1.) Use what you know about converging geometric series to write each power series as an ordinary function.

\[ \sum_{n=2}^{\infty} \frac{x^n}{3^n} \quad \text{b.) } \sum_{n=0}^{\infty} \frac{2^{n-1}(x + 3)^{n+1}}{5^n} \]

\[ x^2 - x^{5/2} + x^3 - x^{7/2} + x^4 - x^{9/2} + \ldots \quad \text{d.) } \sum_{n=0}^{\infty} (n + 1)x^n \]

2.) Recall that if \( y = f(x) \) is a function and

\[ a_0 + a_1(x - a) + a_2(x - a)^2 + a_3(x - a)^3 + a_4(x - a)^4 + \ldots = \sum_{n=0}^{\infty} a_n(x - a)^n \]

is the Taylor Series (or Maclaurin series if \( a = 0 \)) centered at \( x = a \) for \( y = f(x) \), then \( a_n = \frac{f^{(n)}(a)}{n!} \). Use this formula to compute the first four nonzero terms and the general formula for the Taylor series expansion for each function about the given value of \( a \).

\[ \text{a.) } f(x) = e^x \text{ centered at } x = 0 \quad \text{b.) } f(x) = e^x \text{ centered at } x = \ln 2 \]

\[ \text{c.) } f(x) = \frac{1}{1 - x} \text{ centered at } x = 0 \quad \text{d.) } f(x) = \sin x \text{ centered at } x = 0 \]

\[ \text{e.) } f(x) = \frac{1}{x} \text{ centered at } x = 1 \quad \text{f.) } f(x) = \sqrt{x + 5} \text{ centered at } x = -1 \]

3.) Use the suggested method to find the first four nonzero terms of the Maclaurin series for each function.

\[ \text{a.) } f(x) = \frac{1}{1 + x^2} \quad \text{(Substitute } -x^2 \text{ into the Maclaurin series for } \frac{1}{1 - x}. \text{)} \]

\[ \text{b.) } f(x) = x^3e^{-3x} \quad \text{(Substitute } -3x \text{ into the Maclaurin series for } e^x \text{ and then multiply by } x^3 \text{.)} \]

\[ \text{c.) } f(x) = \frac{e^x}{1 - x} = e^x \frac{1}{1 - x} \quad \text{(Multiply the Maclaurin series for } e^x \text{ and } \frac{1}{1 - x} \text{ term by term and then group like powers of } x \text{.)} \]

\[ \text{d.) } f(x) = \frac{e^x}{1 - x} \quad \text{(Use polynomial division. Divide the Maclaurin series for } e^x \text{ by } 1 - x \text{.)} \]

\[ \text{e.) } f(x) = 3x^2 \cos(x^3) \quad \text{(Substitute } x^3 \text{ into the Maclaurin series for } \sin x \text{ then differentiate term by term.)} \]

\[ \text{f.) } f(x) = \arctan x \quad \text{(Integrate the Maclaurin series for } \frac{1}{1 + t^2} \text{ from 0 to } x \text{.)} \]
4.) The Maclaurin series for \( f(x) = \frac{1}{1+x} \) is \( 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \)

a.) Show that \( f(x) = \frac{1}{1+x} \) and \( 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \) have the same value at \( x = 0 \).

b.) Show that \( f(x) = \frac{1}{1+x} \) and \( 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \) have the same value at \( x = 1/2 \).

c.) Show that \( f(x) = \frac{1}{1+x} \) and \( 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \) do not have the same value at \( x = 1 \).

d.) For what x-values is \( f(x) = \frac{1}{1+x} \) defined?

e.) For what x-values is the Maclaurin series \( 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \) defined?

NOTE: It can be shown that \( f(x) = \frac{1}{1+x} \) and its Maclaurin series \( 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \) are equal on the interval \((-1,1)\).

5.) Determine (Use shortcuts.) the third-degree Taylor polynomial, \( P_3(x;0) \), for the function \( f(x) = \frac{x}{1+x} \). Use \( \int_0^1 P_3(x;0) \, dx \) to estimate the value of \( \int_0^1 \frac{x}{1+x} \, dx \). Now evaluate \( \int_0^1 \frac{x}{1+x} \, dx \) directly to see how good the estimate is.

6.) The following definite integral cannot be evaluated using the Fundamental Theorem of Calculus. Use the Maclaurin series for \( \cos x \) and the absolute error \( |R_n| \) for an alternating series to estimate the value of this integral with error at most 0.0001: \( \int_0^1 \cos(x^2) \, dx \)

7.) Write each Maclaurin series as an ordinary function.

a.) \( (3x) - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \frac{(3x)^7}{7!} + \frac{(3x)^9}{9!} - \cdots \) (HINT: Use \( \sin x \)).

b.) \( x^2 - x^3 + x^4 - x^5 + x^6 - \cdots \) (HINT: Factor.)

c.) \( \frac{1}{2!} + \frac{x}{3!} + \frac{x^2}{4!} + \frac{x^3}{5!} + \frac{x^4}{6!} + \cdots \) (HINT: Use \( e^x \)).

d.) \( x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \cdots \) (Challenging)

"In mathematics you don’t understand things. You just get used to them.” – Johann von Neumann