

Section 6.6

Page
444

$$1.) \quad \int_0^{\infty} e^{-x} dx = \lim_{b \rightarrow \infty} \int_0^b e^{-x} dx$$

$$= \lim_{b \rightarrow \infty} -e^{-x} \Big|_0^b = \lim_{b \rightarrow \infty} \left(\frac{-1}{e^b} - \frac{-1}{e^0} \right) = 0 + 1 = \textcircled{1}$$

$$2.) \quad \int_{-\infty}^0 e^{2x} dx = \lim_{b \rightarrow -\infty} \int_b^0 e^{2x} dx$$

$$= \lim_{b \rightarrow -\infty} \frac{1}{2} e^{2x} \Big|_b^0 = \lim_{b \rightarrow -\infty} \frac{1}{2} e^0 - \frac{1}{2} e^{2b}$$

$$= \frac{1}{2} - 0 = \textcircled{\frac{1}{2}}$$

$$3.) \quad \int_1^{\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \int_1^b x^{-2} dx$$

$$= \lim_{b \rightarrow \infty} -x^{-1} \Big|_1^b = \lim_{b \rightarrow \infty} \frac{-1}{x} \Big|_1^b$$

$$= \lim_{b \rightarrow \infty} \left(\frac{-1}{b} - \frac{-1}{1} \right) = 0 + 1 = \textcircled{1}$$

$$4.) \quad \int_1^{\infty} \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow \infty} \int_1^b x^{-1/2} dx$$

$$= \lim_{b \rightarrow \infty} 2x^{1/2} \Big|_1^b = \lim_{b \rightarrow \infty} (2\sqrt{b} - 2\sqrt{1}) = +\infty$$

so diverges

$$5.) \quad \int_0^{\infty} e^{\frac{x}{3}} dx = \lim_{b \rightarrow \infty} \int_0^b e^{\frac{x}{3}} dx$$

$$= \lim_{b \rightarrow \infty} 3e^{\frac{x}{3}} \Big|_0^b = \lim_{b \rightarrow \infty} (3e^{\frac{b}{3}} - 3e^0)$$

$$= +\infty - 3 \quad \text{so } \textcircled{\text{diverges}}$$

$$6.) \quad \int_0^{\infty} \frac{5}{e^{2x}} dx = \lim_{b \rightarrow \infty} \int_0^b 5 \cdot e^{-2x} dx$$

$$= \lim_{b \rightarrow \infty} -\frac{5}{2} e^{-2x} \Big|_0^b = \lim_{b \rightarrow \infty} \left(-\frac{5}{2} \cdot \frac{1}{e^{2b}} - -\frac{5}{2} \cdot e^0 \right)$$

$$= -\frac{5}{2} (0) + \frac{5}{2} = \left(\frac{5}{2} \right)$$

$$7.) \quad \int_5^{\infty} \frac{x}{\sqrt{x^2-16}} dx = \lim_{b \rightarrow \infty} \int_5^b x(x^2-16)^{-\frac{1}{2}} dx$$

$$= \lim_{b \rightarrow \infty} (x^2-16)^{\frac{1}{2}} \Big|_5^b = \lim_{b \rightarrow \infty} (\sqrt{b^2-16} - \sqrt{9})$$

$$= +\infty \quad \text{so } \text{diverges}$$

$$8.) \quad \int_{\frac{1}{2}}^{\infty} \frac{1}{\sqrt{2x-1}} dx = \int_{\frac{1}{2}}^1 \frac{1}{\sqrt{2x-1}} dx + \int_1^{\infty} \frac{1}{\sqrt{2x-1}} dx$$

$$= A + B;$$

$$A = \lim_{x \rightarrow \frac{1}{2}^+} \int_{\frac{1}{2}}^1 \frac{1}{\sqrt{2x-1}} dx = \lim_{x \rightarrow \frac{1}{2}^+} (2x-1)^{\frac{1}{2}} \Big|_{\frac{1}{2}}^1$$

$$= \lim_{x \rightarrow \frac{1}{2}^+} (1 - (2b-1)^{\frac{1}{2}}) = \underline{1} \quad \text{and}$$

$$B = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{\sqrt{2x-1}} dx = \lim_{b \rightarrow \infty} (2x-1)^{\frac{1}{2}} \Big|_1^b$$

$$= \lim_{b \rightarrow \infty} (\sqrt{2b-1} - 1) = \underline{+\infty - 1 = \infty} \quad \text{so } \text{diverges}$$

