Math 21C
Vogler
Examples Using Star (*) and Double Star (*)(*)

EXAMPLE : The series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges by the p-series test since $p = 4 > 1$. What should n be in order that the partial sum

$$S_n = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots + \frac{1}{n^4} = \sum_{i=1}^{n} \frac{1}{i^4}$$

estimates the exact value of the series with error at most 0.000001 ? (i.e., we want error R_n to satisfy $R_n \leq 0.000001$)

SOLUTION : Use (*)(*) : error satisfies $R_n < \int_n^{\infty} f(x)\, dx$

so require that $\int_n^{\infty} f(x)\, dx \leq 0.000001 \quad \longrightarrow$

EXAMPLE : The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by the p-series test since $p = 1/2 \leq 1$. What should n be in order that the partial sum

$$S_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} = \sum_{i=1}^{n} \frac{1}{\sqrt{i}}$$

be at least 100 ? (i.e., we want partial sum S_n to satisfy $100 \leq S_n$)

SOLUTION : Use (*) : partial sum satisfies $\int_1^{n+1} f(x)\, dx < S_n$

so require that $100 \leq \int_1^{n+1} f(x)\, dx \quad \longrightarrow$