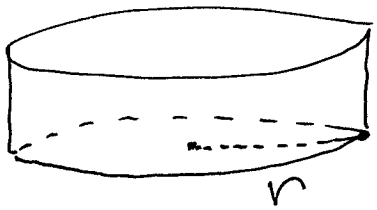
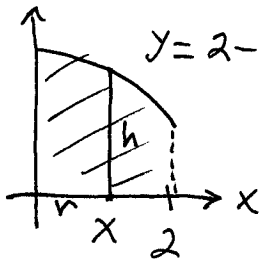
Section 6.2

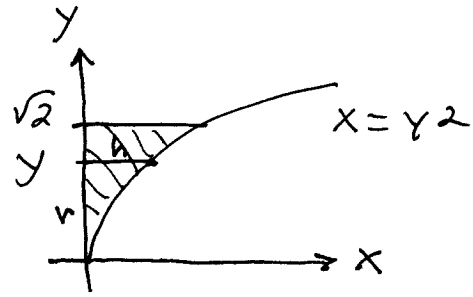


SHELL METHOD:

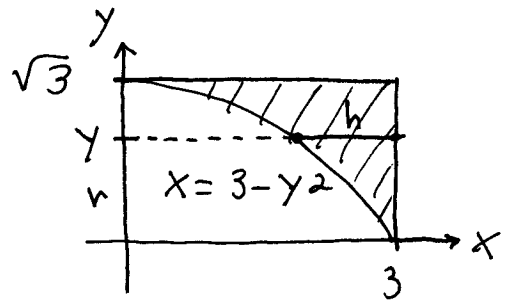
Volume $= 2\pi \int_a^b (\text{radius})(\text{height})\,dx$

2.) 

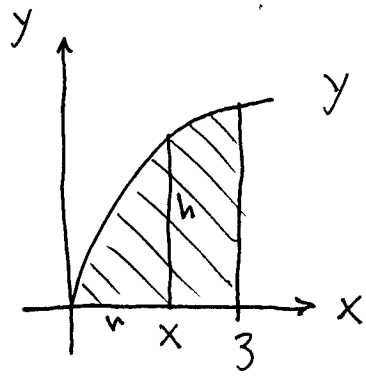
$\text{Vol} = 2\pi \int_0^2 (x)\cdot\left(2 - \frac{x^2}{4}\right) dx$



3.) $\text{Vol} = 2\pi \int_0^{\sqrt{2}} (y)(y^2)\,dy$

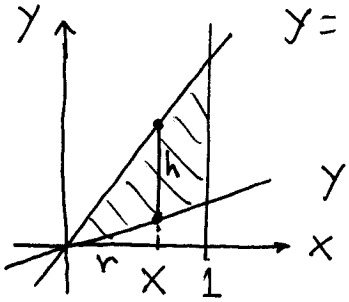


4.) $\text{Vol} = 2\pi \int_0^{\sqrt{3}} (y)(3 - (3 - y^2))\,dy$

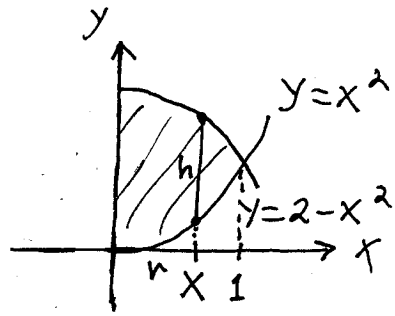


6.) $y = \dfrac{9x}{\sqrt{x^3+9}}$

$\text{Vol} = 2\pi \int_0^3 (x)\cdot \dfrac{9x}{\sqrt{x^3+9}}\,dx$

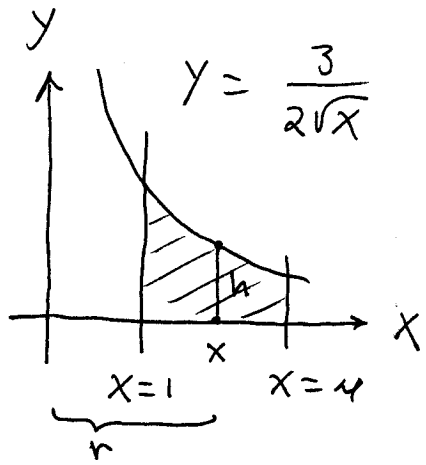


8.) $\text{Vol} = 2\pi \int_0^1 (x)\left(2x - \frac{1}{2}x\right) dx$



10.) $\text{Vol} = 2\pi \int_0^1 (x)\left((2 - x^2) - x^2\right) dx$

12.) $Vol = 2\pi \int_1^4 (x)\left(\frac{3}{2\sqrt{x}}\right) dx$



13.) $f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } 0 < x \le \pi \\ 1 & \text{if } x = 0 \end{cases}$

a.) $x \cdot f(x) = \begin{cases} x \cdot \frac{\sin x}{x} & \text{if } 0 < x \le \pi \\ x & \text{if } x = 0 \end{cases}$

