

Math 21C  
Vogler  
Discussion Sheet 1

1.) Compute the first five terms (starting with  $n=1$ ) of each sequence. Determine whether each sequence converges or diverges.

- a.)  $\{3\}$    b.)  $\{3^n\}$    c.)  $\left\{\frac{3}{n}\right\}$    d.)  $\left\{\left(\frac{1}{3}\right)^n\right\}$   
e.)  $\{3^{1/n}\}$    f.)  $\left\{\frac{n+5}{n+2}\right\}$    g.)  $\{n(3-n)\}$    h.)  $\left\{\frac{n^3+n^2-n+7}{4n^3+5n^2-2}\right\}$   
i.)  $\{(0.9999)^n\}$    j.)  $\{(1.00001)^n\}$    k.)  $\left\{\left(\frac{-2}{3}\right)^n\right\}$    l.)  $14/3, 15/5, 16/7, 17/9, \dots$   
m.)  $\left\{\frac{\sin 3n}{n}\right\}$    n.)  $\left\{\left(\frac{\sqrt{7}}{\ln 14}\right)^n\right\}$    o.)  $\{\cos(2n\pi)\}$    p.)  $\{(1+1/n)^n\}$   
q.)  $\left\{\frac{3^n}{n!}\right\}$    r.)  $\{\sin(\pi/2 + n\pi)\}$    s.)  $\left\{\frac{1000^n}{n!}\right\}$    t.)  $\left\{\frac{n^2}{e^n}\right\}$   
u.)  $\{(n-1)(n-2)(n-3)(n-4)(n-5)\}$    v.)  $\{3+(-1)^n\}$    w.)  $\left\{\sum_{i=1}^n i^2\right\}$   
x.)  $\left\{\sum_{i=1}^n (2/3)^{i-1}\right\}$    y.)  $\left\{\sum_{i=1}^n (1+i/n)^3(1/n)\right\}$

2.) Determine whether the following series converge or diverge.

- a.)  $\sum_{n=0}^{\infty} 2^n$    b.)  $\sum_{n=1}^{\infty} 2$    c.)  $\sum_{n=2}^{\infty} \frac{1}{2^n}$    d.)  $\sum_{n=1}^{\infty} 0.000001$   
e.)  $\sum_{n=1}^{\infty} (0.98)^{n+3}$    f.)  $\sum_{n=1}^{\infty} 5(-2/3)^{n-1}$    g.)  $\sum_{n=0}^{\infty} (-1)^n$    h.)  $\sum_{n=0}^{\infty} (1/4)(3/2)^{n+3}$   
i.)  $\sum_{n=1}^{\infty} \frac{n+2}{n+1000}$    j.)  $\sum_{n=2}^{\infty} (1-1/n)^n$    k.)  $\sum_{n=1}^{\infty} \cos n\pi$   
l.)  $\sum_{n=7}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)}\right)$    m.)  $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$    HINT: Use partial fractions first.

3.) Compute the exact value of the following convergent series :

$$6 - 2 + \frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \frac{2}{81} + \dots$$

4.) Find the 25th term in the following sequence :   1, 1, 2, 3, 5, 8, 13, 21, 34, ...

5.) Find the sum of the first 200 terms in the following sequence :   3, 7, 11, 15, 19, 23, ...

4.) A ball bearing is dropped from a building 200 feet high. Each time the ball bearing

rebounds to 40% of its falling distance. What is the total distance the ball bearing will travel ?

5.) Determine the limit of the following sequence :

$$2, \quad 2 - \frac{1}{2}, \quad 2 - \frac{1}{2 - \frac{1}{2}}, \quad 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}, \quad 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}} \quad \dots$$

6.) Start at the origin and move 10 units along the positive y-axis. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Continue this process. At what point (x,y) will you "end" ?

"The mind is not a vessel to be filled, but a fire to be ignited." – Plutarch