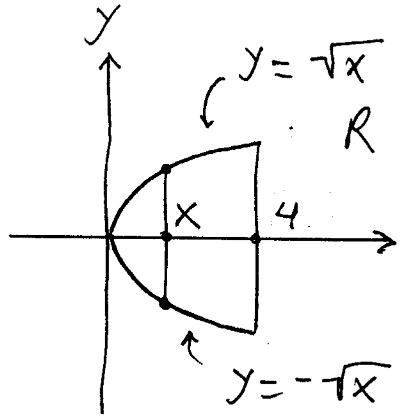
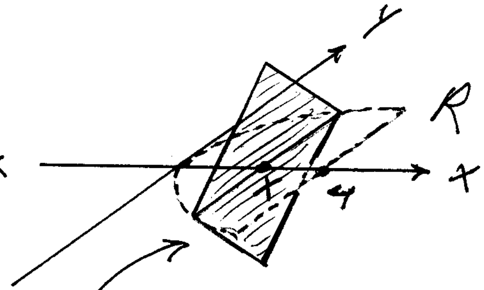
Section 6.1

1.)



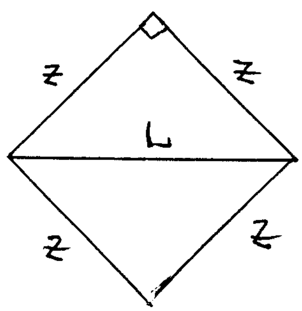


cross-section at x is square with diagonal in region R : area

$A(x) = (\text{edge})^2$

$= \left(\frac{1}{\sqrt{2}} \cdot 2\sqrt{x}\right)^2$

$= 2x$;



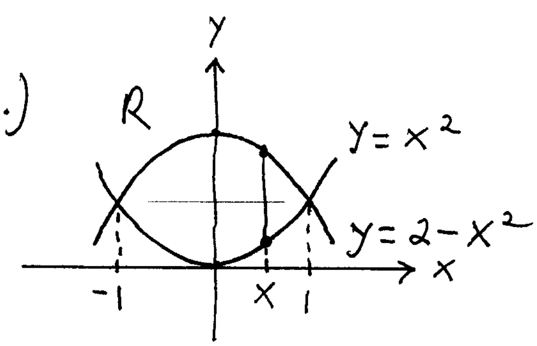
$z^2 + z^2 = L^2 \rightarrow$

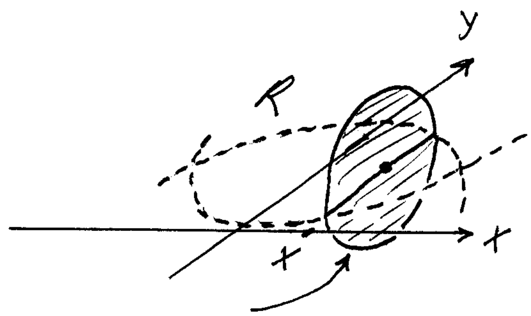
$2z^2 = L^2 \rightarrow$

$z = \frac{1}{\sqrt{2}} L$

$$\text{Volume} = \int_0^4 2x \, dx$$

2.)





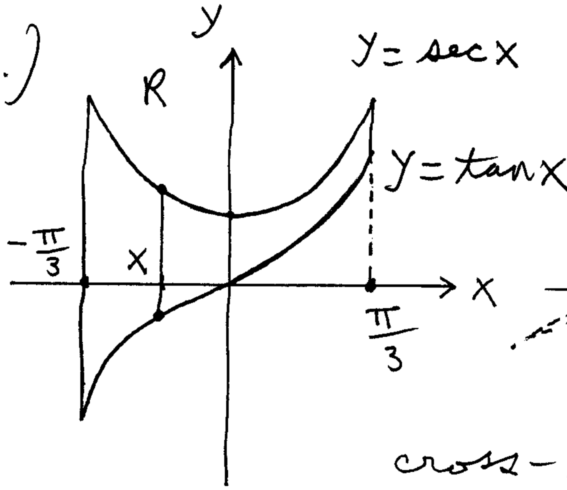
cross-section at x is circle : area

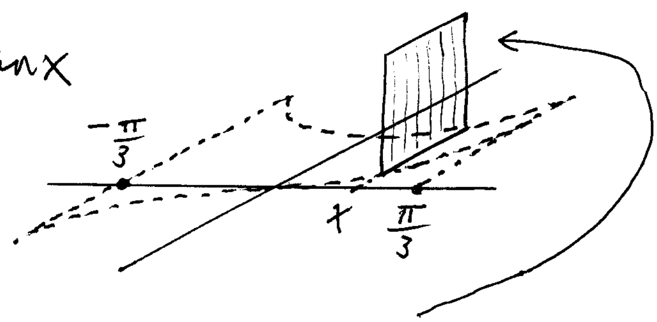
$A(x) = \pi r^2 = \pi\left(\frac{1}{2}\left((2-x^2) - x^2\right)\right)^2$

$= \pi\left(\frac{1}{2}(2 - 2x^2)\right)^2 = \pi(1-x^2)^2$;

$$\text{Volume} = \int_{-1}^{1} \pi (1-x^2)^2 \, dx$$

6.) b.)



