

Section 5.5

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$$2.) \text{ Area} = \int_{-2}^2 [(2x+5) - (x^2+2x+1)] dx$$

$$= \int_{-2}^2 (-x^2+4) dx = \left(-\frac{x^3}{3}+4x\right)\bigg|_{-2}^2 = \left(-\frac{8}{3}+8\right) - \left(\frac{8}{3}-8\right) = \left(\frac{32}{3}\right)$$

$$3.) \text{ Area} = \int_0^3 [(-x^2+2x+3) - (x^2-4x+3)] dx$$

$$= \int_0^3 (-2x^2+6x) dx = \left(-\frac{2}{3}x^3+3x^2\right)\bigg|_0^3 = -18+27 = (9)$$

$$4.) \text{ Area} = \int_0^1 (x^2-x^3) dx = \left(\frac{x^3}{3}-\frac{x^4}{4}\right)\bigg|_0^1 = \frac{1}{3}-\frac{1}{4} = \left(\frac{1}{12}\right)$$

$$6.) \text{ Area} = \int_0^1 [(x-1)^3 - (x-1)] dx + \int_1^2 [(x-1) - (x-1)^3] dx$$

$$= 2 \int_0^1 [(x-1)^3 - (x-1)] dx = 2 \int_0^1 (x^3 - 3x^2 + 3x - 1 - x + 1) dx$$

$$= 2 \int_0^1 (x^3 - 3x^2 + 2x) dx = 2 \left(\frac{x^4}{4} - x^3 + x^2\right)\bigg|_0^1$$

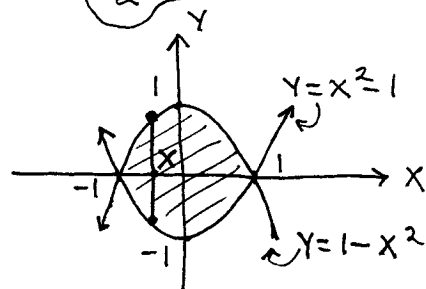
$$= 2 \left(\frac{1}{4} - 1 + 1\right) = \left(\frac{1}{2}\right)$$

$$8.) \text{ Area} = \int_1^2 \left[(3-x) - \frac{2}{x}\right] dx = \left(3x - \frac{x^2}{2} - 2\ln|x|\right)\bigg|_1^2$$

$$= (6 - 2 - 2\ln 2) - \left(3 - \frac{1}{2} - 2\ln 1\right) = \frac{3}{2} - \ln 2^2 = \left(\frac{3}{2} - \ln 4\right)$$

10.)

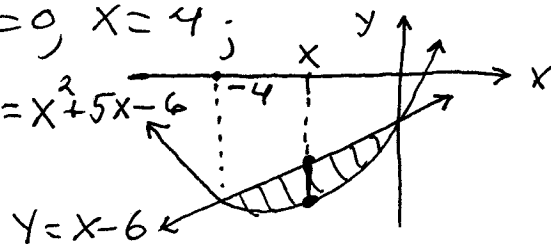
$$\text{Area} = \int_{-1}^1 [(1-x^2) - (x^2-1)] dx$$

