Section 5.6

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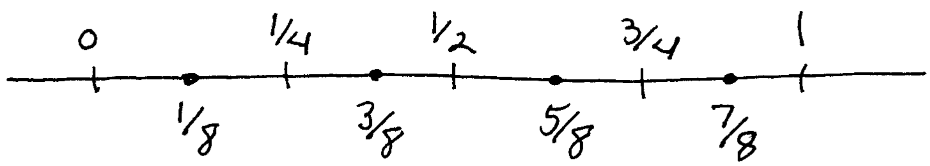
$$M_n = \frac{b-a}{n}\left[f(x_1)+f(x_2)+\cdots+f(x_n)\right]$$

1.) $f(x) = 3-2x$ on $[0,1]$, $n=4$

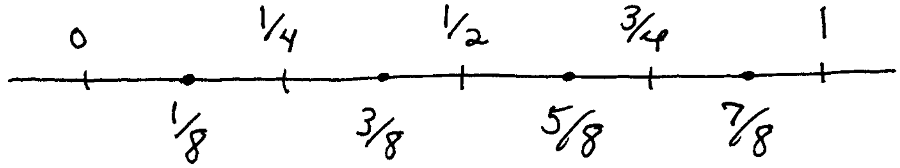


$$M_4 = \frac{1-0}{4}\left[f(1/8)+f(3/8)+f(5/8)+f(7/8)\right]$$

$$= \frac{1}{4}\left[\frac{11}{4}+\frac{9}{4}+\frac{7}{4}+\frac{5}{4}\right] = \frac{1}{4}\left[\frac{32}{4}\right] = 2 \; ;$$

Exact: $\int_0^1 (3-2x)\,dx = (3x-x^2)\Big|_0^1 = (3-1)-(0-0) = 2$.

3.) $f(x) = \sqrt{x}$ on $[0,1]$, $n=4$



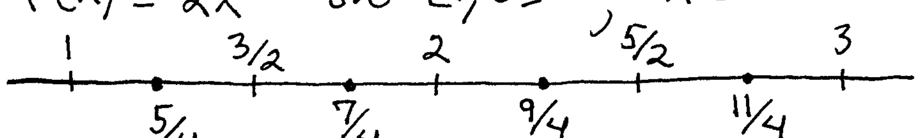
$$M_4 = \frac{1-0}{4}\left[f\left(\tfrac{1}{8}\right)+f\left(\tfrac{3}{8}\right)+f\left(\tfrac{5}{8}\right)+f\left(\tfrac{7}{8}\right)\right]$$

$$= \frac{1}{4}\left[\sqrt{\tfrac{1}{8}}+\sqrt{\tfrac{3}{8}}+\sqrt{\tfrac{5}{8}}+\sqrt{\tfrac{7}{8}}\right] \approx 0.670 \; ;$$

Exact: $\int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^1 = \frac{2}{3}(1)^{3/2} - \frac{2}{3}(0)^{3/2}$

$$= \frac{2}{3}(1) = \frac{2}{3} \approx 0.667$$

7.) $f(x) = 2x^2$ on $[1,3]$, $n=4$



$$M_4 = \frac{3-1}{4}\left[f(5/4) + f(7/4) + f(9/4) + f(11/4)\right]$$

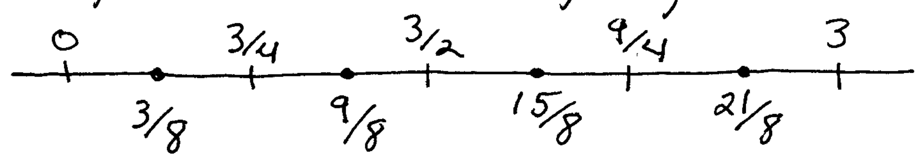
$$= \frac{1}{\cancel{2}}\left[\cancel{2}\left(\frac{5}{4}\right)^2 + \cancel{2}\left(\frac{7}{4}\right)^2 + \cancel{2}\left(\frac{9}{4}\right)^2 + \cancel{2}\left(\frac{11}{4}\right)^2\right]$$

$$= \frac{25}{16} + \frac{49}{16} + \frac{81}{16} + \frac{121}{16} = \frac{276}{16} = 17.25;$$