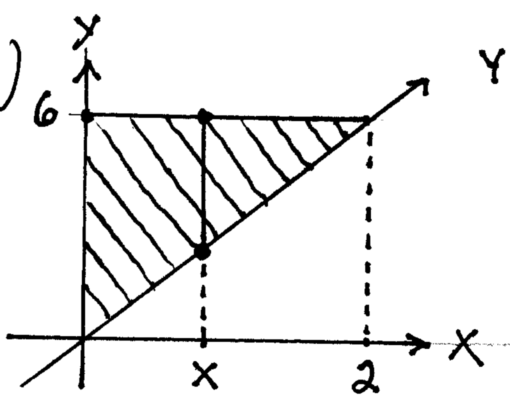


cross-section at x is square : area

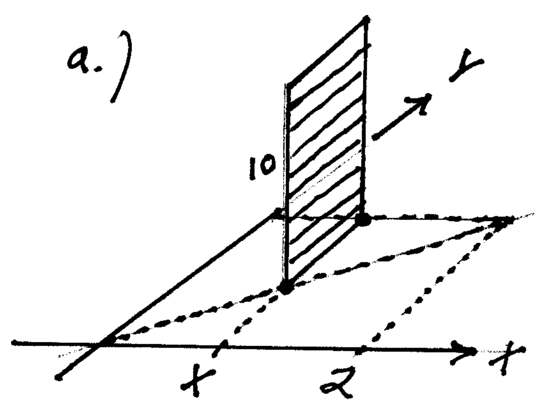
$A(x) = (\text{edge})^2 = (\sec x - \tan x)^2$;

Volume $= \int_{-\pi/3}^{\pi/3} (\sec x - \tan x)^2 \, dx$

7.)



a.)



cross-sections at x are rectangles of height 10 :

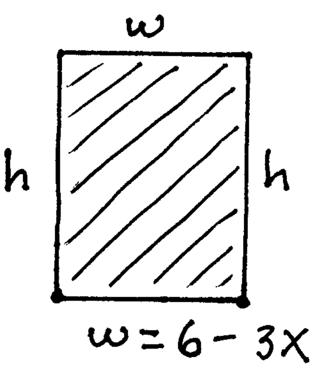
area $A(x) = (\text{base})(\text{height})$

$= (6 - 3x)(10)$

$= 60 - 30x$ →

Volume $= \int_0^2 (60 - 30x) \, dx$

b.)

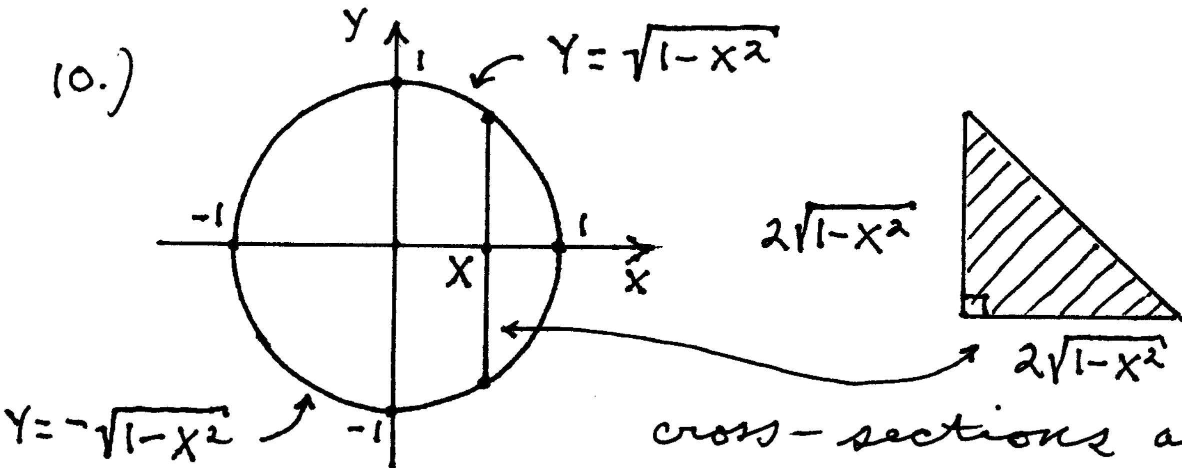


cross-sections at x are rectangles of perimeter 20 : $2h + 2w = 20$ →

$h + w = 10 \rightarrow h = 10 - w = 10 - (6-3x) \rightarrow$
$h = 4 + 3x$, then area
$A(x) = hw = (4+3x)(6-3x) \rightarrow$

$$\text{Volume} = \int_0^2 (4+3x)(6-3x)\,dx$$

10.)

