

Section 6.2

Page 403

$$\begin{aligned} 2.) \int x e^{-x} dx & \quad (\text{Let } u=x, dv=e^{-x} dx \\ & \quad \rightarrow du=dx, v=-e^{-x}) \\ & = -x e^{-x} - \int e^{-x} dx \\ & = -x e^{-x} + \frac{e^{-x}}{(-1)} + C = -x e^{-x} - e^{-x} + C \end{aligned}$$

$$\begin{aligned} 5.) \int \ln 2x dx & \quad (\text{Let } u=\ln 2x, dv=dx \\ & \quad \rightarrow du=\frac{2}{2x} dx = \frac{1}{x} dx, v=x) \\ & = x \ln 2x - \int \frac{1}{x} x dx = x \ln 2x - \int 1 dx \\ & = x \ln 2x - x + C \end{aligned}$$

$$7.) \int e^{4x} dx = \frac{1}{4} e^{4x} + C$$

$$\begin{aligned} 9.) \int x e^{4x} dx & \quad (\text{Let } u=x, dv=e^{4x} dx \\ & \quad \rightarrow du=dx, v=\frac{1}{4} e^{4x}) \\ & = \frac{1}{4} x e^{4x} - \frac{1}{4} \int e^{4x} dx \\ & = \frac{1}{4} x e^{4x} - \frac{1}{4} \cdot \frac{1}{4} e^{4x} + C \end{aligned}$$

$$\begin{aligned} 11.) \int x e^{x^2} dx & \quad (\text{Let } u=x^2 \rightarrow du=2x dx \\ & \quad \rightarrow \frac{1}{2} du = x dx) \\ & = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C \end{aligned}$$

$$\begin{aligned}
 13.) \int x^2 e^x dx & \quad (\text{Let } u = x^2, dv = e^x dx \\
 & \rightarrow du = 2x dx, v = e^x) \\
 & = x^2 e^x - 2 \int x e^x dx \quad (\text{Let } u = x, dv = e^x dx \\
 & \rightarrow du = dx, v = e^x) \\
 & = x^2 e^x - 2 [x e^x - \int e^x dx] \\
 & = x^2 e^x - 2x e^x + 2 \cdot e^x + C
 \end{aligned}$$

$$\begin{aligned}
 16.) \int x^3 \ln x dx & \quad (\text{Let } u = \ln x, dv = x^3 dx \\
 & \rightarrow du = \frac{1}{x} dx, v = \frac{1}{4} x^4) \\
 & = \frac{1}{4} x^4 \ln x - \frac{1}{4} \int \frac{1}{x} x^4 dx \\
 & = \frac{1}{4} x^4 \ln x - \frac{1}{4} \int x^3 dx \\
 & = \frac{1}{4} x^4 \ln x - \frac{1}{4} \cdot \frac{1}{4} x^4 + C
 \end{aligned}$$

$$\begin{aligned}
 17.) \int \frac{e^{1/t}}{t^2} dt & \quad (\text{Let } u = \frac{1}{t} = t^{-1} \rightarrow du = -t^{-2} dt \\
 & \rightarrow -du = \frac{1}{t^2} dt) \\
 & = - \int e^u du = -e^u + C = -e^{1/t} + C
 \end{aligned}$$

$$\begin{aligned}
 18.) \int \frac{1}{x(\ln x)^3} dx & \quad (\text{Let } u = \ln x \rightarrow du = \frac{1}{x} dx) \\
 & = \int \frac{1}{u^3} du = \int u^{-3} du = \frac{u^{-2}}{-2} + C = \frac{(\ln x)^{-2}}{-2} + C
 \end{aligned}$$

$$19.) \int x(\ln x)^2 dx \quad (\text{Let } u = (\ln x)^2, dv = x dx$$

$$\rightarrow du = \frac{2(\ln x)}{x} dx, \quad v = \frac{1}{2}x^2$$

$$= \frac{1}{2}x^2(\ln x)^2 - \int x \ln x dx$$

$$\begin{aligned} & (\text{Let } u = \ln x, \quad dv = x dx \\ & \rightarrow du = \frac{1}{x} dx, \quad v = \frac{1}{2}x^2) \end{aligned}$$

$$= \frac{1}{2}x^2(\ln x)^2 - \left[\frac{1}{2}x^2 \ln x - \frac{1}{2} \int x dx \right]$$