Exact: $\int_1^3 2x^2\,dx = \frac{2}{3}x^3\Big|_1^3 = \frac{2}{3}(27) - \frac{2}{3}(1)$

$$= \frac{52}{3} \approx 17.333$$

12.) $f(x) = 3x^2 - x^3$ on $[0, 3]$, $n = 4$



$$M_4 = \frac{3-0}{4}\left[f\left(\frac{3}{8}\right) + f\left(\frac{9}{8}\right) + f\left(\frac{15}{8}\right) + f\left(\frac{21}{8}\right)\right]$$

$$= \frac{3}{4}\Big[3\left(\frac{3}{8}\right)^2 - \left(\frac{3}{8}\right)^3 + 3\left(\frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^3$$

$$+ 3\left(\frac{15}{8}\right)^2 - \left(\frac{15}{8}\right)^3 + 3\left(\frac{21}{8}\right)^2 - \left(\frac{21}{8}\right)^3\Big]$$

$$= \frac{3}{4}\Big[3\left(\frac{9}{64} + \frac{81}{64} + \frac{225}{64} + \frac{441}{64}\right)$$

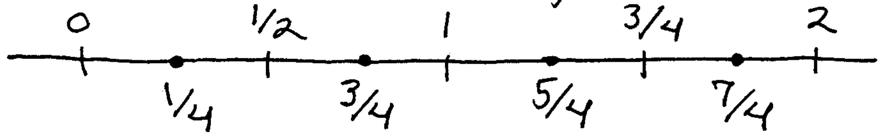
$$- \left(\frac{27}{512} + \frac{729}{512} + \frac{3375}{512} + \frac{9261}{512}\right)\Big]$$

$$= \frac{9}{4}\left(\frac{756}{64}\right) - \frac{3}{4}\left(\frac{13{,}392}{512}\right) \approx 6.961;$$

Exact: $\int_0^3 (3x^2 - x^3)\,dx = \left(x^3 - \frac{1}{4}x^4\right)\Big|_0^3$

$$= 27 - \frac{81}{4} = \frac{27}{4} = 6.75$$

18.) $f(y) = 2y$ on $[0, 2]$, $n = 4$

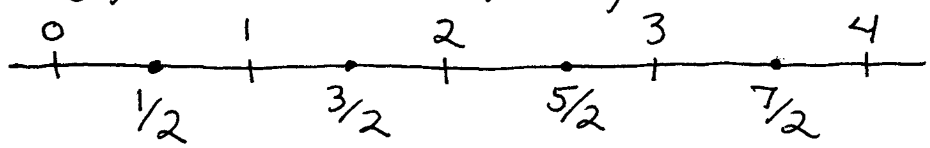


$$M_4 = \frac{2-0}{4}\left[f\left(\tfrac{1}{4}\right) + f\left(\tfrac{3}{4}\right) + f\left(\tfrac{5}{4}\right) + f\left(\tfrac{7}{4}\right)\right]$$

$$= \frac{1}{2}\left[\frac{2}{4} + \frac{6}{4} + \frac{10}{4} + \frac{14}{4}\right] = \frac{1}{2}\left[\frac{32}{4}\right] = 4 \; ;$$

Exact: $\int_0^2 2y\,dy = y^2\Big|_0^2 = 4 - 0 = 4$.

19.) $f(y) = y^2 + 1$ on $[0, 4]$, $n = 4$



$$M_4 = \frac{4-0}{4}\left[f\left(\tfrac{1}{2}\right) + f\left(\tfrac{3}{2}\right) + f\left(\tfrac{5}{2}\right) + f\left(\tfrac{7}{2}\right)\right]$$

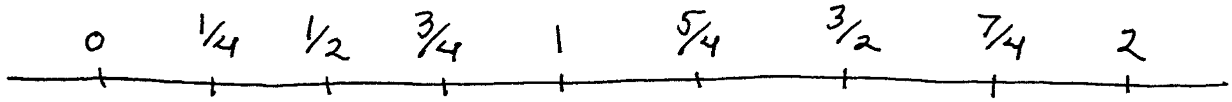
$$= \left[\frac{5}{4} + \frac{13}{4} + \frac{29}{4} + \frac{53}{4}\right] = \frac{100}{4} = 25 \; ;$$