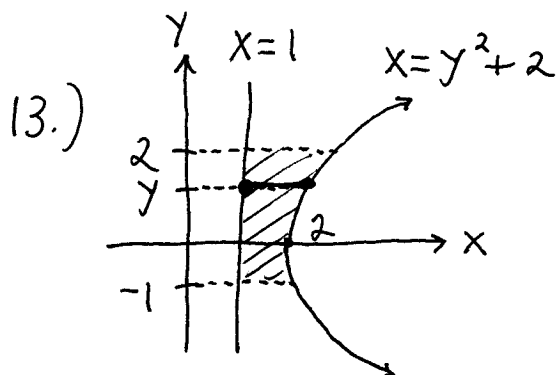


$$12.) \quad x-6 = x^2+5x-6 \rightarrow$$

$$0 = x^2 - 4x = x(x-4) \rightarrow x=0, x=4;$$

$$\text{Area} = \int_{-4}^0 [(x-6) - (x^2+5x-6)] dx$$

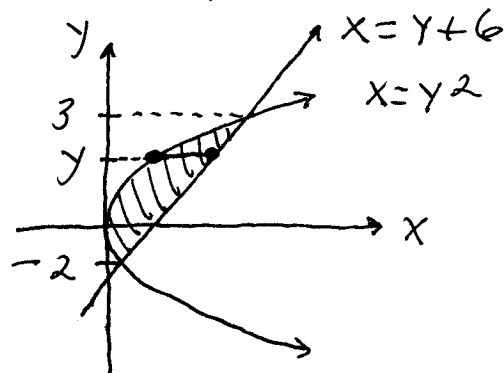




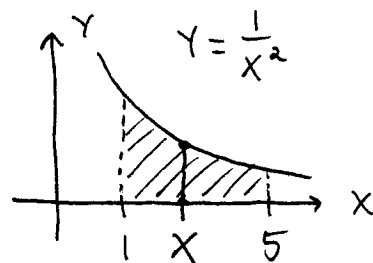
$$\text{Area} = \int_{-1}^2 [(y^2 + 2) - 1] dy$$

14.) $y + 6 = y^2 \rightarrow 0 = y^2 - y - 6 = (y - 3)(y + 2) \rightarrow$
 $y = 3, y = -2$;

$$\text{Area} = \int_{-2}^3 [(y + 6) - y^2] dy$$

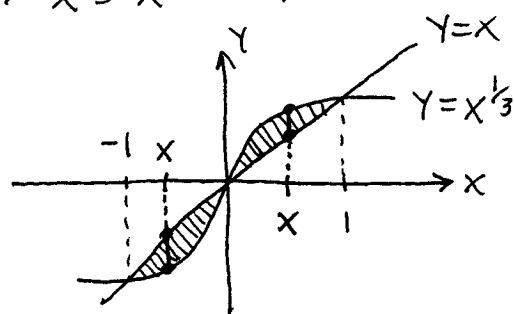


15.) $\text{Area} = \int_1^5 \frac{1}{x^2} dx = \int_1^5 x^{-2} dx$
 $= -x^{-1} \Big|_1^5 = -\frac{1}{5} - \left(-\frac{1}{1}\right) = \left(\frac{4}{5}\right)$

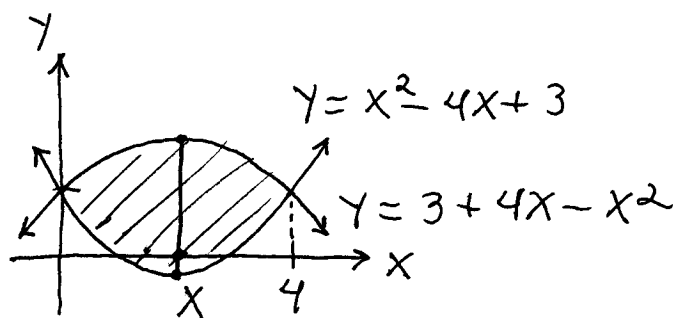


17.) intersection : $x^{1/3} = x \rightarrow x = x^3 \rightarrow$
 $x - x^3 = x(1 - x^2) = x(1 - x)(1 + x) = 0$
 $\rightarrow x = 0, 1, -1$ then

$$\begin{aligned} \text{Area} &= \int_{-1}^0 (x - x^{1/3}) dx + \int_0^1 (x^{1/3} - x) dx \\ &= 2 \int_0^1 (x^{1/3} - x) dx = 2 \left(\frac{3}{4} x^{4/3} - \frac{x^2}{2} \right) \Big|_0^1 \\ &= 2 \left(\frac{3}{4} - \frac{1}{2} \right) = \left(\frac{1}{2} \right) \end{aligned}$$

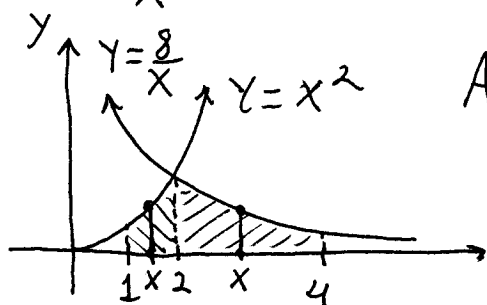


19.) $x^2 - 4x + 3 = 3 + 4x - x^2 \rightarrow$
 $2x^2 - 8x = 0 \rightarrow 2x(x - 4) = 0 \rightarrow$
 $x = 0, x = 4$



$$\begin{aligned}
 \text{Area} &= \int_0^4 [(3 + 4x - x^2) - (x^2 - 4x + 3)] dx \\
 &= \int_0^4 (8x - 2x^2) dx = \left(4x^2 - \frac{2}{3}x^3\right) \Big|_0^4 \\
 &= \left(64 - \frac{2}{3} \cdot 64\right) - (0 - 0) = \boxed{\frac{64}{3}}
 \end{aligned}$$

