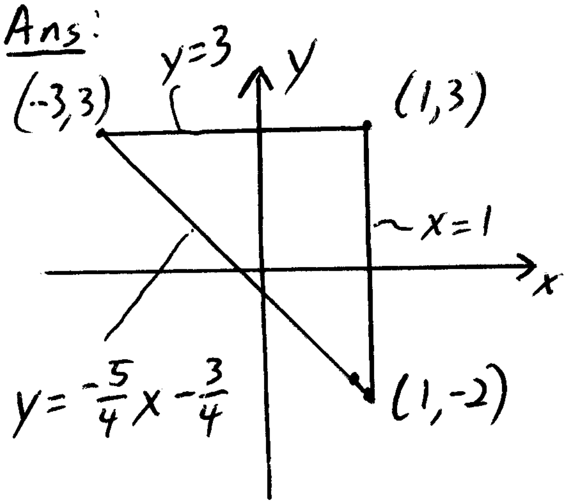
Math 21C
Vogler

Extrema on Restricted Domains

Ex: Find absolute maximum and minimum values of $f(x,y) = y^2 - x^2$ on and inside the triangle with vertices $(-3,3)$, $(1,3)$, and $(1,-2)$.

Ans:



Interior

$f_x = -2x = 0 \Rightarrow x = 0$

$f_y = 2y = 0 \Rightarrow y = 0$

so $\boxed{(0,0)}$ is critical point (C.P.)

Corner: $\boxed{(-3,3)}$, $\boxed{(1,3)}$, & $\boxed{(1,-2)}$

Along path $x=1$: $z = y^2 - 1 \overset{D}{\Rightarrow} z' = 2y = 0 \Rightarrow y = 0$

so $\boxed{(1,0)}$ is C.P.

Along path $y=3$: $z = 3 - x^2 \overset{D}{\Rightarrow} z' = -2x = 0 \Rightarrow x = 0$

so $\boxed{(0,3)}$ is C.P.

Along path $y = -\frac{5}{3}x - \frac{4}{3}$ $\quad z = y^2 - x^2 = \left(-\frac{5}{3}x - \frac{4}{3}\right)^2 - x^2$

$\overset{D}{\Rightarrow} z' = \cancel{2}\left(-\frac{5}{3}x - \frac{4}{3}\right)\left(-\frac{5}{3}\right) - \cancel{2}x = 0$

$\Rightarrow \frac{25}{9}x + \frac{20}{9} - \frac{9}{9}x = 0 \Rightarrow \frac{16}{9}x + \frac{20}{9} = 0 \Rightarrow x = \frac{-5}{4}$

so $\boxed{\left(-\frac{5}{4}, \frac{3}{4}\right)}$ is C.P.

Build Table

C.P.s & Corners	Values of $f(x,y) = y^2 - x^2$	
$(0,0)$	0	
$(-3,3)$	0	
$(1,3)$	$+8$	
$(1,-2)$	$+3$	
$(1,0)$	-1	min
$(0,3)$	$+9$	max
$\left(-\frac{5}{4}, \frac{3}{4}\right)$	-1	Min