Exact: $\int_0^4 (y^2 + 1)\,dy = \left(\frac{1}{3}y^3 + y\right)\Big|_0^4$

$$= \frac{64}{3} + 4 = \frac{76}{3} \approx 25.333$$

$$T_n = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

21.) $f(x) = x^3$ on $[0, 2]$, $n = 8$



$$T_8 = \frac{2-0}{2(8)}\left[f(0) + 2f\left(\tfrac{1}{4}\right) + 2f\left(\tfrac{1}{2}\right) + 2f\left(\tfrac{3}{4}\right) + 2f(1) + 2f\left(\tfrac{5}{4}\right) + 2f\left(\tfrac{3}{2}\right) + 2f\left(\tfrac{7}{4}\right) + f(2)\right]$$

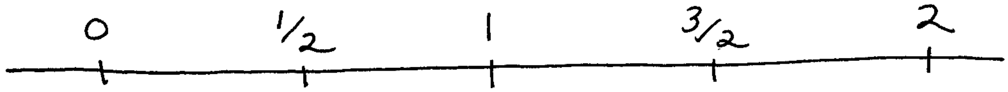
$$= \frac{1}{8}\left[0 + 2\cdot\frac{1}{64} + 2\cdot\frac{1}{8} + 2\cdot\frac{27}{64} + 2(1) + 2\cdot\frac{125}{64} + 2\cdot\frac{27}{8} + 2\cdot\frac{343}{64} + 8\right]$$

$$= \frac{1}{8}\left[10 + \frac{2+16+54+250+432+686}{64}\right]$$

$$= \frac{1}{8}\left[10 + \frac{1440}{64}\right] = 4.0625 \quad ;$$

$$\text{Exact}: \int_0^2 x^3\,dx = \frac{1}{4}x^4\Big|_0^2 = \frac{16}{4} = 4$$

23.) $f(x) = \frac{1}{x+1}$ on $[0,2]$, $n=4$



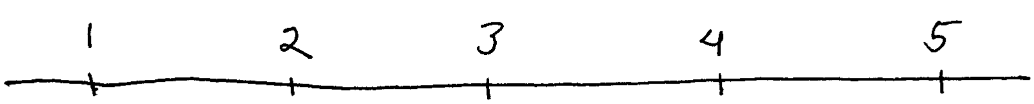
$$T_4 = \frac{2-0}{2(4)}\left[f(0) + 2f\left(\tfrac{1}{2}\right) + 2f(1) + 2f\left(\tfrac{3}{2}\right) + f(2)\right]$$

$$= \frac{1}{4}\left[1 + 2\cdot\frac{2}{3} + 2\cdot\frac{1}{2} + 2\cdot\frac{2}{5} + \frac{1}{3}\right] \approx 1.117 \quad ;$$

$$\text{Exact}: \int_0^2 \frac{1}{x+1}\,dx = \ln|x+1|\Big|_0^2$$

$$= \ln 3 - \cancel{\ln 1}^{\,0} \approx 1.099$$

26.) $f(x) = \frac{\sqrt{x-1}}{x}$ on $[1,5]$, $n=4$



$$T_4 = \frac{5-1}{2(4)} \cdot \left[f(1) + 2f(2) + 2f(3) + 2f(4) + f(5) \right]$$

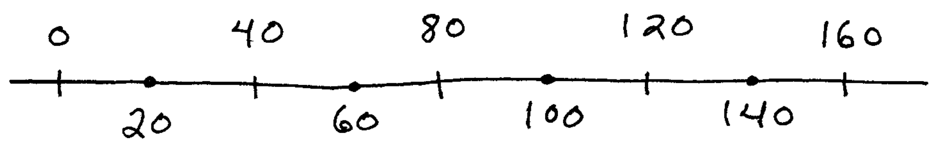
$$= \frac{1}{2}\left[0 + 2\cdot\frac{1}{2} + 2\cdot\frac{\sqrt{2}}{3} + 2\cdot\frac{\sqrt{3}}{4} + \frac{2}{5} \right] \approx 1.604$$