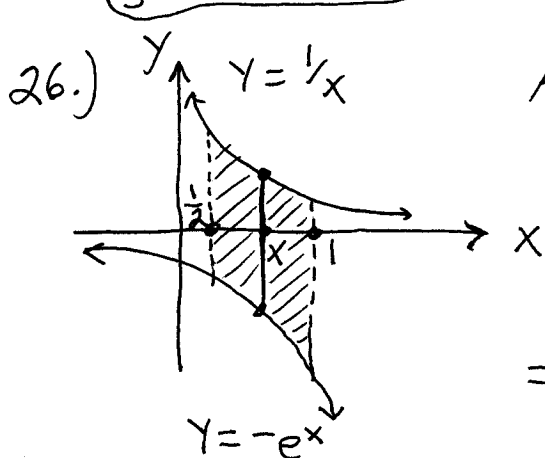
23.) $\frac{8}{x} = x^2 \rightarrow x^3 = 8 \rightarrow x = 2$;



$$\begin{aligned}
 \text{Area} &= \int_1^2 x^2 dx + \int_2^4 \frac{8}{x} dx \\
 &= \frac{1}{3}x^3 \Big|_1^2 + 8 \ln|x| \Big|_2^4 \\
 &= \left(\frac{8}{3} - \frac{1}{3}\right) + (8 \ln 4 - 8 \ln 2)
 \end{aligned}$$

$$= \frac{7}{3} + 8 \ln 2^2 - 8 \ln 2 = \frac{7}{3} + 16 \ln 2 - 8 \ln 2$$

$$= \boxed{\frac{7}{3} + 8 \ln 2}$$



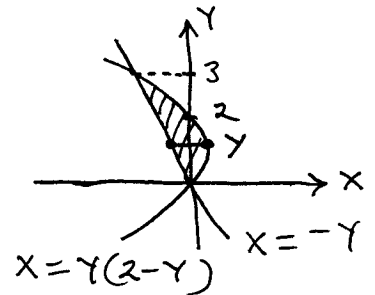
$$\begin{aligned}
 \text{Area} &= \int_{\frac{1}{2}}^1 \left(\frac{1}{x} - (-e^x)\right) dx \\
 &= \int_{\frac{1}{2}}^1 \left(\frac{1}{x} + e^x\right) dx \\
 &= (\ln|x| + e^x) \Big|_{\frac{1}{2}}^1
 \end{aligned}$$

$$= (\ln 1 + e) - (\ln(\frac{1}{2}) + e^{\frac{1}{2}}) = \boxed{e - \ln(\frac{1}{2}) - e^{\frac{1}{2}}}$$

28.) intersection: $y(2-y) = -y \rightarrow 2y - y^2 = -y \rightarrow$
 $0 = y^2 - 3y = y(y-3) \rightarrow y=0, 3$ then

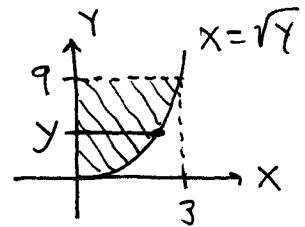
$$\text{Area} = \int_0^3 [y(2-y) - (-y)] dy = \int_0^3 (3y - y^2) dy$$

$$= \left(\frac{3}{2}y^2 - \frac{y^3}{3} \right) \Big|_0^3 = \frac{27}{2} - 9 = \left(\frac{9}{2} \right)$$

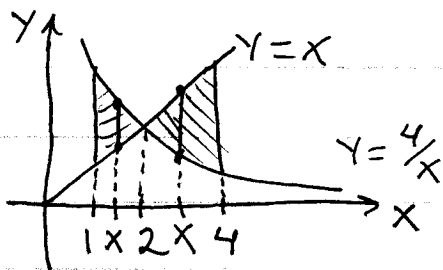


29.) $\text{Area} = \int_0^9 \sqrt{y} dy = \frac{2}{3} y^{3/2} \Big|_0^9$

$$= \frac{2}{3} (9)^{3/2} = \frac{2}{3} (27) = \left(18 \right)$$



33.) $\frac{4}{x} = x \rightarrow x^2 = 4 \rightarrow x = \pm 2 \rightarrow x = 2;$



$$\text{Area} = \int_1^2 \left(\frac{4}{x} - x \right) dx + \int_2^4 \left(x - \frac{4}{x} \right) dx$$