$$9.) \quad \int_{-\infty}^0 e^{-x} dx = \lim_{b \rightarrow -\infty} \int_b^0 e^{-x} dx$$

$$= \lim_{b \rightarrow -\infty} -e^{-x} \Big|_b^0 = \lim_{b \rightarrow -\infty} (-e^0 - -e^{-b})$$

$$= -1 + \infty \quad \text{so } \text{diverges}$$

$$10.) \quad \int_{-\infty}^{-1} \frac{1}{x^2} dx = \lim_{b \rightarrow -\infty} \int_b^{-1} x^{-2} dx$$

$$= \lim_{b \rightarrow -\infty} -x^{-1} \Big|_b^{-1} = \lim_{b \rightarrow -\infty} \left(\frac{-1}{(-1)} - \frac{-1}{(b)} \right) = 1 + 0 = \textcircled{1}$$

$$\begin{aligned} 12.) \quad \int_{-\infty}^0 \frac{x}{x^2+1} dx &= \lim_{A \rightarrow -\infty} \int_A^0 \frac{x}{x^2+1} dx \\ &= \lim_{A \rightarrow -\infty} \frac{1}{2} \ln(x^2+1) \Big|_A^0 = \lim_{A \rightarrow -\infty} \left(\frac{1}{2} \ln 1 - \frac{1}{2} \ln(A^2+1) \right) \\ &= -\infty \quad \text{diverges} \end{aligned}$$

$$\begin{aligned} 14.) \quad \int_{-\infty}^{\infty} x^2 e^{-x^3} dx &= \int_{-\infty}^0 x^2 e^{-x^3} dx + \int_0^{\infty} x^2 e^{-x^3} dx \\ &= A + B; \end{aligned}$$

$$B = \lim_{b \rightarrow \infty} \int_0^b x^2 e^{-x^3} dx = \lim_{b \rightarrow \infty} \left. -\frac{1}{3} e^{-x^3} \right|_0^b$$

$$= \lim_{b \rightarrow \infty} \left(-\frac{1}{3} \cdot \frac{1}{e^{b^3}} - \left(-\frac{1}{3} e^0 \right) \right) = (0) + \frac{1}{3} = \frac{1}{3} \quad \text{and}$$

$$A = \lim_{b \rightarrow -\infty} \int_b^0 x^2 e^{-x^3} dx = \lim_{b \rightarrow -\infty} \left. -\frac{1}{3} e^{-x^3} \right|_b^0$$

$$= \lim_{b \rightarrow -\infty} \left(-\frac{1}{3} e^0 - \left(-\frac{1}{3} e^{-b^3} \right) \right) = \left[-\frac{1}{3} + \infty \right] \quad \text{so}$$

diverges

$$15.) \quad \int_0^4 \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow 0^+} \int_b^4 x^{-1/2} dx$$

$$= \lim_{b \rightarrow 0^+} 2\sqrt{x} \Big|_b^4 = \lim_{b \rightarrow 0^+} (4 - 2\sqrt{b}) = \textcircled{4}$$

$$16.) \int_3^4 \frac{1}{\sqrt{x-3}} dx = \lim_{b \rightarrow 3^+} \int_b^4 (x-3)^{-1/2} dx$$

$$= \lim_{b \rightarrow 3^+} 2\sqrt{x-3} \Big|_b^4 = \lim_{b \rightarrow 3^+} (2 - 2\sqrt{b-3}) = \textcircled{2}$$

$$18.) \int_0^2 \frac{1}{(x-1)^2} dx = \int_0^1 (x-1)^{-2} dx + \int_1^2 (x-1)^{-2} dx$$

$$= A + B ;$$

$$A = \int_0^1 (x-1)^{-2} dx = \lim_{b \rightarrow 1^-} \int_0^b (x-1)^{-2} dx$$

$$= \lim_{b \rightarrow 1^-} -(x-1)^{-1} \Big|_0^b = \lim_{b \rightarrow 1^-} \frac{-1}{x-1} \Big|_0^b$$

$$= \lim_{b \rightarrow 1^-} \frac{-1}{b-1} - \frac{-1}{-1} = +\infty - 1 \text{ so } \textcircled{\text{diverges}}$$

(note that $B = +\infty$ also!)

