Section 6.3

1.) $\text{Arc} = \int_0^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^3 \sqrt{1+\left(\frac{1}{3}\cdot\frac{3}{2}(x^2+2)^{1/2}\cdot 2x\right)^2}\,dx$

$= \int_0^3 \sqrt{1+(x^2+2)x^2}\,dx = \int_0^3 \sqrt{x^4+2x^2+1}\,dx$

$= \int_0^3 \sqrt{(x^2+1)^2}\,dx = \int_0^3 (x^2+1)\,dx$

$= \left(\frac{x^3}{3}+x\right)\Big|_0^3 = 9+3 = 12$

2.) $\text{Arc} = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^4 \sqrt{1+\left(\frac{3}{2}x^{1/2}\right)^2}\,dx$

$= \int_0^4 \sqrt{1+\frac{9}{4}x}\,dx = \frac{4}{9}\,\frac{\left(1+\frac{9}{4}x\right)^{3/2}}{3/2}\Big|_0^4$

$= \frac{8}{27}\left(10^{3/2}-1^{3/2}\right) = \frac{8}{27}\left(10^{3/2}-1\right)$

3.) $\text{Arc} = \int_1^3 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

$= \int_1^3 \sqrt{1+\left(y^2-\frac{1}{4y^2}\right)^2}\,dy$

$= \int_1^3 \sqrt{1+y^4-\frac{1}{2}+\frac{1}{16y^4}}\,dy$

$= \int_1^3 \sqrt{y^4+\frac{1}{2}+\frac{1}{16y^4}}\,dy$

$= \int_1^3 \sqrt{\frac{16y^8}{16y^4}+\frac{8y^4}{16y^4}+\frac{1}{16y^4}}\,dy$

$= \int_1^3 \sqrt{\frac{16y^8+8y^4+1}{16y^4}}\,dy$

$$= \int_1^3 \frac{\sqrt{(4y^4+1)^2}}{4y^2}\, dy$$

$$= \int_1^3 \frac{4y^4+1}{4y^2}\, dy$$

$$= \int_1^3 \left(y^2 + \frac{1}{4}y^{-2}\right) dy$$

$$= \left(\frac{y^3}{3} - \frac{1}{4y}\right)\Big|_1^3$$

$$= \left(9 - \frac{1}{12}\right) - \left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= 9 - \frac{1}{6} = \frac{53}{6}$$

4.) $\text{Arc} = \int_1^9 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

$$= \int_1^9 \sqrt{1+\left(\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{2}y^{1/2} - \frac{1}{2}y^{-1/2}\right)^2}\, dy$$

$$= \int_1^9 \sqrt{1 + \frac{y}{4} - \frac{1}{2} + \frac{1}{4y}}\, dy$$

$$= \int_1^9 \sqrt{\frac{y}{4} + \frac{1}{2} + \frac{1}{4y}}\, dy$$

$$= \int_1^9 \sqrt{\frac{y^2}{4y} + \frac{2y}{4y} + \frac{1}{4y}}\, dy$$

$$= \int_1^9 \sqrt{\frac{y^2+2y+1}{4y}}\, dy$$

$$= \int_1^9 \frac{\sqrt{(y+1)^2}}{2\sqrt{y}}\, dy$$

$$= \int_1^9 \frac{y+1}{2\sqrt{y}}\, dy = \int_1^9 \left(\frac{1}{2}y^{1/2} + \frac{1}{2}y^{-1/2}\right) dy$$

$$= \left(\frac{1}{\cancel{2}} \frac{y^{3/2}}{3/\cancel{2}} + \frac{1}{\cancel{2}} \cdot \frac{y^{1/2}}{\cancel{1/2}} \right) \Big|_1^9$$

$$= \left(\frac{1}{3} 9^{3/2} + 3 \right) - \left(\frac{1}{3} + 1 \right)$$

$$= 12 - \frac{4}{3} \quad = \quad \frac{32}{3}$$

8.) $\text{Arc} = \int_0^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

$$= \int_0^2 \sqrt{1 + \left(x^2 + 2x + 1 + {}^-4(4x+4)^{-2}\right)^2} \, dx$$

$$= \int_0^2 \sqrt{1 + \left[\frac{(x+1)^2}{1} - \frac{\cancel{4}\,1}{4^{\cancel{2}}(x+1)^2} \right]^2} \, dx$$