$= \begin{cases} \sin x & \text{if } 0 < x \le \pi \\ 0 & \text{if } x = 0 \end{cases}$

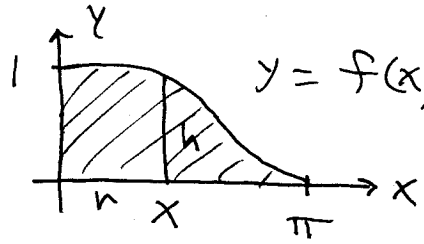
$= \sin x \quad \text{for } 0 \le x \le \pi.$

b.) $Vol = 2\pi \int_0^\pi x \cdot f(x)\, dx$

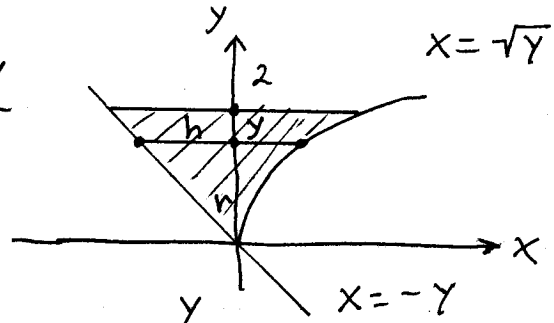
$= 2\pi \int_0^\pi \sin x\, dx$

$= -2\pi \cos x \Big|_0^\pi$

$= -2\pi(\cos\pi - \cos 0) = -2\pi(-1-1) = 4\pi$



15.) $Vol = 2\pi \int_0^2 (y)(\sqrt{y} - (-y))\, dy$

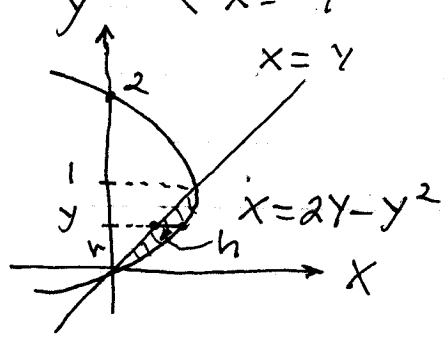


18.) $2y - y^2 = y \rightarrow$

$y - y^2 = 0 \rightarrow y(1-y) = 0$

$\rightarrow y = 0,\ y = 1$

$Vol = 2\pi \int_0^1 (y)((2y - y^2) - y)\, dy$

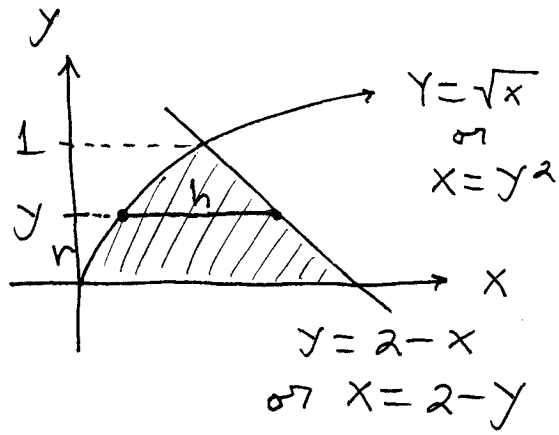


22.) $y^2 = 2-y \rightarrow$

$y^2 + y - 2 = 0 \rightarrow$

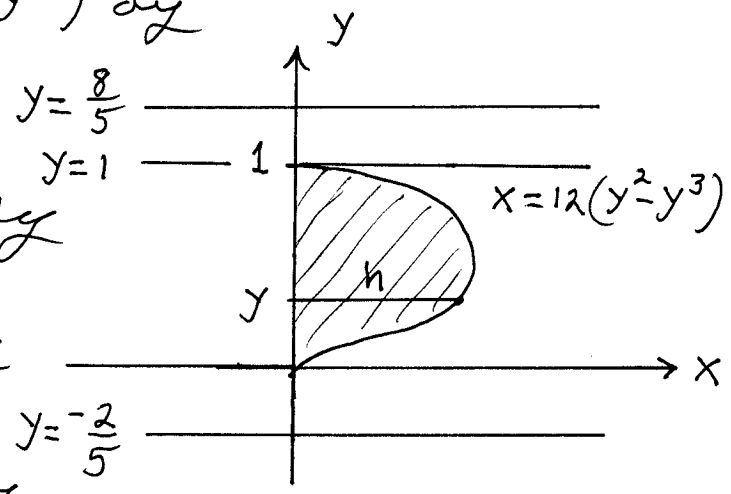
$(y-1)(y+2) = 0 \rightarrow$

$y = 1, \; y \neq -2$