Exact (using calculator): $\int_1^5 \frac{\sqrt{x-1}}{x}\,dx \approx 1.786$

31.) $T_4 = \frac{20-0}{2(4)} \cdot \left[v(0) + 2\cdot v(5) + 2\cdot v(10) + 2\cdot v(15) + v(20) \right]$

$$= \frac{5}{2}\cdot\left[0.0 + 2(29.3) + 2(51.3) + 2(66.0) + 73.3 \right]$$

$$= \frac{5}{2}\cdot\left[366.5\right] = 916.25 \text{ ft.}$$

32.) a.) 

$$M_4 = \frac{160-0}{4}\cdot\left[f(20) + f(60) + f(100) + f(140) \right]$$

$$= 40\left[50 + 82 + 73 + 80 \right] = 40(285) = 11{,}400 \text{ ft.}^2$$

b.) $T_8 = \frac{160-0}{2(8)} \cdot \left[f(0) + 2f(20) + 2f(40) + 2f(60) \right.$

$$\left. + 2f(80) + 2f(100) + 2f(120) + 2f(140) + f(160) \right]$$

$$= 10\cdot\left[0 + 2(50) + 2(54) + 2(82) + 2(82) + 2(73) \right.$$

$$\left. + 2(75) + 2(80) + 0 \right]$$

$$= 10\left[992\right] = 9920 \text{ ft.}^2$$

Section 5.5

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51.) If C is cost in millions of \$/yr., then $\int_4^{10} C(t)\,dt$ is total # of millions of \$ spent from 2004 to 2010:

Projected SAVINGS is

$$\int_4^{10} C_1(t)\,dt - \int_4^{10} C_2(t)\,dt = \int_4^{10} \left(C_1(t) - C_2(t)\right) dt$$

$$= \int_4^{10} \left[(568.5 + 7.15t) - (525.6 + 6.43t)\right] dt$$

$$= \int_4^{10} \left[42.9 + 0.72t\right] dt$$

$$= (42.9t + 0.36t^2) \Big|_4^{10}$$

$$= (429 + 36) - (171.6 + 5.76) = 287.64$$ or

\$287,640,000 saved.

52.) $N_1(t) = 0.1t^2 + 0.5t + 150$ are infected on week t; for example, $N(0) = 150$ people, $N(10) = 165$ people, $N(25) \doteq 225$, etc. Thus, $\int_{25}^{50} N_1(t)\,dt$ is total # infected from week 25 to week 50.

N_1 ↑ people per week; dt ↑ week

But $N_2(t) = -0.2t^2 + 6t + 200$ is a new infection rate model starting on week 25; for example $N_2(25) = 225$ people, $N(40) = 120$ people, and $N(50) \doteq 0$ people. Thus, the total # of people prevented from infection is

$$T = \int_{25}^{50} (N_1(t) - N_2(t))\,dt$$

$$= \int_{25}^{50} [(0.1t^2 + 0.5t + 150) - (-0.2t^2 + 6t + 200)]\,dt$$

$$= \int_{25}^{50} [0.3t^2 - 5.5t - 50]\,dt$$

$$= (0.1t^3 - 2.75t^2 - 50t) \Big|_{25}^{50}$$

$$= 3125 - (-1406.25) \approx 4531 \text{ people}$$

Handout 8

1.) a.) $\text{Ave.} = \frac{1}{\pi - 0}\int_0^{\pi} \sin x \, dx$

$= \frac{1}{\pi}\left(-\cos x\right)\Big|_0^{\pi}$

$= \frac{1}{\pi}(-\cos \pi) - \frac{1}{\pi}(-\cos 0)$

$= \frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}$

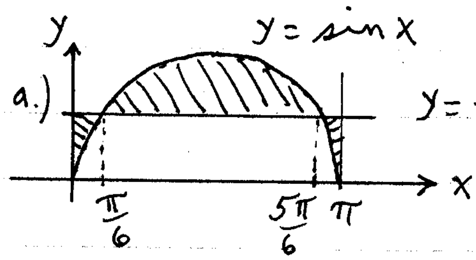
b.) $\text{Ave.} = \frac{1}{2\pi - 0}\int_0^{2\pi} \cos x \, dx$

