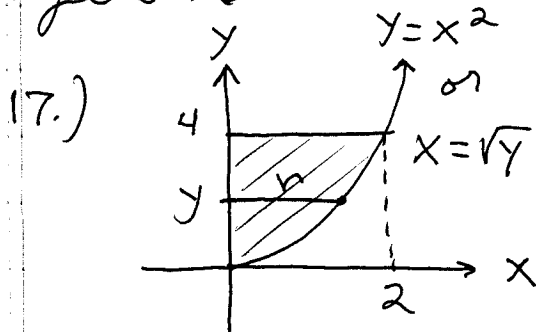


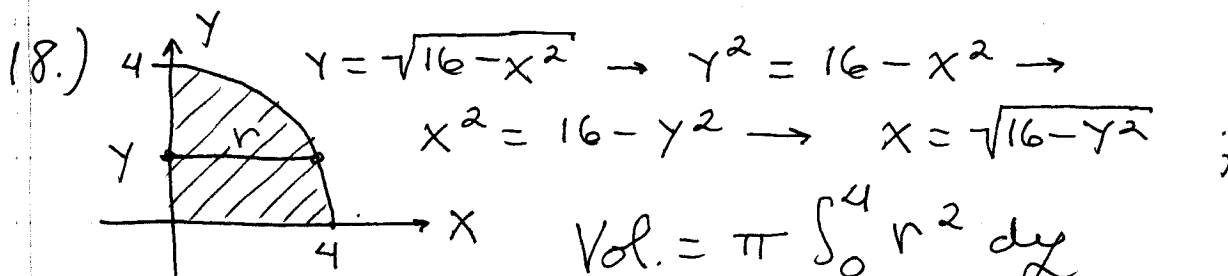
Section 5.7

Page 376

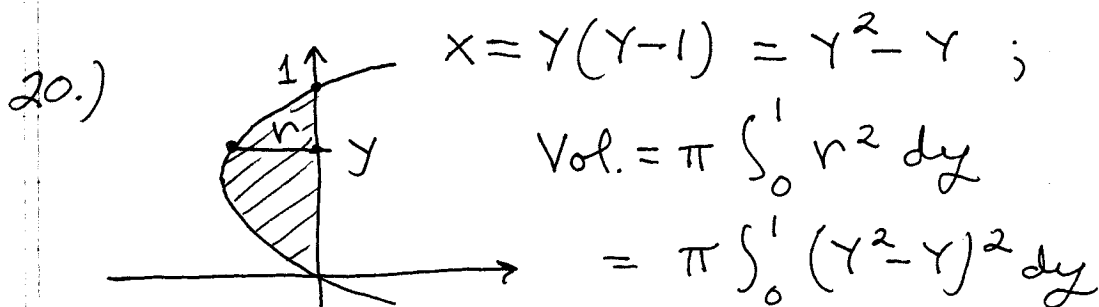


$$= \pi \cdot \frac{1}{2} y^2 \Big|_0^4 = \boxed{8\pi}$$

$$\begin{aligned} \text{Vol.} &= \pi \int_0^4 r^2 dy \\ &= \pi \int_0^4 (\sqrt{y})^2 dy \\ &= \pi \int_0^4 y dy \end{aligned}$$

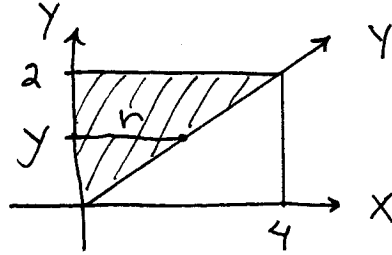


$$\begin{aligned} \text{Vol.} &= \pi \int_0^4 r^2 dy \\ &= \pi \int_0^4 (\sqrt{16-y^2})^2 dy = \pi \int_0^4 (16-y^2) dy \\ &= \pi \left(16y - \frac{1}{3} y^3 \right) \Big|_0^4 = \pi \left(64 - \frac{1}{3} \cdot 64 \right) \\ &= \frac{2}{3} \cdot 64 \pi = \boxed{\frac{128}{3} \pi} \end{aligned}$$