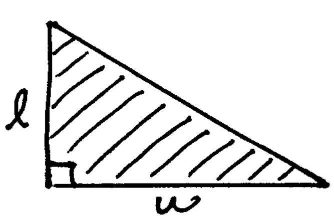


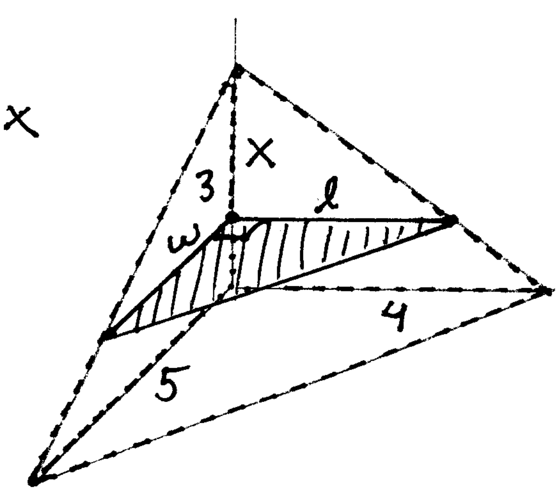
cross-sections at x are isosceles right triangles : area

$$A(x) = \tfrac{1}{2}(\text{base})(\text{height}) = \tfrac{1}{2}\cdot 2\sqrt{1-x^2}\cdot 2\sqrt{1-x^2} = 2(1-x^2)$$

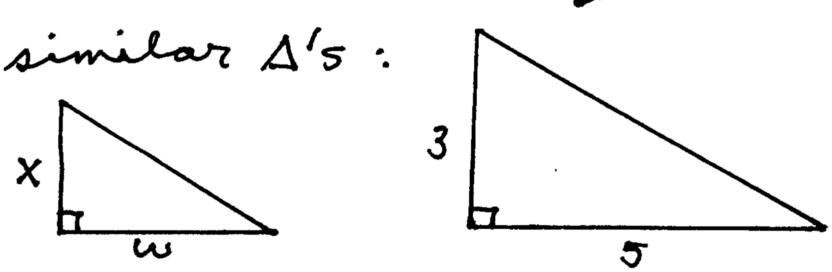
$$\rightarrow \text{Volume} = \int_{-1}^{1} 2(1-x^2)\,dx$$

11.) cross-section at x is right triangle :

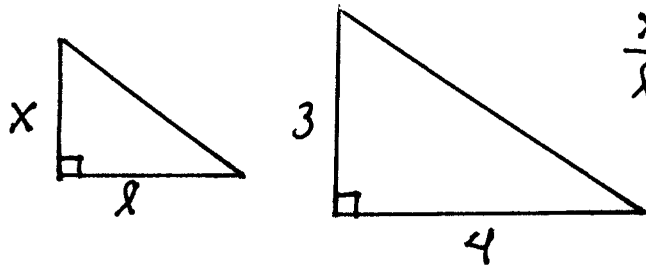




by similar Δ's :



$\frac{x}{w} = \frac{3}{5} \rightarrow \boxed{w = \frac{5}{3}x}$;



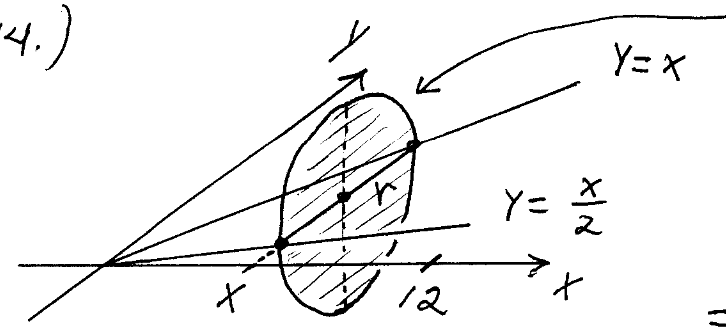
$\frac{x}{l} = \frac{3}{4} \rightarrow \boxed{l = \frac{4}{3}x}$;

then area of cross-section is

$A(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2} w l = \frac{1}{2}\left(\frac{5}{3}x\right)\left(\frac{4}{3}x\right)$

$\rightarrow A(x) = \frac{10}{9}x^2 \longrightarrow \text{Volume} = \int_0^3 \frac{10}{9}x^2\,dx$

14.)



cross-section at x is circle: area

$A(x) = \pi r^2$

$= \pi\left(\frac{1}{2}\left(x - \frac{1}{2}x\right)\right)^2$

$= \pi \cdot \left(\frac{1}{4}x\right)^2$

$= \frac{\pi}{16}x^2$;

$$\text{Volume} = \int_0^{12} \frac{\pi}{16} x^2\,dx = \frac{\pi}{16}\cdot \frac{x^3}{3}\Big|_0^{12}$$

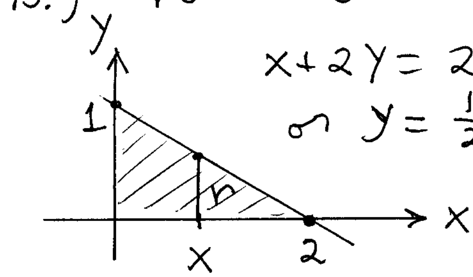
$$= \frac{\pi}{16}\frac{(12)^3}{3} = 36\pi \;;$$