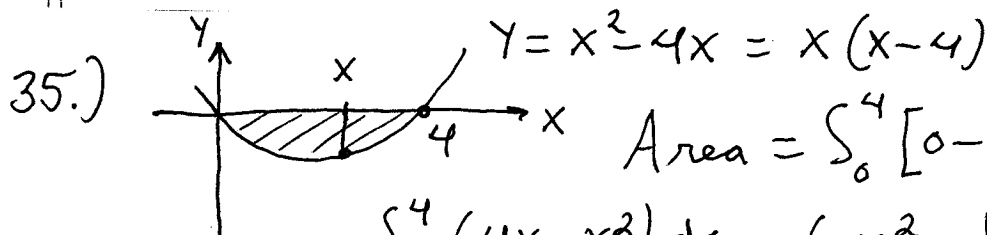
$$= \left(4 \ln x - \frac{x^2}{2} \right) \Big|_1^2 + \left(\frac{x^2}{2} - 4 \ln x \right) \Big|_2^4$$

$$= \left((4 \ln 2 - 2) - (4 \ln 1 - \frac{1}{2}) \right)$$

$$+ \left((8 - 4 \ln 4) - (2 - 4 \ln 2) \right) = 4 \ln 2 - 2 + \frac{1}{2}$$

$$+ 8 - 4 \ln 2^2 - 2 + 4 \ln 2$$

$$= 4 + \frac{1}{2} = \left(\frac{9}{2} \right)$$

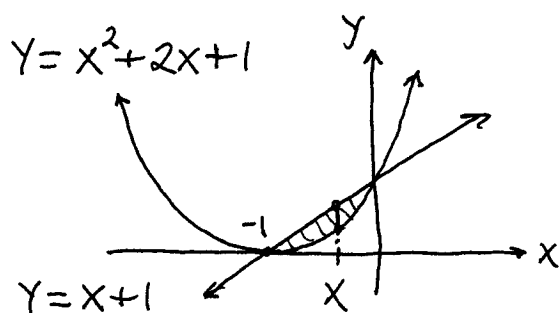


$$\text{Area} = \int_0^4 [0 - (x^2 - 4x)] dx$$

$$= \int_0^4 (4x - x^2) dx = \left(2x^2 - \frac{1}{3}x^3 \right) \Big|_0^4$$

$$= \left(32 - \frac{1}{3} \cdot 64 \right) - (0 - 0) = \left(\frac{32}{3} \right)$$

37.) $x^2 + 2x + 1 = x + 1 \rightarrow x^2 + x = 0 \rightarrow$
 $x(x+1) = 0 \rightarrow x = 0, x = -1;$

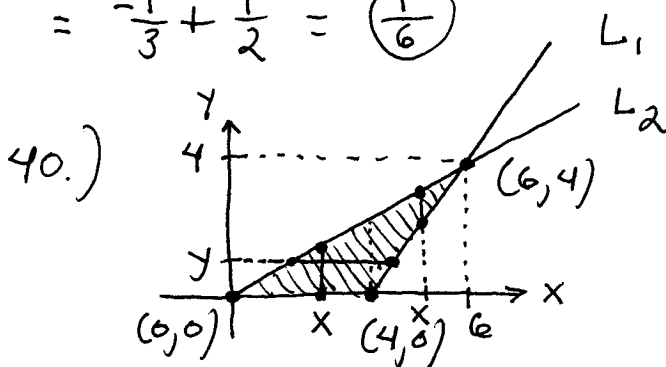


$$\text{Area} = \int_{-1}^0 [(x+1) - (x^2 + 2x + 1)] dx$$

$$= \int_{-1}^0 (-x^2 - x) dx$$

$$= \left(-\frac{1}{3}x^3 - \frac{1}{2}x^2 \right) \Big|_{-1}^0 = (0 - 0) - \left(-\frac{1}{3}(-1) - \frac{1}{2}(1) \right)$$

$$= -\frac{1}{3} + \frac{1}{2} = \left(\frac{1}{6} \right)$$



Find L_1 :