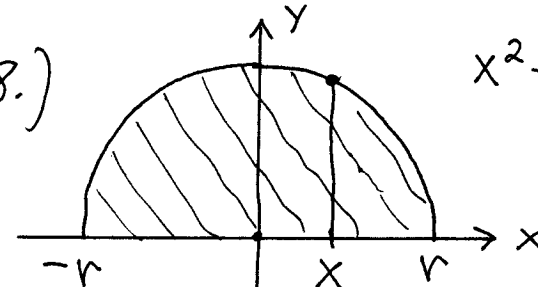


$$\begin{aligned} \text{Vol.} &= \pi \int_0^1 r^2 dy \\ &= \pi \int_0^1 (y^2-y)^2 dy \\ &= \pi \int_0^1 (y^4 - 2y^3 + y^2) dy \\ &= \pi \left[\frac{1}{5} y^5 - \frac{1}{2} y^4 + \frac{1}{3} y^3 \right] \Big|_0^1 \\ &= \pi \left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3} \right) = \pi \left(\frac{6}{30} - \frac{15}{30} + \frac{10}{30} \right) \\ &= \boxed{\frac{\pi}{30}} \end{aligned}$$

$$\begin{aligned}
 22.) \quad \text{Vol.} &= \pi \int_1^4 r^2 dy = \pi \int_1^4 (4y - y^2)^2 dy \\
 &= \pi \int_1^4 (16y^2 - 8y^3 + y^4) dy \\
 &= \pi \left(\frac{16}{3} y^3 - 2y^4 + \frac{1}{5} y^5 \right) \Big|_1^4 \\
 &= \pi \left[\left(\frac{1024}{3} - 512 + \frac{1024}{5} \right) - \left(\frac{16}{3} - 2 + \frac{1}{5} \right) \right] \\
 &= \pi \left[\frac{1008}{3} - 510 + \frac{1023}{5} \right] \\
 &= \pi \left[\frac{5040}{15} - \frac{7650}{15} + \frac{3069}{15} \right] \\
 &= \frac{459}{15} \pi = \frac{153}{5} \pi
 \end{aligned}$$

26.)  $y = \frac{1}{2}x$ or $x = 2y$;

$$\begin{aligned}
 \text{Vol.} &= \pi \int_0^2 r^2 dy \\
 &= \pi \int_0^2 (2y)^2 dy = \pi \int_0^2 4y^2 dy \\
 &= \pi \cdot \frac{4}{3} y^3 \Big|_0^2 = \frac{32}{3} \pi
 \end{aligned}$$

28.)  $x^2 + y^2 = r^2 \rightarrow y^2 = r^2 - x^2$
 $\rightarrow y = \sqrt{r^2 - x^2}$;

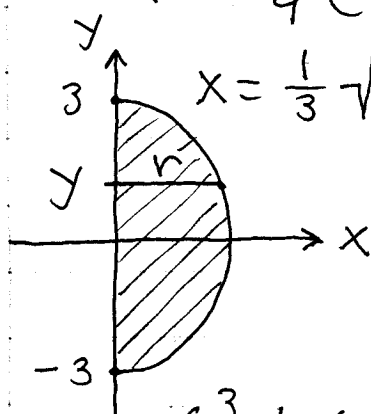
$$\begin{aligned}
 \text{Vol.} &= \pi \int_{-r}^r (\sqrt{r^2 - x^2})^2 dx \\
 &= \pi \int_{-r}^r (r^2 - x^2) dx
 \end{aligned}$$

$$= \pi \left(r^2 x - \frac{1}{3} x^3 \right) \Big|_{x=-r}^{x=r}$$

$$= \pi \left(r^3 - \frac{1}{3} r^3 \right) - \pi \left(-r^3 + \frac{1}{3} r^3 \right)$$

$$= \pi \left(\frac{2}{3} r^3 \right) - \pi \left(-\frac{2}{3} r^3 \right) = \boxed{\frac{4}{3} \pi r^3}$$

