Math 21C
Vogler
Discussion Sheet 8

1.) Compute $z_x$ and $z_y$ for each of the following functions.

a.) $z = xy^2 + \ln x + e^y + 5$ b.) $z = xe^{2y} \arctan x$ c.) $z = \sqrt{x - y^2}$

d.) $z = \dfrac{x^3}{y^2} + \sin(xy)$ e.) $z = \dfrac{x+4}{x^2+y^2}$ f.) $z = \{e^{x^2 y} + \tan(3y+4x)\}^5$

f.) $z = y^{1+x^3}$

2.) Show that $z = \ln(1 + x^2 + y^2)$ satisfies the equation $z_{xy} + z_x z_y = 0$ .

3.) Verify that $w_{xy} = w_{yx}$ for $w = y + \dfrac{x}{y}$ .

4.) Determine functions $z$ whose partial derivatives are given, or state that this is impossible.

a.) $z_x = 2x$ and $z_y = 3y^2 + 1$ b.) $z_x = xy^2 - y$ and $z_y = x^2 y - x$
c.) $z_x = e^x y - 1$ and $z_y = e^x - x$
d.) $z_x = ye^x \cos(xy) + e^x \sin(xy) - 2$ and $z_y = xe^x \cos(xy) + 1$

5.) Consider the function $f(x,y) = \begin{cases} \dfrac{\sin(x^3+y)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0 \quad , & \text{if } (x,y) = (0,0) \ . \end{cases}$

a.) Determine $f_x(x,y)$ when $(x,y) \neq (0,0)$ .
b.) Determine $f_x(0,0)$ (Use limit definition of partial derivative.).
c.) Determine $f_y(0,0)$ (Use limit definition of partial derivative.).

6.) Plane A, parallel to the $xz$-plane, and plane B, parallel to the $yz$-plane, pass through the surface determined by the equation $z = xy^2 - x^3 + 7$ . Both planes include the point $(1, 0, 6)$ , which lies on the surface.

a.) Determine the slope of the line tangent to the surface at the point $(1, 0, 6)$ if the line lies in

i.) plane A.
ii.) plane B.

b.) Determine an equation of the plane tangent to the surface at the point $(1, 0, 6)$ .

7.) Compute $z_x$ and $z_y$ for each of the following functions.

a.) $z = x^3 y + y^4 - 2x + 5$ b.) $z = f(x) + g(y)$ c.) $z = f(x^3) + g(4y)$
d.) $z = f(x^2 + y^3) + g(xy^2)$ e.) $y^2 + z^2 + \sin(xz) = 4$
f.) $z = f(u, v)$ where $u = \ln(x - y)$ and $v = e^{xy}$

8.) Find $\dfrac{\partial w}{\partial t}$ and $\dfrac{\partial w}{\partial s}$ if $w = f(4t^2 - 3s)$ and $f'(x) = \ln x$ .

9.) Assume that $f$ is differentiable function of one variable with $z = xf(xy)$. Show that $xz_x - yz_y = z$ .

10.) Assume that $f$ and $g$ are twice differentiable functions of one variable. Show that $u = f(x+at) + g(x-at)$ satisfies $a^2 \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}$ , where $a$ is a constant.

11.) Consider the paraboloid given by $f(x,y) = 25 - x^2 - y^2$.

a.) Sketch the surface.

b.) Let point $P = (2,-2)$. Compute the derivative of the function $f$ at the point $P$ in the direction

i.) $\vec{A} = \overrightarrow{(-3,4)}$

ii.) $\vec{A} = \overrightarrow{(3,-4)}$

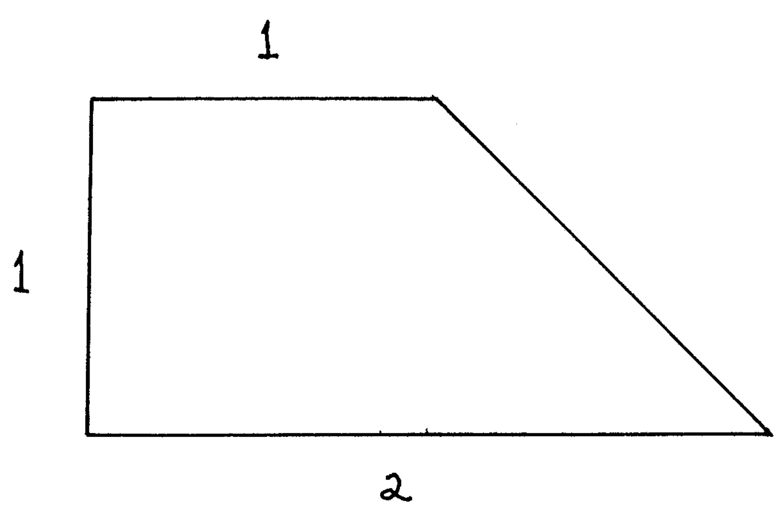
iii.) $\vec{A} = \overrightarrow{(1,0)}$

iv.) $\vec{A} = \overrightarrow{(0,-1)}$

c.) In what directions is the derivative of $f$ at point $P = (2,-2)$ equal to zero ?

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

12.) Divide the given figure into four equal parts, each one of the same size and shape.



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