Volume of right circular cylinder is

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (3)^2 (12) = 36\pi$$

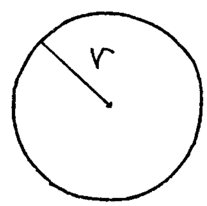
15.) $\text{Volume} = \pi \int_0^2 (\text{radius})^2\,dx$



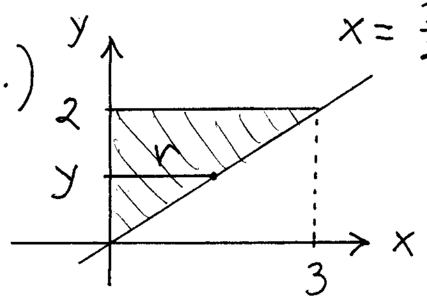
$x + 2y = 2$

or $y = \frac{1}{2}(2-x)$

$$= \pi \int_0^2 \left(\frac{1}{2}(2-x)\right)^2 dx$$



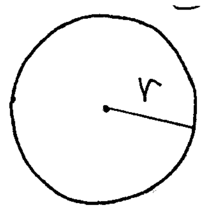
16.)



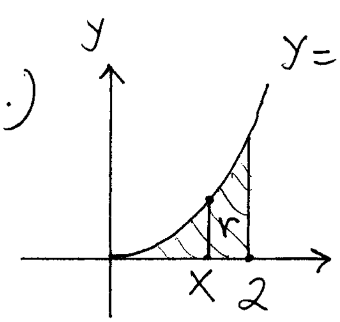
$x = \frac{3}{2}y$

$$\text{Volume} = \pi \int_0^2 (\text{radius})^2\,dy$$

$$= \pi \int_0^2 \left(\frac{3}{2}y\right)^2 dy$$



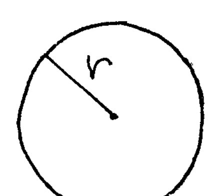
20.)



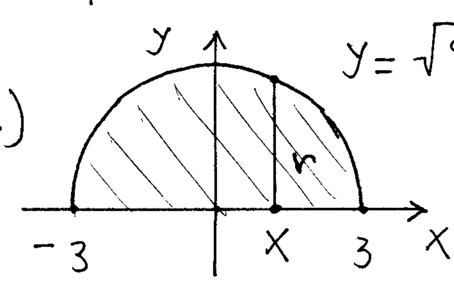
$y = x^3$

$$\text{Volume} = \pi \int_0^2 (\text{radius})^2\,dx$$

$$= \pi \int_0^2 (x^3)^2\,dx$$



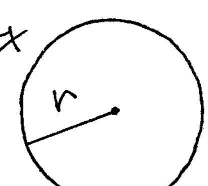
21.)



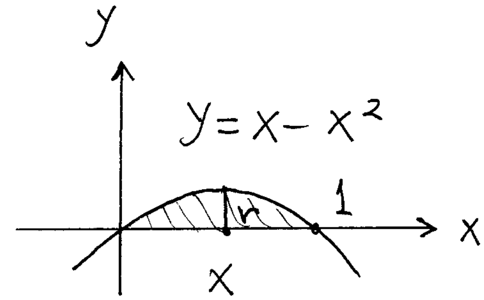
$y = \sqrt{9-x^2}$

$$\text{Volume} = \pi \int_{-3}^{3} (\text{radius})^2\,dx$$

$$= \pi \int_{-3}^{3} \left(\sqrt{9-x^2}\right)^2 dx$$

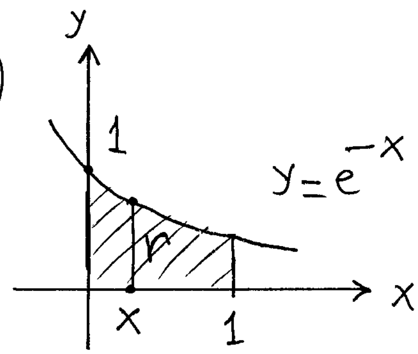


22.) $y = x - x^2 = x(1-x)$

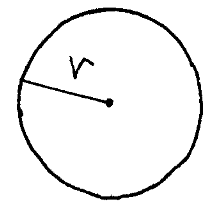


$$\text{Volume} = \pi \int_0^1 (\text{radius})^2\,dx$$
$$= \pi \int_0^1 (x - x^2)^2\,dx$$

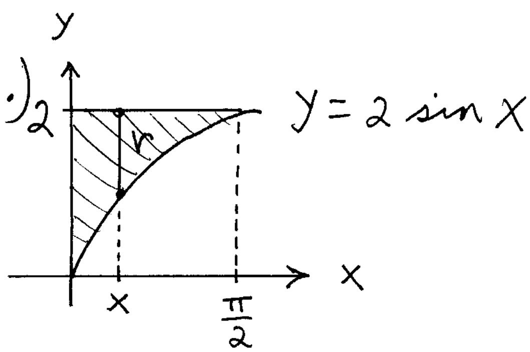
25.)