$= \frac{1}{2\pi}\left(\sin x\right)\Big|_0^{2\pi} = \frac{1}{2\pi}\sin 2\pi - \frac{1}{2\pi}\sin 0 = 0$

c.) $\text{Ave.} = \frac{1}{\frac{\pi}{6} - 0}\int_0^{\frac{\pi}{6}} \sec^2 2x \, dx = \frac{6}{\pi}\cdot\frac{1}{2}\tan 2x\Big|_0^{\frac{\pi}{6}}$

$= \frac{3}{\pi}\tan\frac{\pi}{3} - \frac{3}{\pi}\overset{0}{\tan 0} = \frac{3}{\pi}\cdot\sqrt{3}$

2.) a.)

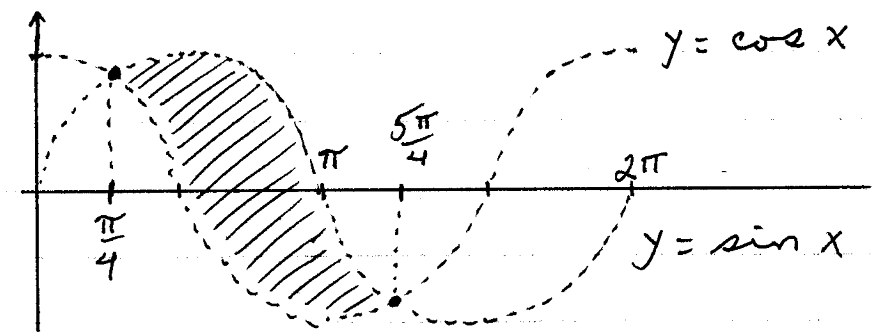


$\text{Area} = 2\int_0^{\frac{\pi}{6}}\left[\frac{1}{2} - \sin x\right]dx + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left[\sin x - \frac{1}{2}\right]dx$

$= 2\left(\frac{1}{2}x + \cos x\right)\Big|_0^{\frac{\pi}{6}} + \left(-\cos x - \frac{1}{2}x\right)\Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

$= 2\left(\frac{\pi}{12} + \frac{\sqrt{3}}{2}\right) - 2(0+1) + \left(\frac{\sqrt{3}}{2} - \frac{5\pi}{12}\right) - \left(\frac{-\sqrt{3}}{2} - \frac{\pi}{12}\right)$

$= \frac{\pi}{6} + \sqrt{3} - 2 + \sqrt{3} - \frac{\pi}{3} = 2\sqrt{3} - 2 - \frac{\pi}{6} \approx .94$

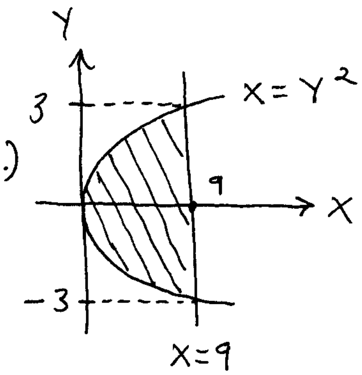
b.) 

$$\text{Area} = \int_{\frac{\pi}{4}}^{\frac{5\pi}{\pi}} [\sin x - \cos x]\, dx$$

$$= (-\cos x - \sin x) \Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

3.) a.)

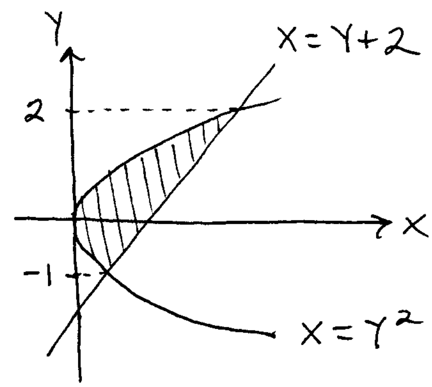


$$\text{Area} = \int_{-3}^{3} (9 - Y^2)\, dy$$

$$= \left(9Y - \tfrac{1}{3}Y^3\right)\Big|_{-3}^{3}$$

$$= (27 - 9) - (-27 + 9) = 36$$

b.)