29.) $9x^2 + 25y^2 = 225 \rightarrow 9x^2 = 225 - 25y^2$
 $\rightarrow x^2 = \frac{1}{9} (225 - 25y^2) \rightarrow x = \frac{1}{3} \sqrt{225 - 25y^2}$;



$$x = \frac{1}{3} \sqrt{225 - 25y^2}$$

$$\text{Vol.} = \pi \int_{-3}^3 r^2 dy$$

$$= \pi \int_{-3}^3 \left(\frac{1}{3} \sqrt{225 - 25y^2} \right)^2 dy$$

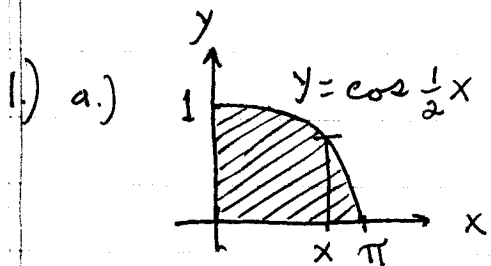
$$= \pi \int_{-3}^3 \frac{1}{9} (225 - 25y^2) dy$$

$$= \frac{\pi}{9} \left(225y - \frac{25}{3} y^3 \right) \Big|_{-3}^3$$

$$= \frac{\pi}{9} (675 - 225) - \frac{\pi}{9} (-675 + 225)$$

$$= \frac{\pi}{9} (450 + 450) = \boxed{100\pi}$$

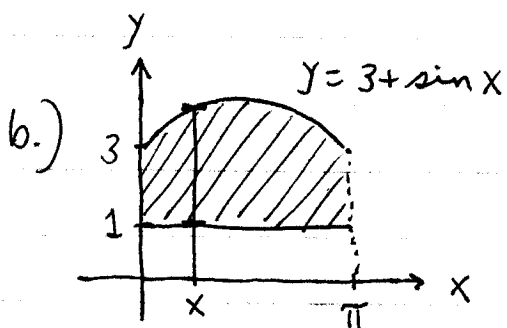
Handout 9



$$\begin{aligned} \text{Vol} &= \pi \int_0^{\pi} \cos^2 \frac{1}{2}x \, dx \\ &= \pi \int_0^{\pi} \frac{1 + \cos x}{2} \, dx \end{aligned}$$

$$= \pi \int_0^{\pi} \left(\frac{1}{2} + \frac{1}{2} \cos x \right) dx = \pi \left(\frac{1}{2}x + \frac{1}{2} \sin x \right) \Big|_0^{\pi}$$

$$= \pi \left(\frac{1}{2}\pi + \frac{1}{2} \sin \pi \right) - \pi \left(0 + \frac{1}{2} \sin 0 \right) = \boxed{\frac{\pi^2}{2}}$$



$$\begin{aligned} \text{Vol} &= \pi \int_0^{\pi} (3 + \sin x)^2 dx \\ &\quad - \pi \int_0^{\pi} (1)^2 dx \end{aligned}$$

$$= \pi \int_0^{\pi} (8 + 6 \sin x + \sin^2 x) dx$$

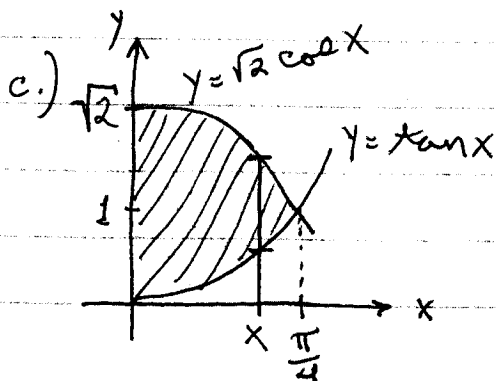
$$= \pi \int_0^{\pi} \left(8 + 6 \sin x + \frac{1 - \cos 2x}{2} \right) dx$$

$$= \pi \int_0^{\pi} \left(\frac{17}{2} + 6 \sin x - \frac{1}{2} \cos 2x \right) dx$$

$$= \pi \left(\frac{17}{2}x - 6 \cos x - \frac{1}{4} \sin 2x \right) \Big|_0^{\pi}$$

