

Math 21C  
Vogler  
The Comparison Tests and Limit Comparison Tests

COMPARISON TESTS: Assume that sequences  $a_n$  and  $b_n$  satisfy  $0 \leq a_n \leq b_n$ . The following statements can now be made.

- I.) If the series  $\sum_{n=1}^{\infty} b_n$  converges, then the series  $\sum_{n=1}^{\infty} a_n$  converges.  
II.) If the series  $\sum_{n=1}^{\infty} a_n$  diverges, then the series  $\sum_{n=1}^{\infty} b_n$  diverges.

LIMIT COMPARISON TESTS: Assume that we know what the series  $\sum_{n=1}^{\infty} b_n$  does (converge or diverge) and we are trying to determine what the series  $\sum_{n=1}^{\infty} a_n$  does. Assume also that  $a_n > 0$ ,  $b_n > 0$ , and  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ , where  $L$  is a *positive, finite* number.

- I.) If the series  $\sum_{n=1}^{\infty} b_n$  converges, then the series  $\sum_{n=1}^{\infty} a_n$  converges.  
II.) If the series  $\sum_{n=1}^{\infty} b_n$  diverges, then the series  $\sum_{n=1}^{\infty} a_n$  diverges.

THE FOLLOWING ARE SPECIAL CASES FOR THE LIMIT COMPARISON TEST.

- III.) a.) If  $L = 0$  and the series  $\sum_{n=1}^{\infty} b_n$  converges, then the series  $\sum_{n=1}^{\infty} a_n$  converges.  
b.) If  $L = 0$  and the series  $\sum_{n=1}^{\infty} b_n$  diverges, then NO CONCLUSION can be made about the series  $\sum_{n=1}^{\infty} a_n$  using this test.  
IV.) a.) If  $L = \infty$  and the series  $\sum_{n=1}^{\infty} b_n$  diverges, then the series  $\sum_{n=1}^{\infty} a_n$  diverges.  
b.) If  $L = \infty$  and the series  $\sum_{n=1}^{\infty} b_n$  converges, then NO CONCLUSION can be made about the series  $\sum_{n=1}^{\infty} a_n$  using this test.