Math 16B
Vogler

# Going from Discrete to Continuous Compound Interest

Recall: I) Discrete Compound Interest: $A = P(1+\frac{r}{n})^{nt}$

II) $e := \lim_{k\to\infty}(1+\frac{1}{k})^k$

To obtain formula for continuous compound interest, the interest must be compounded an $\infty$ number of times, which is equivalent to letting $n\to\infty$ in above formula

$$\lim_{n\to\infty} P(1+\frac{r}{n})^{nt} = \lim_{n\to\infty} P\Big[\underbrace{\Big(1+\frac{1}{(\frac{n}{r})}\Big)^{(\frac{n}{r})}}_{\text{By II) } = e}\Big]^{rt} = Pe^{rt}$$

Hence, continuous compound interest formula is $\boxed{A = Pe^{rt}}$ ■

Note: You don't need to know this arguement for the exams, but it could be an extra credit question.

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Ex 2 You wish an initial deposit of \$800 to grow to \$1,800 in 8 years. If interest is compounded daily, what should the yearly interest rate $r$ be?

Given: $P = \$800$, $A = \$1,800$, $t = 8$ yrs, $n = 365$

Plugging into $A = P(1+\frac{r}{n})^{nt}$, we need to solve for $r$

$\Rightarrow\ 1,800 = 800(1+\frac{r}{365})^{365(8)} \quad \Rightarrow\ {}^{18}/_{8} = (1+\frac{r}{365})^{2920}$

$\Rightarrow\ (\frac{18}{8})^{\frac{1}{2920}} = 1 + \frac{r}{365} \quad \Rightarrow\ 1+\frac{r}{365} \approx 1.000277754$

$\Rightarrow\ r = 0.1013$ or $\boxed{r = 10.13\%}$