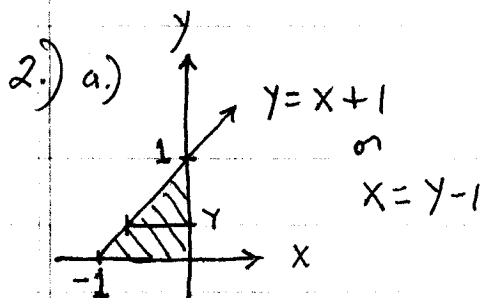
$$= \pi \left(\frac{17}{2}\pi - 6 \cos \pi - \frac{1}{4} \sin 2\pi \right) - \pi \left(0 - 6 \cos 0 - \frac{1}{4} \sin 0 \right)$$

$$= \pi \left(\frac{17}{2}\pi + 6 \right) + \pi \cdot 6 = \boxed{\frac{17}{2}\pi^2 + 12\pi}$$



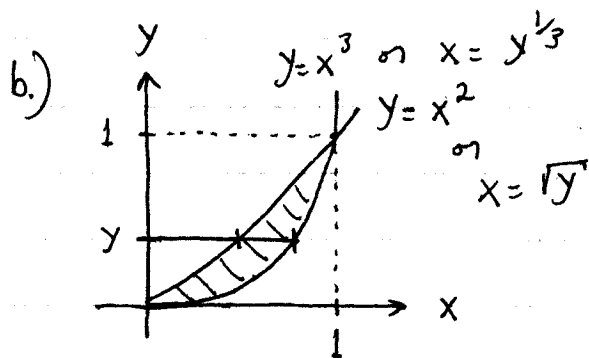
$$\begin{aligned} \text{Vol} &= \pi \int_0^{\frac{\pi}{4}} (\sqrt{2} \cos x)^2 dx \\ &\quad - \pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx \end{aligned}$$

$$\begin{aligned}
 &= \pi \int_0^{\frac{\pi}{4}} 2 \cos^2 x \, dx - \pi \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \, dx \\
 &= \pi \int_0^{\frac{\pi}{4}} \frac{1}{2} (1 + \cos 2x) \, dx - \pi (\tan x - x) \Big|_0^{\frac{\pi}{4}} \\
 &= \pi \left(x + \frac{1}{2} \sin 2x \right) \Big|_0^{\frac{\pi}{4}} - \pi \left(\tan \frac{\pi}{4} - \frac{\pi}{4} \right) + \pi (\tan 0 - 0) \\
 &= \pi \left(\frac{\pi}{4} + \frac{1}{2} \sin \frac{\pi}{2} \right) - \pi \left(0 + \frac{1}{2} \sin 0 \right) \\
 &\quad - \pi \left(1 - \frac{\pi}{4} \right) = \boxed{\frac{\pi^2}{2} - \frac{\pi}{2}}
 \end{aligned}$$



$$\begin{aligned}
 \text{Vol.} &= \pi \int_0^1 (y-1)^2 \, dy \\
 &= \pi \int_0^1 (y^2 - 2y + 1) \, dy \\
 &= \pi \left(\frac{1}{3} y^3 - y^2 + y \right) \Big|_0^1
 \end{aligned}$$

