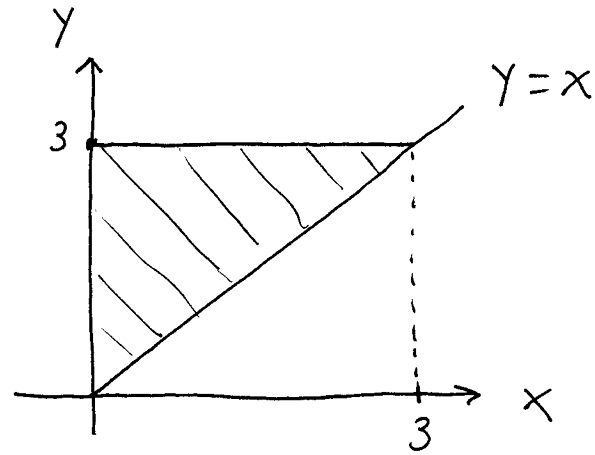
Section 14.7

31.)



$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1 \Rightarrow$

$f_x = 4x - 4 = 4(x-1) = 0 \Rightarrow x = 1$;

$f_y = 2y - 4 = 2(y-2) = 0 \Rightarrow y = 2$,

so $\boxed{(1,2)}$ is critical point;

corners are $\boxed{(0,0)}$, $\boxed{(3,3)}$, and $\boxed{(0,3)}$;

<u>along path $x=0$</u>: $z = y^2 - 4y + 1 \Rightarrow$

$z' = 2y - 4 = 2(y-2) = 0 \Rightarrow y = 2$ so

$\boxed{(0,2)}$ is critical point;

<u>along path $y=3$:</u> $z = 2x^2 - 4x - 2 \Rightarrow$

$z' = 4x - 4 = 4(x-1) = 0 \Rightarrow x = 1$ so

$\boxed{(1,3)}$ is critical point;

<u>along path $y=x$</u>:

$z = 2x^2 - 4x + x^2 - 4x + 1 \Rightarrow$

$z = 3x^2 - 8x + 1 \Rightarrow$

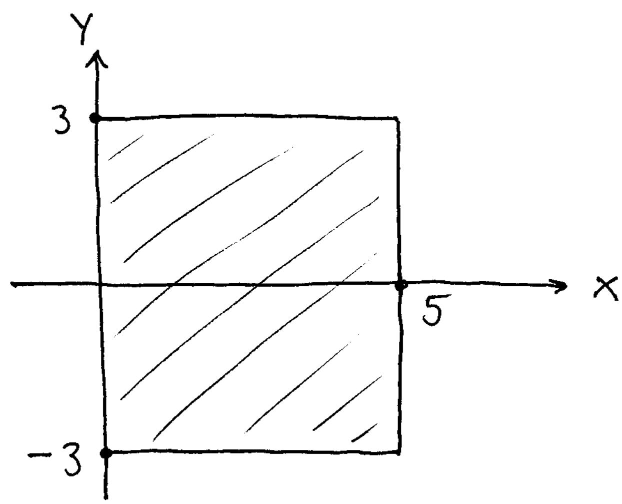
$z' = 6x - 8 = 0 \Rightarrow x = 4/3$, so

$\boxed{(4/3, 4/3)}$ is critical point:

compare function values:

critical points and corners	function values	
$(1,2)$	$f(1,2) = -5$	
$(0,0)$	$f(0,0) = 1$	
$(3,3)$	$f(3,3) = 4$	MAX
$(0,3)$	$f(0,3) = -2$	
$(0,2)$	$f(0,2) = -3$	
$(1,3)$	$f(1,3) = -12$	MIN
$(4/3, 4/3)$	$f(4/3, 4/3) = -37/9$	

34.)