$Y + 2 = Y^2 \rightarrow Y^2 - Y - 2 = 0 \rightarrow$

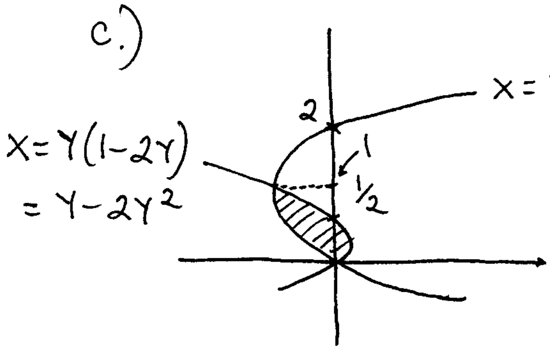
$(Y-2)(Y+1) = 0$

$Y = 2 \qquad Y = -1$

$$\text{Area} = \int_{-1}^{2} \left[(Y+2) - Y^2\right] dy = \left(\tfrac{1}{2}Y^2 + 2Y - \tfrac{1}{3}Y^3\right)\Big|_{-1}^{2}$$

$$= \left(2 + 4 - \tfrac{8}{3}\right) - \left(\tfrac{1}{2} - 2 + \tfrac{1}{3}\right) = \frac{9}{2}$$

c.)



$X = Y(Y-2) = Y^2 - 2Y$

$Y^2 - 2Y = Y - 2Y^2 \rightarrow 3Y^2 - 3Y = 0 \rightarrow$

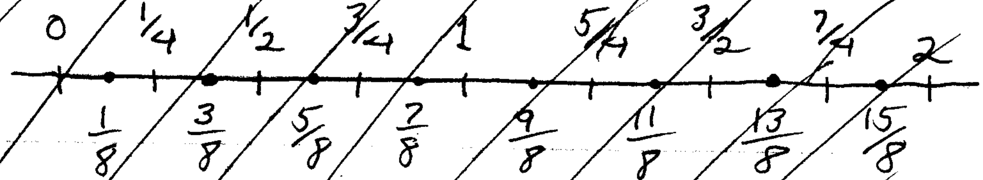
$3Y(Y-1) = 0$

$Y = 0 \qquad Y = 1$

$$\text{Area} = \int_0^1 \left[(Y - 2Y^2) - (Y^2 - 2Y)\right] dy$$

$$= \int_0^1 (3Y - 3Y^2)\, dy = \left(\tfrac{3}{2}Y^2 - Y^3\right)\Big|_0^1$$

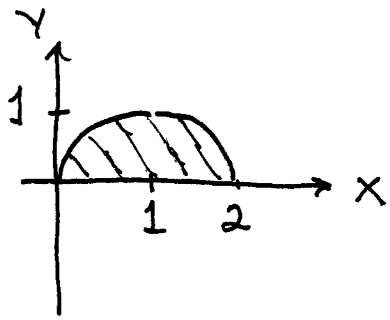
$$= \frac{3}{2} - 1 = \frac{1}{2}$$



$$M_8 = \frac{2-0}{8}\left[\frac{1}{8}\sqrt{\frac{513}{512}} + \frac{3}{8}\sqrt{\frac{539}{512}} + \frac{5}{8}\sqrt{\frac{637}{512}} + \frac{7}{8}\sqrt{\frac{855}{512}} + \frac{9}{8}\sqrt{\frac{1241}{512}} + \frac{11}{8}\sqrt{\frac{1843}{512}} + \frac{13}{8}\sqrt{\frac{2709}{512}} + \frac{15}{8}\sqrt{\frac{3887}{512}}\right] \approx 3.901$$

$\boxed{\text{SA11}}$: $Y = \sqrt{1-(x-1)^2} \rightarrow Y^2 = 1-(x-1)^2 \rightarrow$

$(x-1)^2 + Y^2 = 1 \rightarrow$ top half of circle only!



Solid formed by revolving region about the x-axis is a sphere of radius 1 :

$$V = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi$$