Logarithmic Functions

**Defn**

The logarithm base $b$ of $x$ equals $y$, written $\log_b x = y$, is equivalent to expression $x = b^y$.

(i.e. $\log_b x = y \iff x = b^y$)

**Notes**

1. **Domain:** $x > 0$
2. **Range:** All real numbers
3. $\log_b x$ has a vertical asymptote at $x = 0$.

**Base Graph**

```
Y
/  \\
|___
VA: x = 0
```

**Rules**

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x$
5. $\log_b (x \cdot y) = \log_b x + \log_b y$
6. $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
7. $\log_b x^y = y \log_b x$
8. $\log_b x = \frac{\log_e x}{\log_e b}$ (Change of Base Formula)

**Notation**: 1) $\log x := \log_{10} x$ 2) $\ln x := \log_e x$

**Notes**:

a) Rules (3) and (4) show logarithms cancel with exponentiation, as long as the bases match.

b) Rule 5) can be viewed as 'inside multiplication' is 'outside addition'.

c) Rule 6) can be viewed as 'inside division' is 'outside subtraction'.

d) Rule 7) can be viewed as 'inside powers' is 'outside multiplication'.