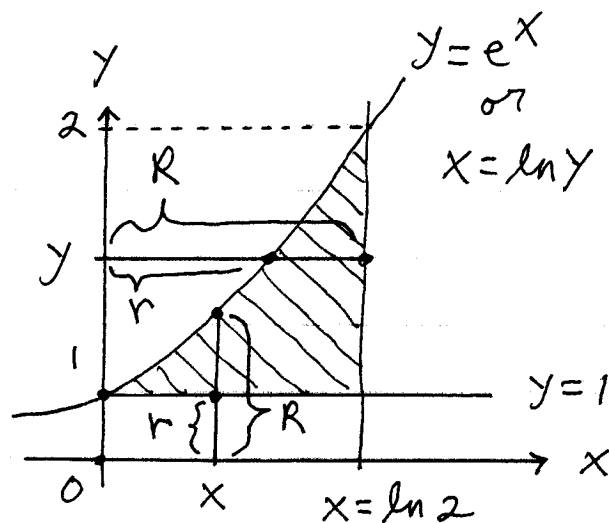
$$= \pi \left(\frac{1}{3} \right) - \pi(0) = \boxed{\frac{1}{3} \pi}$$



$$\begin{aligned}
 \text{Vol.} &= \pi \int_0^1 (y^{1/3})^2 \, dy \\
 &\quad - \pi \int_0^1 (\sqrt{y})^2 \, dy \\
 &= \pi \int_0^1 (y^{2/3} - y) \, dy
 \end{aligned}$$

$$= \pi \left(\frac{3}{5} y^{5/3} - \frac{1}{2} y^2 \right) \Big|_0^1 = \pi \left(\frac{3}{5} - \frac{1}{2} \right) = \boxed{\frac{\pi}{10}}$$

$$\begin{aligned}
 3.) \text{ a.) } Vol &= \pi \int_0^{\ln 2} R^2 dx \\
 &\quad - \pi \int_0^{\ln 2} r^2 dx \\
 &= \pi \int_0^{\ln 2} (e^x)^2 dx \\
 &\quad - \pi \int_0^{\ln 2} (1)^2 dx
 \end{aligned}$$



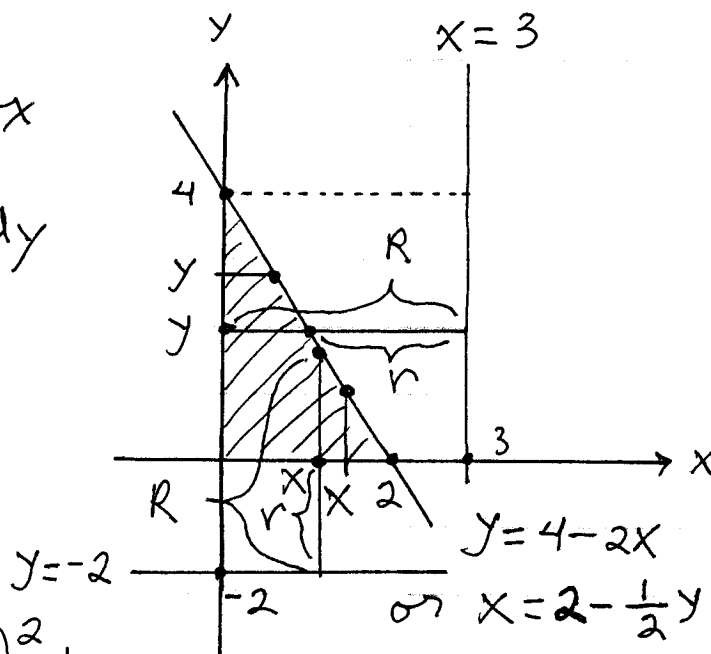
$$\begin{aligned}
 \text{b.) } Vol &= \pi \int_1^2 R^2 dy - \pi \int_1^2 r^2 dy \\
 &= \pi \int_1^2 (\ln 2)^2 dy - \pi \int_1^2 (\ln y)^2 dy
 \end{aligned}$$

$$4.) \text{ a.) } Vol = \pi \int_0^2 (4-2x)^2 dx$$

$$\text{b.) } Vol = \pi \int_0^4 (2 - \frac{1}{2}y)^2 dy$$

$$\begin{aligned}
 \text{c.) } Vol &= \pi \int_0^4 R^2 dy \\
 &\quad - \pi \int_0^4 r^2 dy
 \end{aligned}$$