$T(x,y) = x^2 + xy + y^2 - 6x \Rightarrow$

$T_x = 2x + y - 6 = 0 \Rightarrow \underline{y = -2x + 6}$;

$T_y = x + 2y = 0 \Rightarrow \underline{x = -2y}$; substitute

$\Rightarrow y = -2x + 6 = -2(-2y) + 6 = 4y + 6 \Rightarrow$

$0 = 3y + 6 \Rightarrow y = -2 \Rightarrow x = 4$ so

$\boxed{(4,-2)}$ is critical point ;

corners are $(0,3)$, $(0,-3)$, $(5,3)$, $(5,-3)$;

along path $x=0$: $z = y^2 \Rightarrow z' = 2y = 0 \Rightarrow$ $y = 0$ so $\boxed{(0,0)}$ is critical point;

along path $x=5$: $z = y^2 + 5y - 5 \Rightarrow$ $z' = 2y + 5 = 0 \Rightarrow y = -5/2$, so $\boxed{(5, -5/2)}$ is critical point;

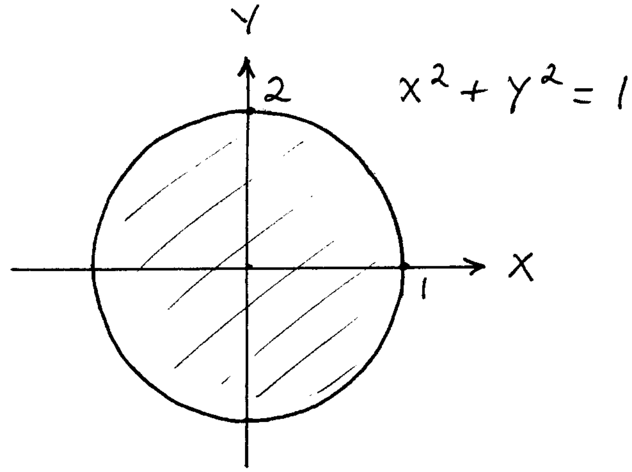
along path $y=3$: $z = x^2 - 3x + 9 \Rightarrow$ $z' = 2x - 3 = 0 \Rightarrow x = 3/2$, so $\boxed{(3/2, 3)}$ is critical point;

along path $y=-3$: $z = x^2 - 9x + 9 \Rightarrow$ $z' = 2x - 9 = 0 \Rightarrow x = 9/2$, so $\boxed{(9/2, 3)}$ is critical point;

compare function values:

critical points and corners	function values	
$(4,-2)$	$T(4,-2) = -12$	MIN
$(0,3)$	$T(0,3) = 9$	
$(0,-3)$	$T(0,-3) = 9$	
$(5,3)$	$T(5,3) = 19$	MAX
$(5,-3)$	$T(5,-3) = -11$	
$(0,0)$	$T(0,0) = 0$	
$(5,-5/2)$	$T(5,-5/2) = -45/4$	
$(3/2,3)$	$T(3/2,3) = 27/4$	
$(9/2,3)$	$T(9/2,3) = 63/4$	

41.)



$T(x,y) = x^2 + 2y^2 - x \Rightarrow$

$T_x = 2x - 1 = 0 \Rightarrow x = 1/2$;

$T_y = 4y = 0 \Rightarrow y = 0$, so

$\boxed{(1/2, 0)}$ is <u>critical point</u> ;

<u>along path $x^2+y^2=1$</u> : $\begin{cases} x = \cos t \\ y = \sin t \end{cases}$

for $0 \le t \le 2\pi$:

$z = (\cos t)^2 + 2(\sin t)^2 - \cos t$

$= \underline{\cos^2 t + \sin^2 t} + \sin^2 t - \cos t$

$= 1 + \sin^2 t - \cos t \Rightarrow$

$z' = 2\sin t \cos t + \sin t$

$= \sin t\,(2\cos t + 1) = 0 \Rightarrow$

$\sin t = 0 \Rightarrow \underline{t = 0^\circ}$ or $\underline{t = 180^\circ}$ OR

$\cos t = -1/2 \Rightarrow \underline{t = 120^\circ}$ or $\underline{t = 240^\circ}$;

so <u>critical points</u> are :