$$19.) \int_0^1 \frac{1}{1-x} dx = \lim_{b \rightarrow 1^-} \int_0^b \frac{1}{1-x} dx$$

$$= \lim_{b \rightarrow 1^-} -\ln|1-x| \Big|_0^b = \lim_{b \rightarrow 1^-} (-\ln|1-b| + \ln 1)$$

$$= +\infty \quad \textcircled{\text{diverges}}$$

$$22.) \int_0^2 \frac{x}{\sqrt{4-x^2}} dx = \lim_{b \rightarrow 2^-} \int_0^b \frac{x}{\sqrt{4-x^2}} dx$$

$$= \lim_{b \rightarrow 2^-} -(4-x^2)^{1/2} \Big|_0^b = \lim_{b \rightarrow 2^-} (-\sqrt{4-b^2} + 2) = \textcircled{2}$$

$$23.) \int_0^1 \frac{1}{x^2} dx = \lim_{b \rightarrow 0^+} \int_b^1 \frac{1}{x^2} dx = \lim_{b \rightarrow 0^+} \frac{-1}{x} \Big|_b^1$$

$$= \lim_{b \rightarrow 0^+} \left(\frac{-1}{1} - \frac{-1}{b} \right) = -1 + \infty \quad \textcircled{\text{diverges}}$$

$$24.) \quad \int_0^1 \frac{1}{x} dx = \lim_{b \rightarrow 0^+} \int_b^1 \frac{1}{x} dx = \lim_{b \rightarrow 0^+} \ln|x| \Big|_b^1$$

$$= \lim_{b \rightarrow 0^+} (\ln 1 - \ln b) = \infty \quad \text{diverges}$$

$$25.) \quad \int_0^2 (x-1)^{-1/3} dx = \int_0^1 (x-1)^{-1/3} dx + \int_1^2 (x-1)^{-1/3} dx$$

$$= A + B ;$$

$$A = \lim_{b \rightarrow 1^-} \int_0^b (x-1)^{-1/3} dx = \lim_{b \rightarrow 1^-} \frac{3}{2} (x-1)^{2/3} \Big|_0^b$$

$$= \lim_{b \rightarrow 1^-} \left\{ \frac{3}{2} (b-1)^{2/3} - \frac{3}{2} (-1)^{2/3} \right\} = 0 - \frac{3}{2} = \underline{-\frac{3}{2}} ;$$

$$B = \lim_{b \rightarrow 1^+} \int_b^2 (x-1)^{-1/3} dx = \lim_{b \rightarrow 1^+} \frac{3}{2} (x-1)^{2/3} \Big|_b^2$$

$$= \lim_{b \rightarrow 1^+} \left\{ \frac{3}{2} (1)^{2/3} - \frac{3}{2} (b-1)^{2/3} \right\} = \frac{3}{2} - 0 = \underline{\frac{3}{2}} ; \text{ so}$$

$$\int_0^2 (x-1)^{-1/3} dx = A + B = -\frac{3}{2} + \frac{3}{2} = \boxed{0}$$

$$26.) \quad \int_0^2 (x-1)^{-4/3} dx = \int_0^1 (x-1)^{-4/3} dx + \int_1^2 (x-1)^{-4/3} dx$$

$$= A + B ;$$

$$A = \lim_{b \rightarrow 1^-} \int_0^b (x-1)^{-4/3} dx$$

$$= \lim_{b \rightarrow 1^-} -3 (x-1)^{-1/3} \Big|_0^b$$

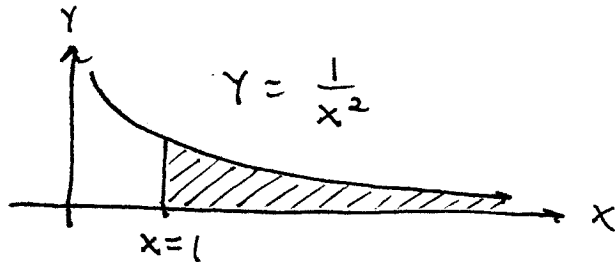
$$= \lim_{b \rightarrow 1^-} \left\{ \frac{-3}{(b-1)^{1/3}} - \frac{-3}{(-1)^{1/3}} \right\} = +\infty - 3$$

so

diverges

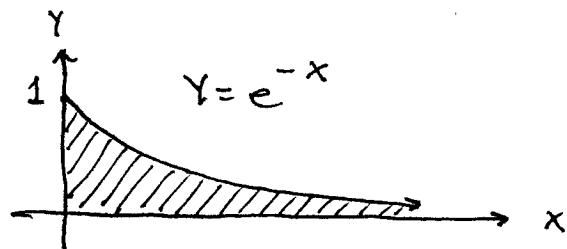
(Note that $B = +\infty$ also!)