$$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{2} \cdot \frac{1}{2}x^2 + c$$

$$21.) \int \frac{(\ln x)^2}{x} dx \quad (\text{Let } u = \ln x \rightarrow du = \frac{1}{x} dx)$$

$$= \int u^2 du = \frac{1}{3}u^3 + c = \frac{1}{3}(\ln x)^3 + c$$

$$22.) \int \frac{1}{x \ln x} dx \quad (\text{Let } u = \ln x \rightarrow du = \frac{1}{x} dx)$$

$$= \int \frac{1}{u} du = \ln|u| + c = \ln|\ln x| + c$$

$$24.) \int \frac{x}{\sqrt{x-1}} dx \quad (\text{Let } u = x-1 \rightarrow du = dx \text{ and } x = u+1)$$

$$= \int \frac{u+1}{\sqrt{u}} du = \int \left(\frac{u}{u^{1/2}} + \frac{1}{u^{1/2}} \right) du$$

$$= \int (u^{1/2} + u^{-1/2}) du = \frac{2}{3}u^{3/2} + 2u^{1/2} + c$$

$$= \frac{2}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + c$$

$$27.) \int \frac{x e^{2x}}{(2x+1)^2} dx \quad (\text{Let } u = x e^{2x}, \quad dv = \frac{1}{(2x+1)^2} dx)$$

$$\rightarrow du = (x \cdot 2e^{2x} + e^{2x}) dx = (2x+1)e^{2x} dx,$$

$$v = \frac{1}{2} \cdot \frac{1}{2x+1}$$

$$= \frac{xe^{2x}}{2(2x+1)} - \frac{1}{2} \int e^{2x} dx$$

$$= \frac{xe^{2x}}{2(2x+1)} - \frac{1}{2} \cdot \frac{1}{2} e^{2x} + C$$

$$32.) \int_1^e 2x \ln x dx \quad (\text{Let } u = \ln x, dv = 2x dx \\ \rightarrow du = \frac{1}{x} dx, v = x^2)$$

$$= x^2 \ln x \Big|_1^e - \int_1^e x dx$$

$$= e^2 \ln e - 1^2 \ln 1 - \frac{1}{2} x^2 \Big|_1^e$$

$$= e^2 - \left(\frac{1}{2} e^2 - \frac{1}{2} \right) = \frac{1}{2} e^2 + \frac{1}{2}$$

$$33.) \int_{-1}^0 \ln(x+2) dx \quad (\text{Let } u = \ln(x+2), dv = dx \\ \rightarrow du = \frac{1}{x+2} dx, v = x)$$

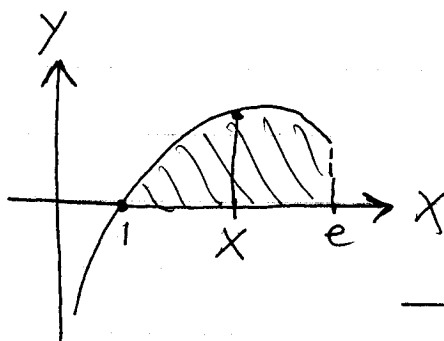
$$= x \ln(x+2) \Big|_{-1}^0 - \int_{-1}^0 \frac{x}{x+2} dx$$

$$= (0) \ln 2 - (-1) \ln 1 - \int_{-1}^0 \frac{x+2-2}{x+2} dx$$

$$= - \int_{-1}^0 \left(\frac{x+2}{x+2} - \frac{2}{x+2} \right) dx$$

$$= - \int_{-1}^0 \left(1 - 2 \cdot \frac{1}{x+2} \right) dx = - (x - 2 \ln(x+2)) \Big|_{-1}^0$$

$$= -(0 - 2 \ln 2) - (-(-1 - 2 \ln 1)) = 2 \ln 2 - 1$$

38.)  Area = $\int_1^e \frac{\ln x}{x^2} dx$
 (Let $u = \ln x$, $dv = \frac{1}{x^2} dx$
 $\rightarrow du = \frac{1}{x} dx$, $v = -\frac{1}{x}$)