$$\begin{aligned}
 &= \pi \int_0^4 (3)^2 dy \\
 &\quad - \pi \int_0^4 (3 - (2 - \frac{1}{2}y))^2 dy
 \end{aligned}$$



$$\text{d.) } Vol = \pi \int_0^2 R^2 dx - \pi \int_0^2 r^2 dx$$

$$= \pi \int_0^2 (2 + (4 - 2x))^2 dx - \pi \int_0^2 (2)^2 dx$$

5.) $x^3 = 4\sqrt{2x} \rightarrow (\text{square})$

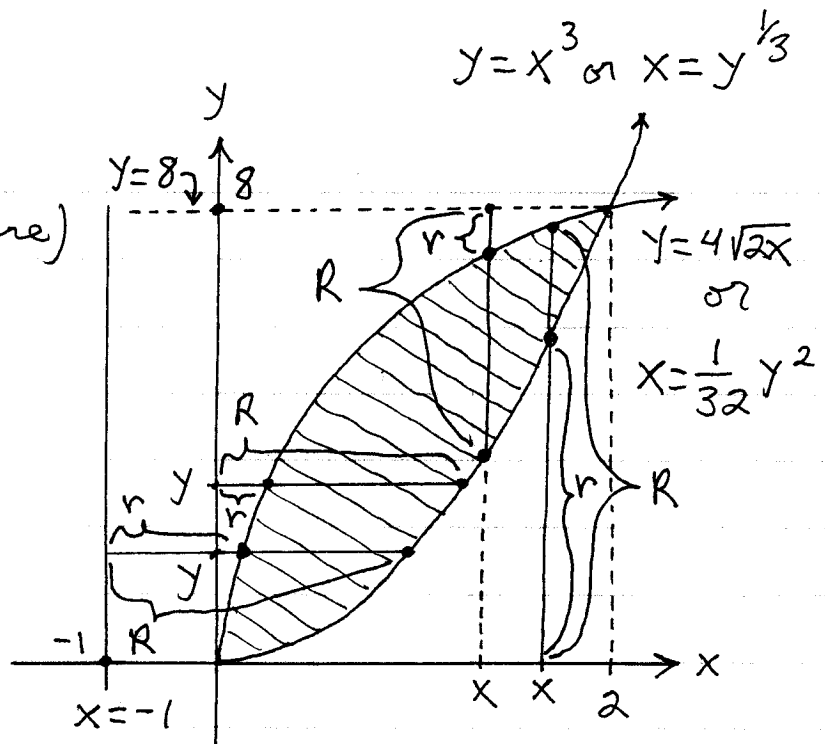
$$x^6 = 16(2x) \rightarrow$$

$$x^6 - 32x = 0 \rightarrow$$

$$x(x^5 - 32) = 0 \rightarrow$$

$$\downarrow \quad \quad \downarrow$$

$$x=0 \quad \quad x=2$$



a.)
$$\text{Vol} = \pi \int_0^2 R^2 dx - \pi \int_0^2 r^2 dx$$

$$= \pi \int_0^2 (4\sqrt{2x})^2 dx - \pi \int_0^2 (x^3)^2 dx$$

b.)
$$\text{Vol} = \pi \int_0^8 R^2 dy - \pi \int_0^8 r^2 dy$$

$$= \pi \int_0^8 (y^{1/3})^2 dy - \pi \int_0^8 (\frac{1}{32} y^2)^2 dy$$

c.)
$$\text{Vol} = \pi \int_0^8 R^2 dy - \pi \int_0^8 r^2 dy$$

$$= \pi \int_0^8 (1 + y^{1/3})^2 dy - \pi \int_0^8 (1 + \frac{1}{32} y^2)^2 dy$$

d.)
$$\text{Vol} = \pi \int_0^2 R^2 dx - \pi \int_0^2 r^2 dx$$

$$= \pi \int_0^2 (8 - x^3)^2 dx - \pi \int_0^2 (8 - 4\sqrt{2x})^2 dx$$