$$Vol = 2\pi \int_0^1 (y)\cdot((2-y)-y^2)\,dy$$

27.)

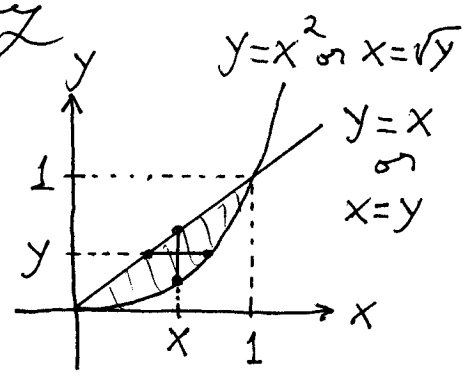


a.) $Vol = 2\pi \int_0^1 (y)\cdot 12(y^2-y^3)\,dy$

b.) $Vol = 2\pi \int_0^1 (1-y)\cdot 12(y^2-y^3)\,dy$

c.) $Vol = 2\pi \int_0^1 \left(\frac{8}{5}-y\right)\cdot 12(y^2-y^3)\,dy$

d.) $Vol = 2\pi \int_0^1 \left(y+\frac{2}{5}\right)\cdot 12(y^2-y^3)\,dy$



29.) a.) (about y-axis)

$Vol = 2\pi \int_0^1 x\cdot(x-x^2)\,dx$

(about x-axis)

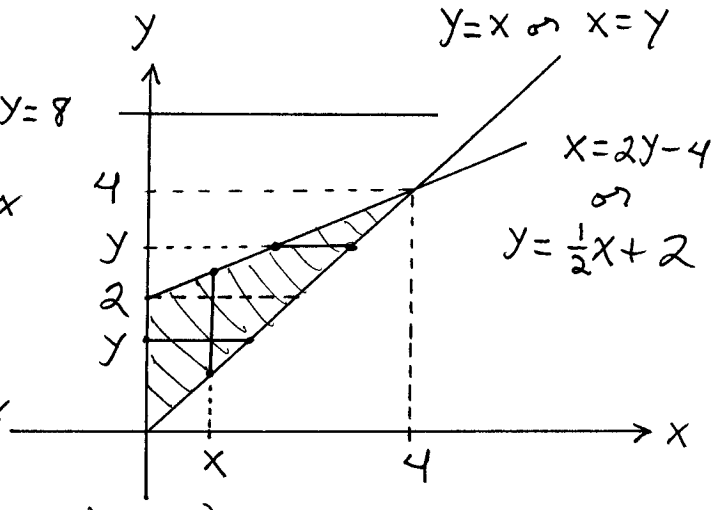
$Vol = 2\pi \int_0^1 y\,(\sqrt{y}-y)\,dy$

b.) (about x-axis)

$Vol = \pi \int_0^1 (x)^2\,dx - \pi \int_0^1 (x^2)^2\,dx$

(about y-axis)

$Vol = \pi \int_0^1 (\sqrt{y})^2\,dy - \pi \int_0^1 (y)^2\,dy$

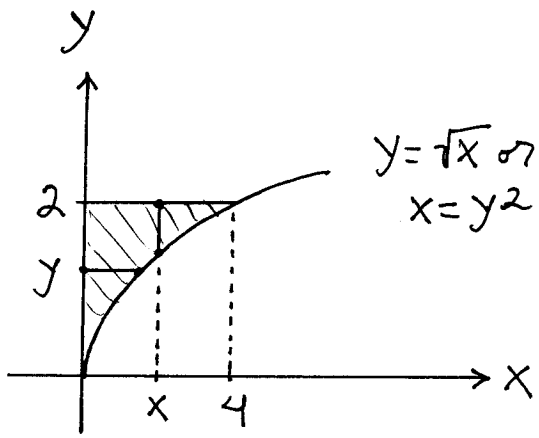


30.) a.) $Vol = \pi \int_0^4 \left(\frac{1}{2}x+2\right)^2 dx - \pi \int_0^4 (x)^2 dx$