$$m = \frac{4-0}{6-4} = \frac{4}{2} = 2 \text{ so}$$

$$y - 4 = 2(x - 6) \rightarrow$$

$$\boxed{y = 2x - 8}$$

Find L_2 : $m = \frac{4-0}{6-0} = \frac{2}{3}$ so $y - 0 = \frac{2}{3}(x - 0)$

$$\rightarrow \boxed{y = \frac{2}{3}x}$$
 ; so

$$\text{Area} = \int_0^4 \frac{2}{3}x dx + \int_4^6 \left(\frac{2}{3}x - (2x - 8) \right) dx$$

$$= \frac{1}{3}x^2 \Big|_0^4 + \int_4^6 \left(8 - \frac{4}{3}x \right) dx$$

$$= \frac{16}{3} + \left(8x - \frac{2}{3}x^2 \right) \Big|_4^6$$

$$= \frac{16}{3} + \left(48 - \frac{72}{3} \right) - \left(32 - \frac{32}{3} \right)$$

$$= \frac{16}{3} + 16 - \frac{40}{3} = \frac{16}{3} + \frac{48}{3} - \frac{40}{3} = \frac{24}{3} = \boxed{8} \text{ OR}$$

$$Y = 2X - 8 \rightarrow 2X = Y + 8 \rightarrow \boxed{X = \frac{1}{2}Y + 4} ; \text{ and}$$

$$Y = \frac{2}{3}X \rightarrow \boxed{X = \frac{3}{2}Y} ; \text{ then}$$

$$\text{Area} = \int_0^4 \left[\left(\frac{1}{2}Y + 4 \right) - \frac{3}{2}Y \right] dy$$

$$= \int_0^4 (4 - Y) dy = \left(4Y - \frac{1}{2}Y^2 \right) \Big|_0^4 = (16 - 8) - (0 - 0)$$

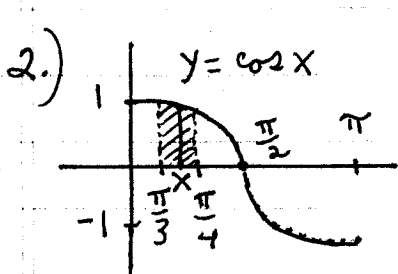
$$= \textcircled{8} .$$

Handout 7

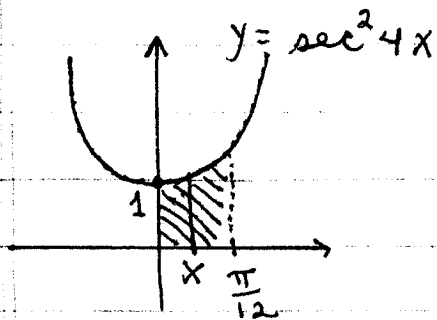
$$1.) \quad a.) \quad \int_0^{\frac{\pi}{6}} (1 + \cos 3x) dx = \left(x + \frac{1}{3} \sin 3x \right) \Big|_0^{\frac{\pi}{6}} \\ = \frac{\pi}{6} + \frac{1}{3} \sin \frac{\pi}{2} = \boxed{\frac{\pi}{6} + \frac{1}{3}}$$

$$b.) \quad \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sec x \tan x - \sin x) dx \\ = (\sec x + \cos x) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

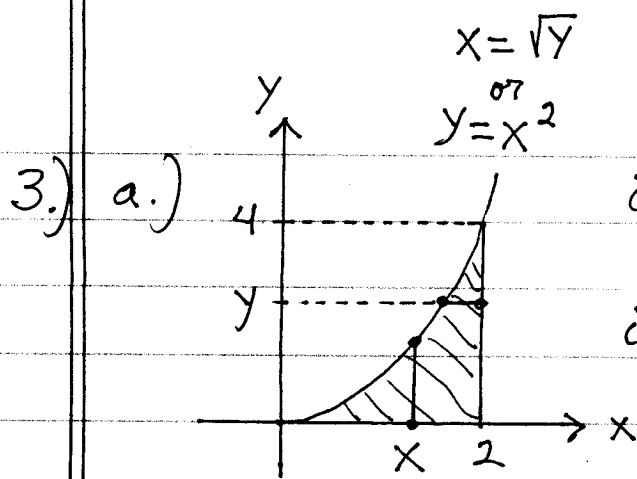
$$= \left(\sec \frac{\pi}{3} + \cos \frac{\pi}{3} \right) - \left(\sec \frac{\pi}{4} + \cos \frac{\pi}{4} \right) \\ = \left(2 + \frac{1}{2} \right) - \left(\sqrt{2} + \frac{1}{\sqrt{2}} \right) = \boxed{\frac{5}{2} - \sqrt{2} - \frac{1}{\sqrt{2}}}$$



$$a.) \quad \text{Area} = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \cos x dx \\ = \sin x \Big|_{\frac{\pi}{3}}^{\frac{\pi}{4}} \\ = \sin \frac{\pi}{4} - \sin \frac{\pi}{3} = \boxed{\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}}$$