$$= \int_0^2 \sqrt{1 + (x+1)^4 - \frac{1}{2} + \frac{1}{16(x+1)^4}} \, dx$$

$$= \int_0^2 \sqrt{(x+1)^4 + \frac{1}{2} + \frac{1}{16(x+1)^4}} \, dx$$

$$= \int_0^2 \sqrt{\left((x+1)^2 + \frac{1}{4(x+1)^2} \right)^2} \, dx$$

$$= \int_0^2 \left((x+1)^2 + \frac{1}{4(x+1)^2} \right) dx$$

$$= \left(\frac{1}{3}(x+1)^3 + \frac{-1}{4(x+1)} \right) \Big|_0^2$$

$$= \left(9 - \frac{1}{12} \right) - \left(\frac{1}{3} - \frac{1}{4} \right)$$

$$= \frac{108}{12} - \frac{1}{12} - \frac{4}{12} + \frac{3}{12} = \frac{106}{12} = \frac{53}{6}$$

19.) a.) $L = \int_1^4 \sqrt{1+\frac{1}{4x}}\,dx = \int_1^4 \sqrt{1+(f'(x))^2}\,dx$

$\rightarrow (f'(x))^2 = \frac{1}{4x} \rightarrow f'(x) = \sqrt{\frac{1}{4x}} \rightarrow$

$f'(x) = \frac{1}{2\sqrt{x}} \rightarrow f(x) = \sqrt{x} + c$

and $x=1,\ y=1 \rightarrow 1 = \sqrt{1} + c \rightarrow c = 0 \rightarrow$

$f(x) = \sqrt{x}$.

20.) a.) $L = \int_1^2 \sqrt{1+\frac{1}{y^4}}\,dy = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

$\rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{y^4} \rightarrow \frac{dx}{dy} = \frac{1}{y^2} \rightarrow$

$x = \frac{-1}{y} + c$ and $x=0,\ y=1 \rightarrow$

$0 = -1 + c \rightarrow c = 1 \rightarrow x = \frac{-1}{y} + 1$.

21.) $y = \int_0^x \sqrt{\cos 2t}\,dt$ for $0 \le x \le \frac{\pi}{4} \xrightarrow{D}$

$y' = \sqrt{\cos 2x} \rightarrow Arc = \int_0^{\frac{\pi}{4}} \sqrt{1+(y')^2}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{1+(\sqrt{\cos 2x})^2}\,dx = \int_0^{\frac{\pi}{4}} \sqrt{1+\cos 2x}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{1+(2\cos^2 x - 1)}\,dx = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 x}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{2}\cdot|\cos x|\,dx = \sqrt{2}\int_0^{\frac{\pi}{4}} \cos x\,dx$

$= \sqrt{2}\sin x\Big|_0^{\frac{\pi}{4}} = \sqrt{2}\cdot\sin\frac{\pi}{4} - \sqrt{2}\cdot\cancelto{0}{\sin 0}$

$= \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \frac{2}{2} = 1$

22.) $Y = (1-X^{2/3})^{3/2}$ for $\frac{\sqrt{2}}{4} \le X \le 1 \xrightarrow{D}$

$$Y' = \frac{3}{2}(1-X^{2/3})^{1/2} \cdot -\frac{2}{3}X^{-1/3} = \frac{-(1-X^{2/3})^{1/2}}{X^{1/3}} \longrightarrow$$

$$\text{Arc} = \int_{\frac{\sqrt{2}}{4}}^{1} \sqrt{1+(Y')^2}\,dx = \int_{\frac{\sqrt{2}}{4}}^{1} \sqrt{1+\left[\frac{-(1-X^{2/3})^{1/2}}{X^{1/3}}\right]^2}\,dx$$