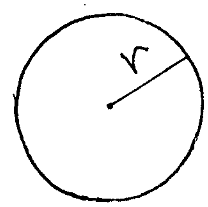


$$\text{Volume} = \pi \int_0^1 (\text{radius})^2\,dx$$
$$= \pi \int_0^1 (e^{-x})^2\,dx$$



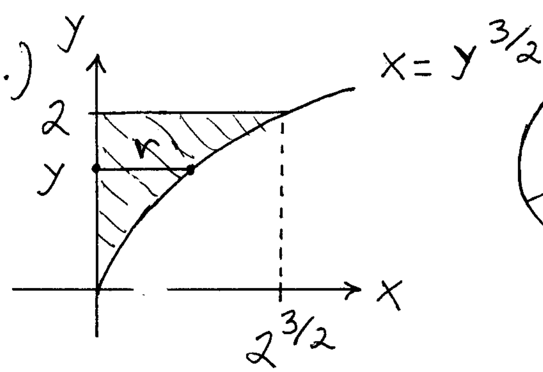
30.)

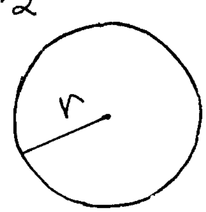




$$\text{Volume} = \pi \int_0^{\frac{\pi}{2}} (\text{radius})^2\,dx$$
$$= \pi \int_0^{\frac{\pi}{2}} (2 - 2\sin x)^2\,dx$$

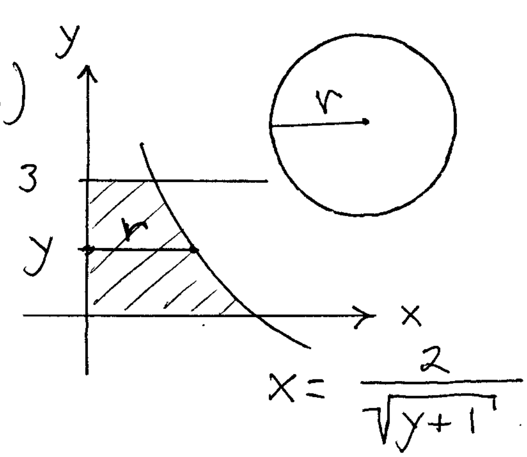
32.)





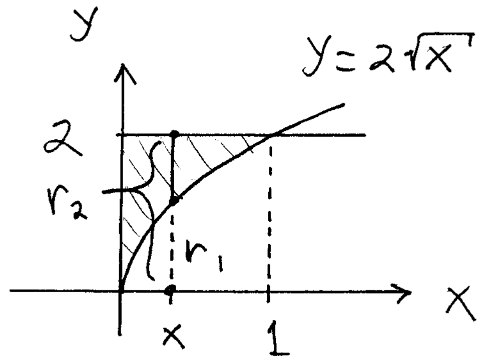
$$\text{Volume} = \pi \int_0^2 (\text{radius})^2\,dy$$
$$= \pi \int_0^2 (y^{3/2})^2\,dy$$

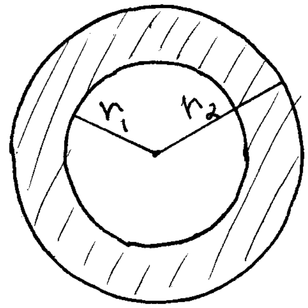
35.)



$$\text{Volume} = \pi \int_0^3 (\text{radius})^2\,dy$$
$$= \pi \int_0^3 \left(\frac{2}{\sqrt{y+1}}\right)^2\,dy$$

40.)

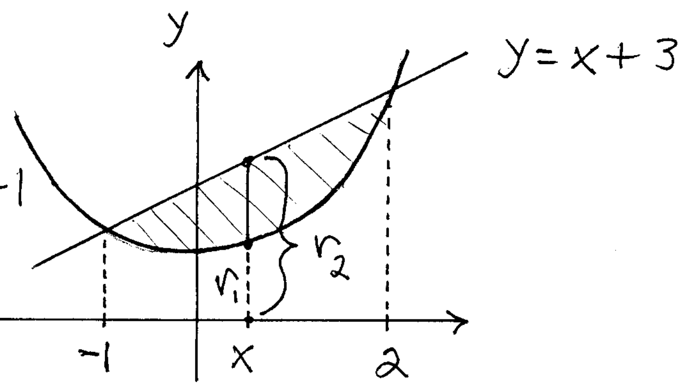




$$\text{Volume} = \pi \int_0^1 (r_2)^2\,dx - \pi \int_0^1 (r_1)^2\,dx$$

$$= \pi \int_0^1 (2)^2\,dx - \pi \int_0^1 (2\sqrt{x})^2\,dx$$

41.)