29.)



$$\begin{aligned}
 \text{a.) Area} &= \int_1^{\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2} dx \\
 &= \lim_{b \rightarrow \infty} \left. -\frac{1}{x} \right|_1^b = \lim_{b \rightarrow \infty} \left(-\frac{1}{b} - \left(-\frac{1}{1} \right) \right) = 0 + 1 = \textcircled{1} \\
 \text{b.) Volume} &= \pi \int_1^{\infty} \left(\frac{1}{x^2} \right)^2 dx = \lim_{b \rightarrow \infty} \pi \int_1^b x^{-4} dx \\
 &= \lim_{b \rightarrow \infty} \pi \cdot \left. \frac{x^{-3}}{-3} \right|_1^b = \lim_{b \rightarrow \infty} -\frac{\pi}{3} \left(\frac{1}{b^3} - \frac{1}{1^3} \right) \\
 &= -\frac{\pi}{3} (0 - 1) = \textcircled{\frac{\pi}{3}}
 \end{aligned}$$

30.)



$$\begin{aligned}
 \text{a.) Area} &= \int_0^{\infty} e^{-x} dx = \lim_{b \rightarrow \infty} \int_0^b e^{-x} dx \\
 &= \lim_{b \rightarrow \infty} \left. -e^{-x} \right|_0^b = \lim_{b \rightarrow \infty} \left(\frac{-1}{e^b} - \frac{-1}{e^0} \right) = 0 + 1 = \textcircled{1} \\
 \text{b.) Volume} &= \pi \int_0^{\infty} (e^{-x})^2 dx = \lim_{b \rightarrow \infty} \pi \int_0^b (e^{-2x}) dx \\
 &= \lim_{b \rightarrow \infty} \pi \cdot \left. \frac{e^{-2x}}{-2} \right|_0^b = \lim_{b \rightarrow \infty} -\frac{\pi}{2} \left(\frac{1}{e^{2b}} - \frac{1}{e^0} \right) \\
 &= -\frac{\pi}{2} (0 - 1) = \textcircled{\frac{\pi}{2}}
 \end{aligned}$$

Improper Integrals

$$\begin{aligned} 1.) \quad \int_1^{\infty} e^{-7x} dx &= \lim_{A \rightarrow \infty} \int_1^A e^{-7x} dx \\ &= \lim_{A \rightarrow \infty} \left. \frac{-1}{7} e^{-7x} \right|_1^A = \lim_{A \rightarrow \infty} \left(\frac{-1}{7} e^{-7A} - \frac{-1}{7} e^{-7} \right) \\ &= \lim_{A \rightarrow \infty} \left(\frac{-1}{7e^{7A}} + \frac{1}{7e^7} \right) = \cancel{\frac{-1}{\infty}} + \frac{1}{7e^7} \\ &= \frac{1}{7e^7} \end{aligned}$$

$$\begin{aligned} 2.) \quad \int_{-\infty}^0 e^{\frac{1}{4}x} dx &= \lim_{A \rightarrow -\infty} \int_A^0 e^{\frac{1}{4}x} dx \\ &= \lim_{A \rightarrow -\infty} \left. 4e^{\frac{1}{4}x} \right|_A^0 = \lim_{A \rightarrow -\infty} (4e^0 - 4e^{\frac{1}{4}A}) \\ &= 4 - 4e^{-\infty} = 4 - 4 \cdot \cancel{\frac{1}{e^{\infty}}} = 4 \end{aligned}$$

$$\begin{aligned} 3.) \quad \int_{-\infty}^{\infty} e^{3x} dx &= \int_{-\infty}^0 e^{3x} dx + \int_0^{\infty} e^{3x} dx \\ &= B + C ; \end{aligned}$$

$$\begin{aligned} B &= \lim_{A \rightarrow -\infty} \int_A^0 e^{3x} dx = \lim_{A \rightarrow -\infty} \left. \frac{1}{3} e^{3x} \right|_A^0 \\ &= \lim_{A \rightarrow -\infty} \left(\frac{1}{3} e^0 - \frac{1}{3} e^{3A} \right) = \frac{1}{3} - \frac{1}{3} e^{-\infty} \\ &= \frac{1}{3} - \frac{1}{3} \cdot \cancel{\frac{1}{e^{\infty}}} = \frac{1}{3} ; \end{aligned}$$