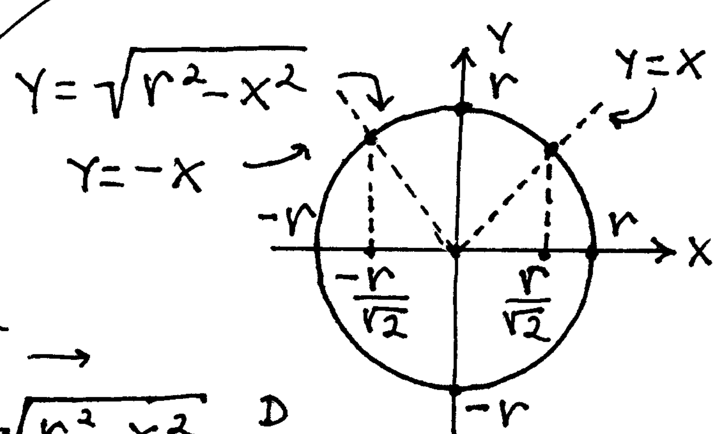
$$= \int_{\frac{\sqrt{2}}{4}}^{1} \sqrt{1+\frac{1-X^{2/3}}{X^{2/3}}}\,dx = \int_{\frac{\sqrt{2}}{4}}^{1} \sqrt{\frac{X^{2/3}}{X^{2/3}}+\frac{1-X^{2/3}}{X^{2/3}}}\,dx$$

$$= \int_{\frac{\sqrt{2}}{4}}^{1} \sqrt{\frac{1}{X^{2/3}}}\,dx = \int_{\frac{\sqrt{2}}{4}}^{1} \frac{1}{X^{1/3}}\,dx = \int_{\frac{\sqrt{2}}{4}}^{1} X^{-1/3}\,dx$$

$$= \frac{3}{2}X^{2/3}\Big|_{\frac{\sqrt{2}}{4}}^{1} = \frac{3}{2}(1) - \frac{3}{2}\left(\frac{\sqrt{2}}{4}\right)^{2\cdot\frac{1}{3}}$$

$$= \frac{3}{2} - \frac{3}{2}\cdot\left(\frac{1}{8}\right)^{1/3} = \frac{3}{2} - \frac{3}{2}\cdot\frac{1}{2}$$

$$= \frac{6}{4} - \frac{3}{4} = \frac{3}{4}$$, then total length is $8\left(\frac{3}{4}\right) = 6$



24.) $X = \sqrt{r^2 - X^2} \longrightarrow$

$X^2 = r^2 - X^2 \longrightarrow$

$2X^2 = r^2 \longrightarrow X^2 = \frac{r^2}{2} \longrightarrow$

$X = \frac{\pm r}{\sqrt{2}}$; then $Y = \sqrt{r^2 - X^2} \xrightarrow{D}$

$Y' = \frac{1}{2}(r^2 - X^2)^{-1/2} \cdot -2X$, so that

$$\text{Arc} = 4\int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \sqrt{1+(y')^2}\,dx = 4\int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}\,dx$$

$$= 4\int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \sqrt{1+\frac{x^2}{r^2-x^2}}\,dx = 4\int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \sqrt{\frac{r^2\cancel{-x^2}\cancel{+x^2}}{r^2-x^2}}\,dx$$

$$= 4\int_{-\frac{r}{\sqrt{2}}}^{\frac{r}{\sqrt{2}}} \frac{r}{\sqrt{r^2-x^2}}\,dx$$

31.) $f(x) = 2x^{3/2} \xrightarrow{D} f'(x) = 2\cdot\frac{3}{2}x^{1/2} = 3x^{1/2}$, ~~then~~

$$\text{Arc} = \int_0^1 \sqrt{1+(f'(x))^2}\,dx =$$

$$= \int_0^1 \sqrt{1+(3x^{1/2})^2}\,dx$$

$$= \int_0^1 \sqrt{1+9x}\,dx$$

$$= \frac{1}{9}\cdot\frac{2}{3}(1+9x)^{3/2}\Big|_0^1$$

$$= \frac{2}{27}(10)^{3/2} - \frac{2}{27}$$