b.) $Vol = 2\pi \int_0^4 x\left(\left(\frac{1}{2}x+2\right)-x\right) dx$

c.) $Vol = 2\pi \int_0^4 (4-x)\left(\left(\frac{1}{2}x+2\right)-x\right) dx$

d.) $Vol = \pi \int_0^4 (8-x)^2 dx - \pi \int_0^4 \left(8-\left(\frac{1}{2}x+2\right)\right)^2 dx$



32.) a.) (DISC)

$Vol = \pi \int_0^4 (2)^2 dx - \pi \int_0^4 (\sqrt{x})^2 dx$

(SHELL)

$Vol = 2\pi \int_0^2 y \cdot (y^2)\, dy$

b.) (DISC) $Vol = \pi \int_0^2 (y^2)^2 dy$

(SHELL) $Vol = 2\pi \int_0^4 x\,(2-\sqrt{x})\, dx$

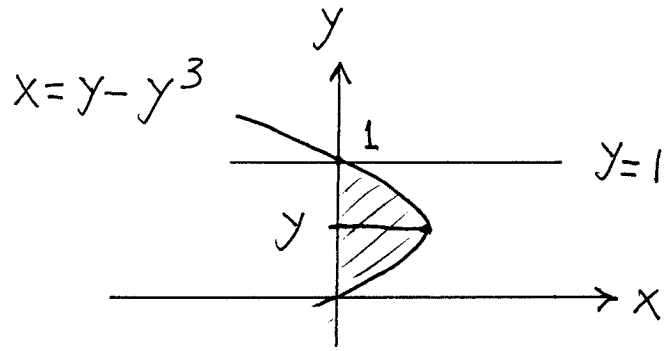
c.) (DISC) $Vol = \pi \int_0^2 (4)^2 dy - \pi \int_0^2 (4-y^2)^2 dy$

(SHELL) $Vol = 2\pi \int_0^4 (4-x)(2-\sqrt{x})\, dx$

d.) (DISC) $Vol = \pi \int_0^4 (2-\sqrt{x})^2 dx$

(SHELL) $Vol = 2\pi \int_0^2 (2-y)\cdot y^2\, dy$

33.) a.) (SHELL)



$$Vol = 2\pi \int_0^1 y(y - y^3)\, dy$$

b.) (SHELL)

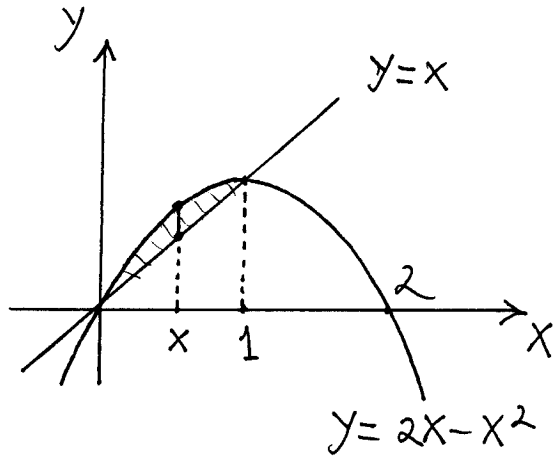
$$Vol = 2\pi \int_0^1 (1-y)(y - y^3)\, dy$$