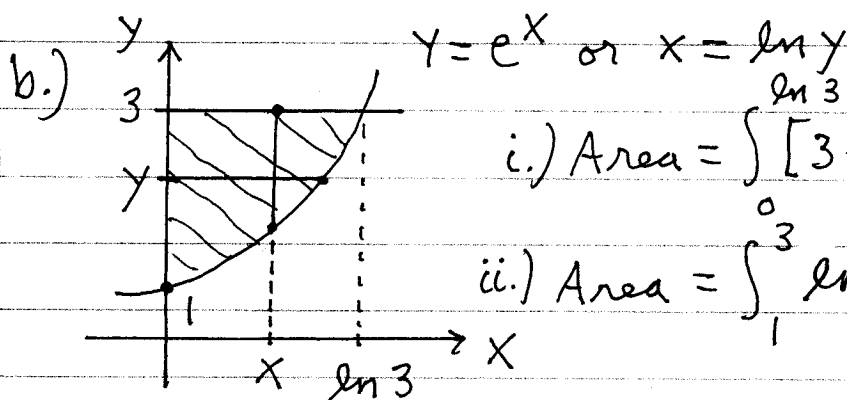


$$b.) \quad \text{Area} = \int_0^{\frac{\pi}{12}} \sec^2 4x dx \\ = \frac{1}{4} \cdot \tan 4x \Big|_0^{\frac{\pi}{12}} \\ = \frac{1}{4} \tan \frac{\pi}{3} - \frac{1}{4} \tan 0 \\ = \frac{1}{4} (\sqrt{3}) = \boxed{\frac{\sqrt{3}}{4}}$$