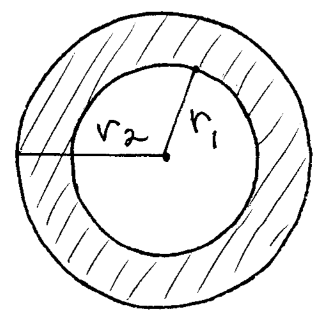
$x^2 + 1 = x + 3 \rightarrow$

$x^2 - x - 2 = 0 \rightarrow$

$(x-2)(x+1) = 0 \rightarrow$

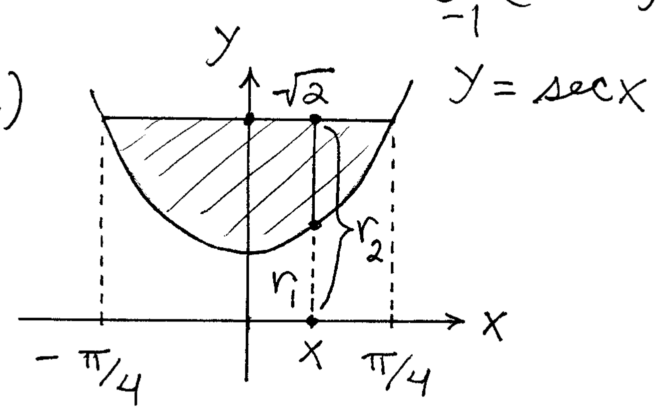
$x = 2, \; x = -1$

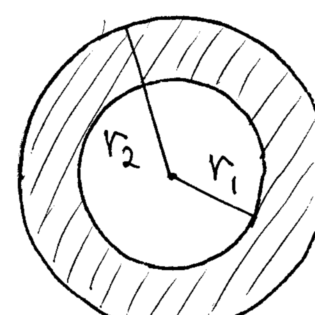
$$\text{Volume} = \pi \int_{-1}^{2} (r_2)^2\,dx - \pi \int_{-1}^{2} (r_1)^2\,dx$$



$$= \pi \int_{-1}^{2} (x+3)^2\,dx - \pi \int_{-1}^{2} (x^2+1)^2\,dx$$

43.)

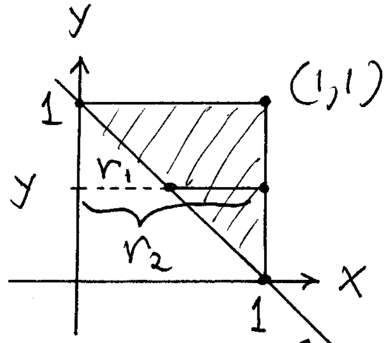




$$\text{Volume} = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (r_2)^2\,dx - \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (r_1)^2\,dx$$

$$= \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sqrt{2})^2 \, dx - \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sec x)^2 \, dx$$

46.)

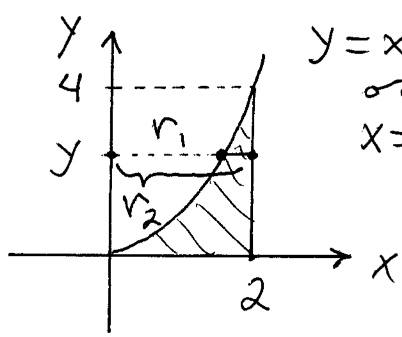


$y = 1 - x$ or
$x = 1 - y$

$$\text{Volume} = \pi \int_0^1 (r_2)^2 \, dy - \pi \int_0^1 (r_1)^2 \, dy$$

$$= \pi \int_0^1 (1)^2 \, dy - \pi \int_0^1 (1-y)^2 \, dy$$

47.)