$t = 0^\circ$: $(1, 0)$

$t = 180^\circ$: $(-1, 0)$

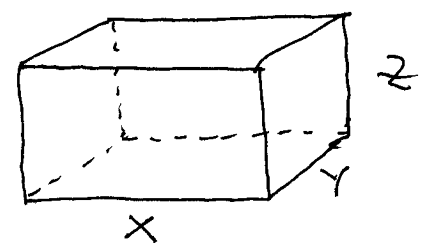
$t = 120^\circ$: $(-1/2, \sqrt{3}/2)$

$t = 240^\circ$: $(-1/2, -\sqrt{3}/2)$

compare function values :

critical points	function values		
$(\frac{1}{2}, 0)$	$T(\frac{1}{2}, 0) = \boxed{-\frac{1}{4}\,°F}$	MIN
$(1, 0)$	$T(1, 0) = 0°F$	
$(-1, 0)$	$T(-1, 0) = 2°F$	
$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$T(-\frac{1}{2}, \frac{\sqrt{3}}{2}) = \boxed{2\frac{1}{4}°F}$	MAX
$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$	$T(-\frac{1}{2}, -\frac{\sqrt{3}}{2}) = \boxed{2\frac{1}{4}°F}$	MAX

I.)



Volume

$V = xyz = 1$ ~~so~~

$$\boxed{z = \frac{1}{xy}} \; ;$$

minimize cost (¢)

$$C = C_{top} + C_{bottom} + C_{sides}$$

$$= 3xy + 3xy + 2(2xz + 2yz)$$

$$= 6xy + 4(x+y) \cdot z$$

$$= 6xy + 4(x+y) \cdot \frac{1}{xy} \Rightarrow$$

$$\boxed{C = 6xy + \frac{4}{y} + \frac{4}{x}} \; ; \text{~~then~~}$$

$$C_x = 6y - \frac{4}{x^2} = 0 \Rightarrow \underline{y = \frac{2}{3x^2}} \; ;$$

$$C_y = 6x - \frac{4}{y^2} = 0 \Rightarrow \underline{x = \frac{2}{3y^2}} \; ;$$

substitute $\Rightarrow$

$$y = \frac{2}{3x^2} = \frac{2}{3\left(\frac{2}{3y^2}\right)^2} = \frac{2}{\frac{4}{3y^4}} = 2 \cdot \frac{3}{4} y^4 \Rightarrow$$

$$Y = \frac{3}{2}Y^4 \Rightarrow 0 = \frac{3}{2}Y^4 - Y = Y\left(\frac{3}{2}Y^3 - 1\right)$$

$$\Rightarrow Y = 0 \text{ (NO!) OR } Y = \left(\frac{2}{3}\right)^{1/3} \text{ ft.} \Rightarrow$$

$$X = \frac{2}{3\left(\frac{2}{3}\right)^{2/3}} = \frac{2}{3 \cdot \frac{2^{2/3}}{3^{2/3}}} = \left(\frac{2}{3}\right)^{1/3} \text{ ft.} \Rightarrow$$

$$Z = \frac{1}{XY} = \frac{1}{\left(\frac{2}{3}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3}} = \frac{1}{\left(\frac{2}{3}\right)^{2/3}} = \left(\frac{3}{2}\right)^{2/3} \text{ ft.}$$

and minimum cost is

$$C = 6XY + \frac{4}{Y} + \frac{4}{X}$$

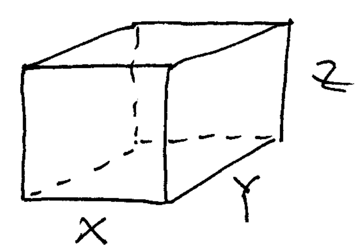
$$= 6\left(\frac{2}{3}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3} + \frac{4}{\left(\frac{2}{3}\right)^{1/3}} + \frac{4}{\left(\frac{2}{3}\right)^{1/3}}$$

$$= 6\left(\frac{2}{3}\right)^{2/3} + 8 \cdot \left(\frac{3}{2}\right)^{1/3}$$

$$= 2^{5/3}3^{1/3} + 2^{5/3} \cdot 3^{1/3}$$

$$= 2^{8/3}3^{1/3} \approx 9.16 ¢$$

II.)