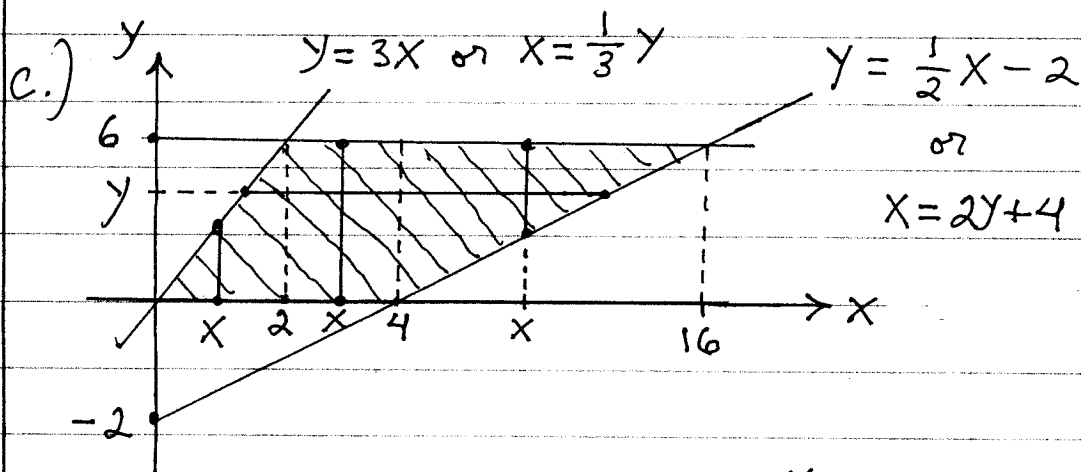
i.) $\text{Area} = \int_0^2 x^2 dx$

ii.) $\text{Area} = \int_0^4 [2 - \sqrt{y}] dy$



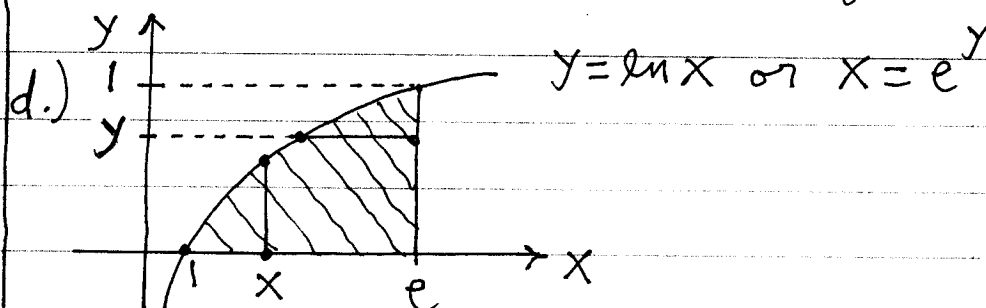
i.) $\text{Area} = \int_0^{\ln 3} [3 - e^x] dx$

ii.) $\text{Area} = \int_1^3 \ln y dy$



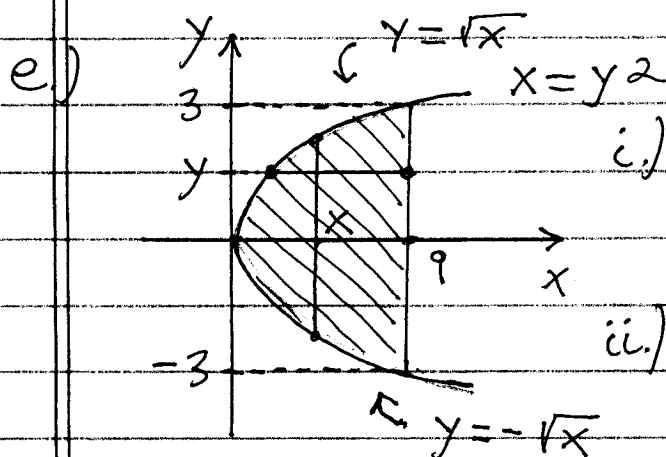
i.) $\text{Area} = \int_0^2 3x dx + \int_2^4 6 dx + \int_4^{16} (6 - (\frac{1}{2}x - 2)) dx$

ii.) $\text{Area} = \int_0^6 ((2y + 4) - (\frac{1}{3}y)) dy$



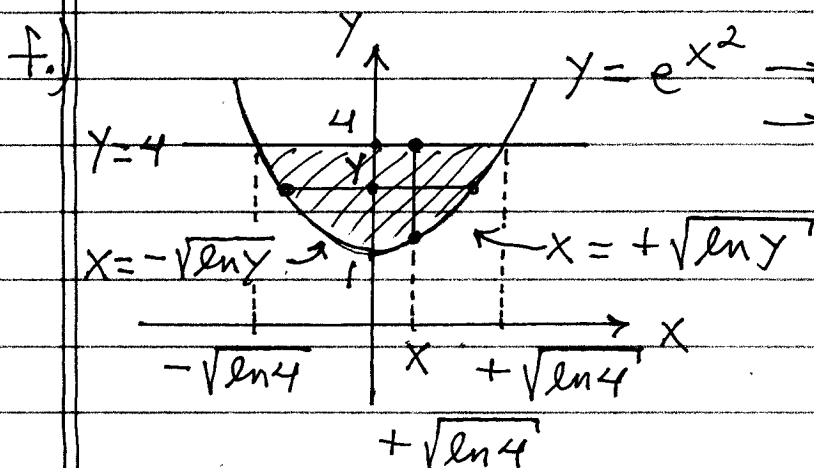
$$i.) \text{ Area} = \int_1^e \ln x \, dx$$

$$ii.) \text{ Area} = \int_0^1 (e - e^y) \, dy$$



$$i.) \text{ Area} = \int_0^9 (\sqrt{x} - (-\sqrt{x})) \, dx$$

$$ii.) \text{ Area} = \int_{-3}^3 (9 - y^2) \, dy$$



$$y = e^{x^2} \rightarrow \ln y = x^2$$

$$\rightarrow x = \pm \sqrt{\ln y}$$

$$4 = e^{x^2} \rightarrow$$

$$\ln 4 = x^2 \rightarrow$$

$$x = \pm \sqrt{\ln 4}$$

$$i.) \text{ Area} = \int_{-\sqrt{\ln 4}}^{+\sqrt{\ln 4}} (4 - e^{x^2}) \, dx$$

$$ii.) \text{ Area} = \int_1^4 (\sqrt{\ln y} - (-\sqrt{\ln y})) \, dy$$