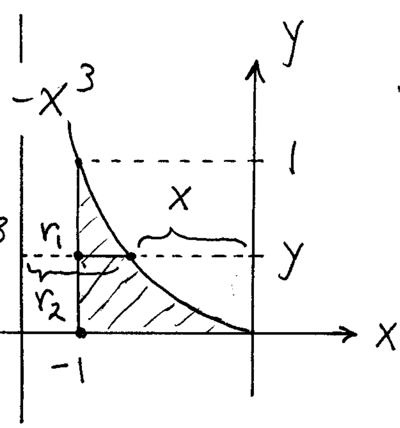


$y = x^2$
or
$x = \sqrt{y}$

$$\text{Volume} = \pi \int_0^4 (r_2)^2 \, dy - \pi \int_0^4 (r_1)^2 \, dy$$

$$= \pi \int_0^4 (2)^2 \, dy - \pi \int_0^4 (\sqrt{y})^2 \, dy$$

50.) $y = -x^3$
or
$x = -y^{1/3}$

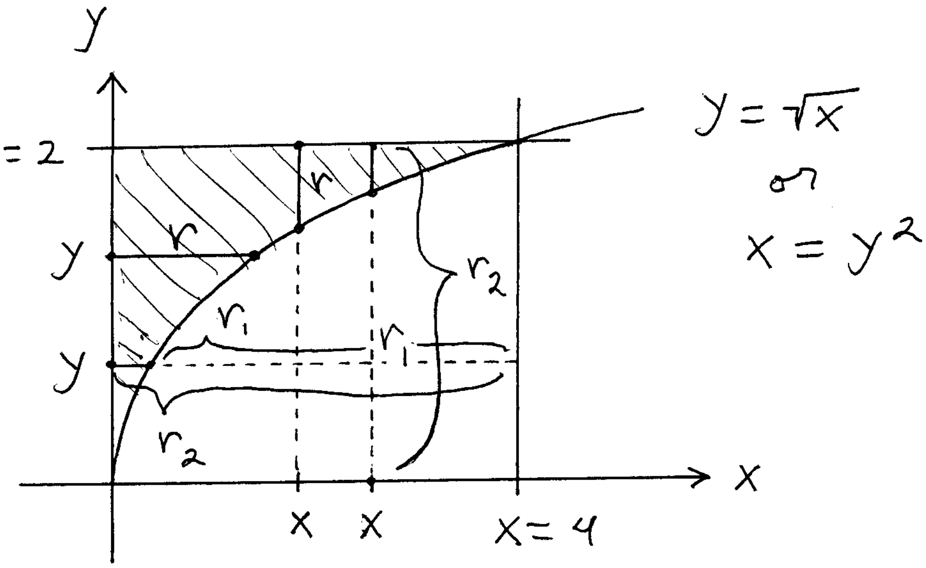


$x = -2$

$$\text{Volume} = \pi \int_0^1 (r_2)^2 \, dy - \pi \int_0^1 (r_1)^2 \, dy$$

$$= \pi \int_0^1 \left(-y^{1/3} - (-2)\right)^2 dy - \pi \int_0^1 (\,1\,)^2 \, dy$$

51.)



a.) Volume $= \pi \int_0^4 (r_2)^2\,dx - \pi \int_0^4 (r_1)^2\,dx$

$= \pi \int_0^4 (2)^2\,dx - \pi \int_0^4 (\sqrt{x})^2\,dx$

b.) Volume $= \pi \int_0^2 (\text{radius})^2\,dy$

$= \pi \int_0^2 (y^2)^2\,dy$

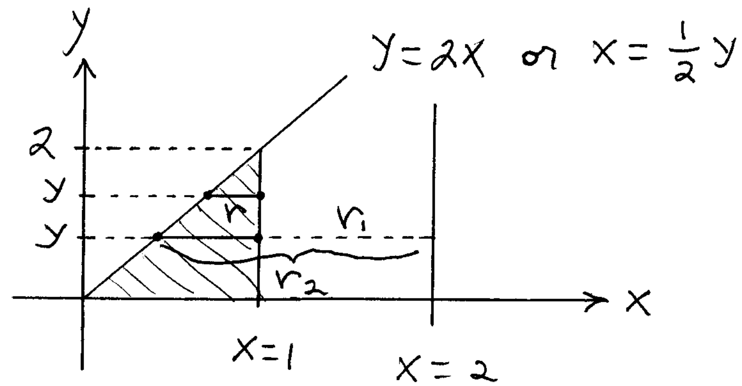
c.) Volume $= \pi \int_0^4 (\text{radius})^2\,dx$

$= \pi \int_0^4 (2-\sqrt{x})^2\,dx$

d.) Volume $= \pi \int_0^2 (r_2)^2\,dy - \pi \int_0^2 (r_1)^2\,dy$

$= \pi \int_0^2 (4)^2\,dy - \pi \int_0^2 (4-y^2)^2\,dy$

52.)



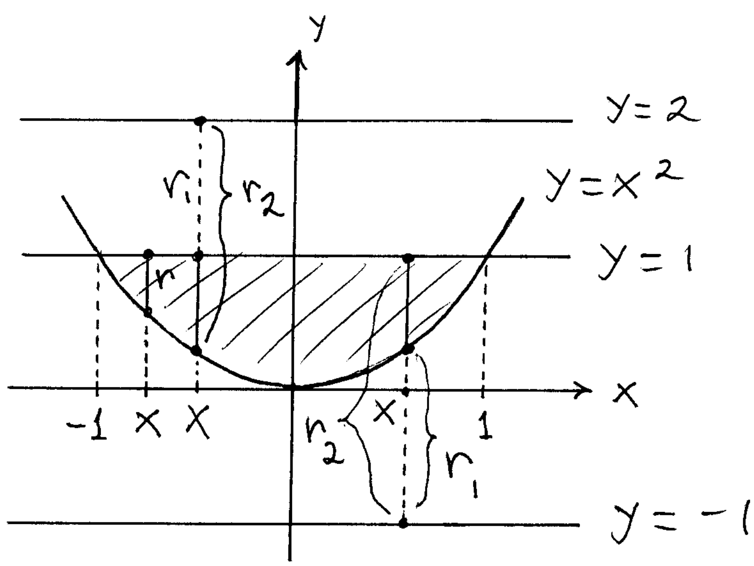
a.) Volume $= \pi \int_0^2 (\text{radius})^2 \, dy$

$= \pi \int_0^2 \left(1 - \frac{1}{2}y\right)^2 dy$

b.) Volume $= \pi \int_0^2 (r_2)^2 dy - \pi \int_0^2 (r_1)^2 dy$

$= \pi \int_0^2 \left(2 - \frac{1}{2}y\right)^2 dy - \pi \int_0^2 (1)^2 dy$

53.)