$$= -\frac{\ln x}{x} \Big|_1^e - \int_1^e \frac{1}{x^2} dx$$

$$= \left(-\frac{\ln e}{e} - -\frac{\ln 1}{1} \right) + \frac{1}{x} \Big|_1^e$$

$$= -\frac{1}{e} - \left(\frac{1}{e} - 1 \right) = 1 - \frac{2}{e}$$

40.) a.) $\int x \sqrt{4+x} dx$ (Let $u = x$, $dv = \sqrt{4+x} dx$
 $\rightarrow du = dx$, $v = \frac{2}{3} (4+x)^{3/2}$)

$$= \frac{2}{3} x (4+x)^{3/2} - \frac{2}{3} \int (4+x)^{3/2} dx$$

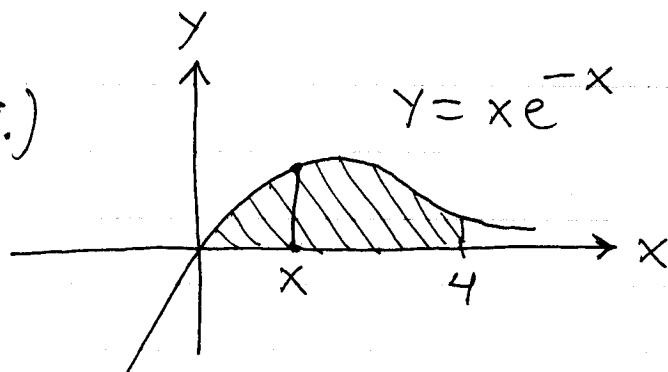
$$= \frac{2}{3} x (4+x)^{3/2} - \frac{2}{3} \cdot \frac{2}{5} (4+x)^{5/2} + C$$

b.) $\int x \sqrt{4+x} dx$ (Let $u = \sqrt{4+x} \rightarrow$
 $du = \frac{1}{2} (4+x)^{-1/2} dx = \frac{1}{2u} dx \rightarrow$
 $2u du = dx$, and $u^2 = 4+x \rightarrow$
 $x = u^2 - 4$)

$$= \int (u^2 - 4) \cdot u \cdot 2u du = \int (2u^4 - 8u^2) du$$

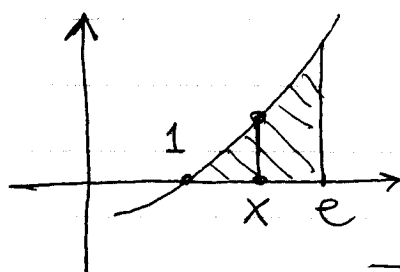
$$= \frac{2}{5} u^5 - \frac{8}{3} u^3 + C = \frac{2}{5} (\sqrt{4+x})^5 - \frac{8}{3} (\sqrt{4+x})^3 + C$$

49.)



$$\begin{aligned}
 \text{Area} &= \int_0^4 x e^{-x} dx \quad (\text{let } u=x, dv=e^{-x} dx \\
 &\quad \rightarrow du=dx, v=-e^{-x}) \\
 &= -x e^{-x} \Big|_0^4 - \int_0^4 e^{-x} dx \\
 &= -4e^{-4} - 0 + \frac{e^{-x}}{(-1)} \Big|_0^4 = \frac{-4}{e^4} + (-e^{-4} - e^0) \\
 &= \frac{-4}{e^4} + \frac{-1}{e^4} + 1 = 1 - \frac{5}{e^4}
 \end{aligned}$$

51.)



$$y = x \ln x$$

$$\text{Area} = \int_1^e x \ln x dx$$

$$\begin{aligned}
 &(\text{let } u=\ln x, dv=x dx \\
 &\rightarrow du=\frac{1}{x} dx, v=\frac{1}{2} x^2)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} x^2 \ln x \Big|_1^e - \frac{1}{2} \int_1^e x dx \\
 &= \frac{1}{2} e^2 \ln e - \frac{1}{2} 1^2 \ln 1 - \frac{1}{2} \cdot \frac{1}{2} x^2 \Big|_1^e \\
 &= \frac{1}{2} e^2 - \frac{1}{4} (e^2 - 1) = \frac{1}{4} e^2 + \frac{1}{4}
 \end{aligned}$$

$$54.) \text{ b.) } \text{Vol.} = \pi \int_0^1 r^2 dx = \pi \int_0^1 (x e^x)^2 dx$$

