Ex: Find absolute maximum and minimum values of $f(x,y) = y^2 - x^2$ on and inside the triangle with vertices $(-3,3)$, $(1,3)$, and $(1,-2)$.

Ans:

```
(-3,3)   y=3
(1,3)

y = \frac{-5}{4}x - \frac{3}{4}

(1,-2)
```

Interior

\[ f_x = -2x = 0 \implies x = 0 \]
\[ f_y = 2y = 0 \implies y = 0 \]

So \((0,0)\) is critical point (C.P.)

Corner:

\([-3,3], [1,3], \& [1,-2]\)

Along path $x = 1$:

\[ z = y^2 - 1 \implies z' = 2y = 0 \implies y = 0 \]

so \((1,0)\) is C.P.

Along path $y = 3$:

\[ z = 3 - x^2 \implies z' = -2x = 0 \implies x = 0 \]

so \((0,3)\) is C.P.

Along path $y = \frac{-5}{3}x - \frac{4}{3}$

\[ z = y^2 - x^2 = \left(\frac{-5}{3}x - \frac{4}{3}\right)^2 - x^2 \]

\[ z' = 2\left(\frac{-5}{3}x - \frac{4}{3}\right)\left(-\frac{5}{3}\right) - 2x = 0 \]

\[ \implies \frac{25}{9}x + \frac{20}{9} - \frac{9}{4}x = 0 \implies \frac{16}{9}x + \frac{20}{9} = 0 \implies x = \frac{5}{4} \]

so \((-\frac{5}{4}, \frac{3}{4})\) is C.P.
Build Table

<table>
<thead>
<tr>
<th>C.P.s &amp; Corners</th>
<th>Values of $f(x,y) = y^2 - x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0</td>
</tr>
<tr>
<td>(-3,3)</td>
<td>0</td>
</tr>
<tr>
<td>(1,3)</td>
<td>+8</td>
</tr>
<tr>
<td>(1,-2)</td>
<td>+3</td>
</tr>
<tr>
<td>(1,0)</td>
<td>-1 Min</td>
</tr>
<tr>
<td>(0,3)</td>
<td>+9 Max</td>
</tr>
<tr>
<td>$\left(-\frac{5}{4}, \frac{3}{4}\right)$</td>
<td>-1 Min</td>
</tr>
</tbody>
</table>