a.) Volume $= \pi \int_{-1}^1 (\text{radius})^2 \, dx$

$= \pi \int_{-1}^1 (1 - x^2)^2 dx$

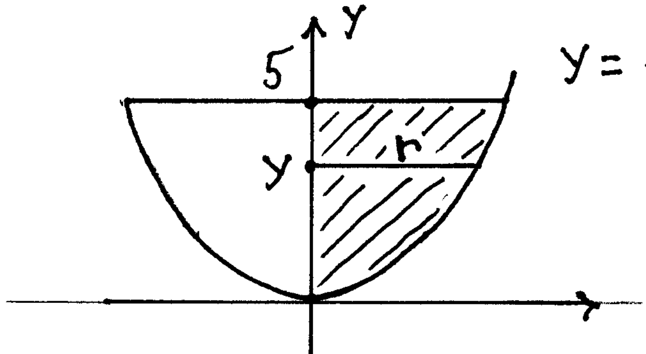
b.) Volume $= \pi \int_{-1}^1 (r_2)^2 dx - \pi \int_{-1}^1 (r_1)^2 dx$

$= \pi \int_{-1}^1 (2 - x^2)^2 dx - \pi \int_{-1}^1 (1)^2 dx$

c.) Volume $= \pi \int_{-1}^{1} (r_2)^2 dx - \pi \int_{-1}^{1} (r_1)^2 dx$

$= \pi \int_{-1}^{1} (2)^2 dx - \pi \int_{-1}^{1} (x^2+1)^2 dx$

56.)



$y = \frac{1}{2}x^2$ or

$x = \sqrt{2y}$

a.)

Volume $= \pi \int_0^5 (\text{radius})^2 dy$

$= \pi \int_0^5 (\sqrt{2y})^2 dy = \pi \int_0^5 2y\, dy = \pi y^2 \Big|_0^5 = 25\pi$

b.) assume depth of H_2O is h, then

Volume $= \pi \int_0^h 2y\, dy = \pi y^2 \Big|_0^h = \pi h^2 \longrightarrow$

$\boxed{V = \pi h^2}$; then $\frac{dV}{dt} = \pi \cdot 2h \cdot \frac{dh}{dt} \longrightarrow$

(Let $\frac{dV}{dt} = 3$, and find $\frac{dh}{dt}$ when $h = 4$.)

$3 = \pi \cdot 2(4) \cdot \frac{dh}{dt} \longrightarrow \frac{dh}{dt} = \frac{3}{8\pi}$ $\frac{\text{units}}{\text{sec.}}$

60.) Volume $= \pi \int_0^6 (\text{radius})^2 dx$

$= \pi \int_0^6 \left(\frac{x}{12}\sqrt{36-x^2}\right)^2 dx = \pi \int_0^6 \frac{x^2}{144}(36-x^2)\, dx$

$= \frac{\pi}{144} \int_0^6 (36x^2 - x^4)\, dx = \frac{\pi}{144}\left(12x^3 - \frac{1}{5}x^5\right)\Big|_0^6$

$$= \frac{\pi}{144}\left(12(6)^3 - \frac{1}{5}(6)^5\right) = 7.2\pi \text{ cm.}^3 ;$$

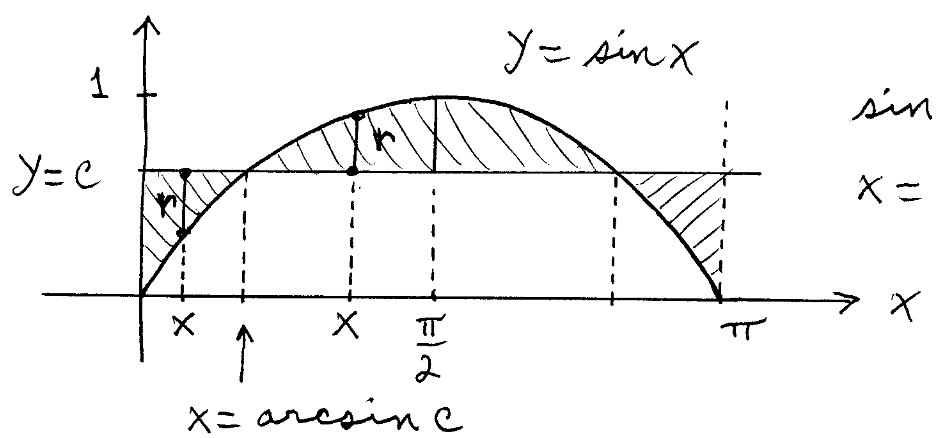
then total mass is

Mass = (density)(volume)

$$= \left(8.5 \frac{\text{g.}}{\text{cm.}^3}\right)\left(7.2\pi \text{ cm.}^3\right)$$

$$\approx 192.27 \text{ g.}$$

62.)



$\sin x = c \rightarrow$
$x = \arcsin c$

Use symmetry about $x = \frac{\pi}{2}$:

Volume

$$V(c) = 2\int_0^{\arcsin c} (c - \sin x)^2\, dx$$

$$+ \; 2\int_{\arcsin x}^{\frac{\pi}{2}} (\sin x - c)^2\, dx$$