$$\begin{aligned}
&= \pi \int_0^1 x^2 e^{2x} dx \quad (\text{let } u = x^2, dv = e^{2x} dx \\
&\quad \rightarrow du = 2x dx, v = \frac{1}{2} e^{2x}) \\
&= \pi \left[\frac{1}{2} x^2 e^{2x} \Big|_0^1 - \int_0^1 x e^{2x} dx \right] \\
&\quad (\text{let } u = x, dv = e^{2x} dx \\
&\quad \rightarrow du = dx, v = \frac{1}{2} e^{2x}) \\
&= \pi \left[\frac{1}{2} e^2 - \left\{ \frac{1}{2} x e^{2x} \Big|_0^1 - \frac{1}{2} \int_0^1 e^{2x} dx \right\} \right] \\
&= \pi \left[\frac{1}{2} e^2 - \left\{ \frac{1}{2} e^2 - \frac{1}{2} \cdot \frac{1}{2} e^{2x} \Big|_0^1 \right\} \right] \\
&= \pi \left[\cancel{\frac{1}{2} e^2} - \cancel{\frac{1}{2} e^2} + \frac{1}{4} (e^2 - 1) \right] = \frac{\pi}{4} (e^2 - 1)
\end{aligned}$$

59.) SEE NEXT PAGE

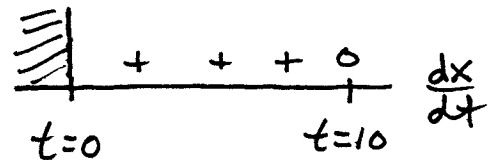
$$\begin{aligned}
61.) a.) \text{ AVE} &= \frac{1}{2-1} \int_1^2 (1 + 1.6t \ln t) dt \\
&= t \Big|_1^2 + 1.6 \int_1^2 t \ln t dt \quad (\text{let } u = \ln t, dv = t dt \\
&\quad \rightarrow du = \frac{1}{t} dt, v = \frac{1}{2} t^2) \\
&= (2-1) + 1.6 \left[\frac{1}{2} t^2 \ln t \Big|_1^2 - \frac{1}{2} \int_1^2 t dt \right] \\
&= 1 + 1.6 \left[(2 \ln 2 - \frac{1}{2} \cancel{\ln 1}) - \frac{1}{2} \cdot \frac{1}{2} t^2 \Big|_1^2 \right] \\
&= 1 + 1.6 \left[2 \ln 2 - \frac{1}{4} (4-1) \right] \\
&= 1 + (3.2) \ln 2 - 1.2 \\
&= (3.2) \ln 2 - 0.2 \approx 2.018
\end{aligned}$$

59.) x : # of units (demand) t : # yrs.

$$x = 500(20 + te^{-0.1t}) \rightarrow$$

$$\frac{dx}{dt} = 500 \cdot [te^{-0.1t}(-0.1) + e^{-0.1t}]$$

$$= 500 \cdot e^{-0.1t} \left(1 - \frac{1}{10}t\right) = 0$$



a.) Since $\frac{dx}{dt} \geq 0$ for $0 \leq t \leq 10$ the demand is increasing.

b.) Total Demand is

$$TOT = \int_0^{10} 500(20 + te^{-\frac{1}{10}t}) dt$$

$$= \int_0^{10} 10,000 dt + 500 \int_0^{10} te^{-\frac{1}{10}t} dt$$

$$\left(\text{Let } u = t, dv = e^{-\frac{1}{10}t} dt \right. \\ \left. du = dt, v = -10e^{-\frac{1}{10}t} \right)$$

$$= 100,000 + 500 \left\{ -10te^{-\frac{1}{10}t} \Big|_0^{10} + 10 \int_0^{10} e^{-\frac{1}{10}t} dt \right\}$$

$$= 100,000 + 500 \left\{ \frac{-100}{e} - 100e^{-\frac{1}{10}t} \Big|_0^{10} \right\}$$

$$= 100,000 + 500 \left\{ \frac{-100}{e} - 100 \left(\frac{1}{e} - 1 \right) \right\} \approx 113,212 \text{ units}$$

c.) Average Demand is

$$AVE = \frac{1}{10-0} \int_0^{10} 500(20 + te^{-\frac{1}{10}t}) dt$$

$$\approx 11,321 \text{ units/yr.}$$