36.) $2x - x^2 = x \rightarrow$

$x - x^2 = 0 \rightarrow$

$x(1-x) = 0 \rightarrow$

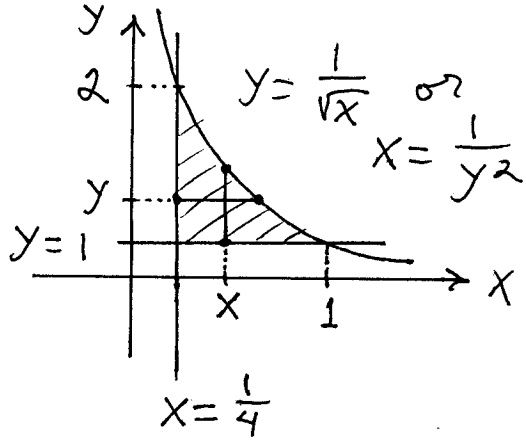
$x = 0,\ x = 1$



a.) $Vol = 2\pi \int_0^1 x((2x - x^2) - x)\, dx$

b.) $Vol = 2\pi \int_0^1 (1-x)((2x - x^2) - x)\, dx$

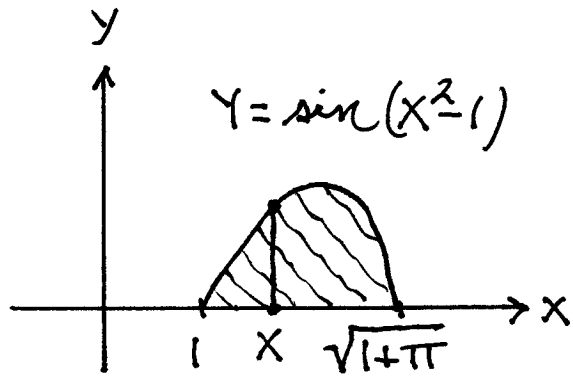
38.) a.)



$$Vol = \pi \int_1^2 \left(\frac{1}{y^2}\right)^2 dy - \pi \int_1^2 \left(\frac{1}{4}\right)^2 dy$$

b.) $Vol = 2\pi \int_{\frac{1}{4}}^1 x \cdot \left(\frac{1}{\sqrt{x}} - 1\right) dx$

42.) shell method around y-axis:

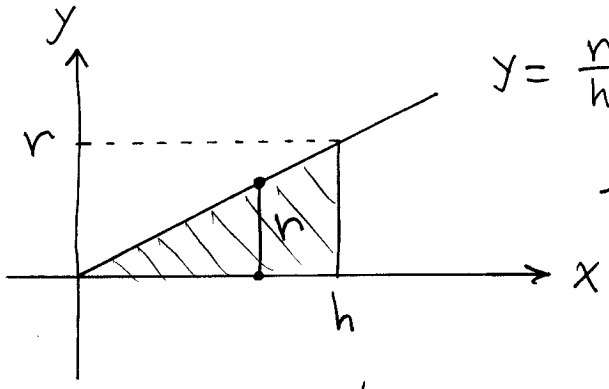


$$Vol = 2\pi \int_1^{\sqrt{1+\pi}} x \cdot \sin(x^2-1)\,dx$$

$$= 2\pi \cdot \frac{-1}{2} \cos(x^2-1) \Big|_1^{\sqrt{1+\pi}} = -\pi \cos\pi - -\pi \cos 0$$

$$= -\pi(-1) + \pi(1) = 2\pi$$

43.)



$y = \frac{r}{h}x$; rotate region about x-axis to form cylinder

$$\text{Volume} = \pi \int_0^h (\text{radius})^2\,dx$$

$$= \pi \int_0^h \left(\frac{r}{h}x\right)^2 dx$$

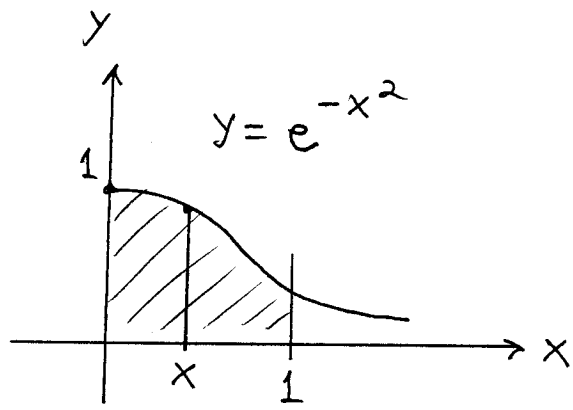
$$= \pi \cdot \frac{r^2}{h^2} \int_0^h x^2\,dx$$

$$= \pi \cdot \frac{r^2}{h^2} \cdot \frac{x^3}{3} \Big|_0^h$$

$$= \pi \cdot \frac{r^2}{h^2} \cdot \frac{h^3}{3}$$

$$= \frac{1}{3}\pi r^2 h$$

47.) (SHELL)



$$\text{Vol} = 2\pi \int_0^1 x \cdot e^{-x^2}\, dx$$

$$= 2\pi \cdot \frac{-1}{2} e^{-x^2} \Big|_0^1$$

$$= -\pi \left(e^{-1} - e^{0}\right)$$

$$= \pi \left(1 - \frac{1}{e}\right)$$