Surface area

$$S = 2XY + 2XZ + 2YZ = 12$$

$$\Rightarrow XY + XZ + YZ = 6$$

$$\Rightarrow XY + (X+Y)Z = 6$$

$$\Rightarrow \boxed{Z = \frac{6 - XY}{X+Y}} \;;$$

maximize volume

$$V = XYZ = XY \cdot \frac{6-XY}{X+Y} = \frac{6XY - X^2Y^2}{X+Y} \Rightarrow$$

$$\boxed{V = \frac{6XY - X^2Y^2}{X+Y}} \quad ; \quad \text{then}$$

$$V_X = \frac{(X+Y)(6Y-2XY^2) - (6XY - X^2Y^2)}{(X+Y)^2} = 0 \Rightarrow$$

$$Y\left[(X+Y)\cdot(6-2XY) - (6X - X^2Y)\right] = 0 \Rightarrow$$

$$Y=0 \ (\text{NO!}) \text{ OR } \cancel{6X} + 6Y - 2X^2Y - 2XY^2 - \cancel{6X} + X^2Y = 0 \Rightarrow$$

$$Y\cdot\left[6 - 2X^2 - 2XY + X^2\right] = 0 \Rightarrow Y = 0 \ (\text{NO!})$$

$$\text{OR} \quad 6 - X^2 - 2XY = 0 \Rightarrow$$

$$2XY = 6 - X^2 \Rightarrow \quad \boxed{Y = \frac{6-X^2}{2X}} \quad ; \quad \text{and}$$

$$V_Y = \frac{(X+Y)(6X - 2X^2Y) - (6XY - X^2Y^2)}{(X+Y)^2} = 0 \Rightarrow$$

$$X\left[(X+Y)(6-2XY) - (6Y - XY^2)\right] = 0 \Rightarrow$$

$$X=0 \ (\text{NO!}) \text{ OR } 6X + \cancel{6Y} - 2X^2Y - 2XY^2 - \cancel{6Y} + XY^2 = 0 \Rightarrow$$

$$X\cdot\left[6 - 2XY - 2Y^2 + Y^2\right] = 0 \Rightarrow X = 0 \ (\text{NO!}) \text{ OR}$$

$$6 - 2XY - Y^2 = 0 \Rightarrow 2XY = 6 - Y^2 \Rightarrow$$

$$\boxed{X = \frac{6-Y^2}{2Y}} \quad ; \quad \text{substitute} \Rightarrow$$

$$X = \frac{6-Y^2}{2Y} = \frac{6 - \left(\frac{6-X^2}{2X}\right)^2}{2\left(\frac{6-X^2}{2X}\right)} \cdot \frac{(2X)^2}{(2X)^2} \Rightarrow$$

$$X = \frac{6(2X)^2 - (6-X^2)^2}{2(2X)(6-X^2)} \Rightarrow$$

$$4x^2(6-x^2) = 24x^2 - (36 - 12x^2 + x^4) \Rightarrow$$

$$\cancel{24x^2} - 4x^4 = \cancel{24x^2} - 36 + 12x^2 - x^4 \Rightarrow$$

$$0 = 3x^4 + 12x^2 - 36$$

$$= 3\left((x^2)^2 + 4(x^2) - 12\right)$$

$$= 3(x^2 - 2)(x^2 + 6) \Rightarrow$$

$x^2 - 2 = 0 \Rightarrow \boxed{x = \sqrt{2}\text{ m.}}$ or $x = -\sqrt{2}$ (NO!);

if $x = \sqrt{2}$, then

$$y = \frac{6 - (\sqrt{2})^2}{2(\sqrt{2})} = \frac{6-2}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \Rightarrow$$

$\boxed{y = \sqrt{2}\text{ m.}}$, then $z = \dfrac{6 - (\sqrt{2})(\sqrt{2})}{\sqrt{2} + \sqrt{2}} = \dfrac{4}{2\sqrt{2}}$

$\Rightarrow \boxed{z = \sqrt{2}\text{ m.}}$; and max. volume is $V = (\sqrt{2})^3 \Rightarrow$

$$\boxed{V = 2\sqrt{2}\text{ m.}^3}$$