$$\begin{aligned}
 C &= \lim_{A \rightarrow \infty} \int_0^A e^{3x} dx = \lim_{A \rightarrow \infty} \left. \frac{1}{3} e^{3x} \right|_0^A \\
 &= \lim_{A \rightarrow \infty} \left(\frac{1}{3} e^{3A} - \frac{1}{3} e^0 \right) = \frac{1}{3} e^{\infty} - \frac{1}{3} \\
 &= \infty - \frac{1}{3} = \infty ; \text{ then}
 \end{aligned}$$

$$\int_{-\infty}^{\infty} e^{3x} dx = B + C = \frac{1}{3} + \infty = \infty \text{ (Diverges)}$$

$$\begin{aligned}
 4.) \quad \int_4^7 \frac{3}{4-x} dx &= \lim_{A \rightarrow 4^+} \int_A^7 \frac{3}{4-x} dx \\
 &= \lim_{A \rightarrow 4^+} \left. -3 \ln|4-x| \right|_A^7 \\
 &= \lim_{A \rightarrow 4^+} \left(-3 \ln 3 - \underbrace{-3 \ln|4-A|}_{-\infty} \right) \\
 &= -3 \ln 3 - \infty = -\infty \text{ (Diverges)}
 \end{aligned}$$

$$\begin{aligned}
 5.) \quad \int_2^{\infty} \frac{1}{\sqrt{x-2}} dx &= \int_2^3 \frac{1}{\sqrt{x-2}} dx + \int_3^{\infty} \frac{1}{\sqrt{x-2}} dx \\
 &= B + C ;
 \end{aligned}$$

$$\begin{aligned}
 B &= \lim_{A \rightarrow 2^+} \int_A^3 \frac{1}{\sqrt{x-2}} dx = \lim_{A \rightarrow 2^+} \left. 2\sqrt{x-2} \right|_A^3 \\
 &= \lim_{A \rightarrow 2^+} (2\sqrt{1} - 2\sqrt{A-2}) = 2(1) - 2(0) = 2
 \end{aligned}$$

$$C = \lim_{A \rightarrow \infty} \int_3^A \frac{1}{\sqrt{x-2}} dx = \lim_{A \rightarrow \infty} 2\sqrt{x-2} \Big|_3^A$$

$$= \lim_{A \rightarrow \infty} (2\sqrt{A-2} - 2\sqrt{1}) = \infty - 2 = \infty ;$$

then $\int_2^{\infty} \frac{1}{\sqrt{x-2}} dx = B + C = 2 + \infty = \infty$
(Diverges)

$$6.) \int_0^{\infty} \frac{1}{(x-1)^3} dx = \int_0^1 \frac{1}{(x-1)^3} dx + \int_1^{\infty} \frac{1}{(x-1)^3} dx$$

$$= \int_0^1 \frac{1}{(x-1)^3} dx + \int_1^2 \frac{1}{(x-1)^3} dx + \int_2^{\infty} \frac{1}{(x-1)^3} dx$$

$$= B + C + D ;$$

$$B = \lim_{A \rightarrow 1^-} \int_0^A (x-1)^{-3} dx = \lim_{A \rightarrow 1^-} \frac{(x-1)^{-2}}{-2} \Big|_0^A$$

$$= \lim_{A \rightarrow 1^-} \left(\frac{(A-1)^{-2}}{-2} - \frac{(-1)^{-2}}{-2} \right)$$

$$= \lim_{A \rightarrow 1^-} \left(\frac{-1}{2} \cdot \frac{1}{(A-1)^2} + \frac{1}{2} \right)$$

$$= \frac{-1}{2} \cdot \left(\frac{1}{0^+} \right) + \frac{1}{2} = -\frac{1}{2}(\infty) + \frac{1}{2}$$

$= -\infty + \frac{1}{2} = -\infty$; since $B = -\infty$
it won't matter whether C and D are finite, $+\infty$, or $-\infty$; we conclude that

$$\int_0^{\infty} \frac{1}{(x-1)^3} dx = B+C+D \quad \text{Diverges.}$$

$$7) \int_5^{\infty} \frac{1}{x^2-9} dx = \int_5^{\infty} \frac{1}{(x-3)(x+3)} dx$$

$$= \int_5^{\infty} \left[\frac{A}{x-3} + \frac{B}{x+3} \right] dx$$

$$(A(x+3) + B(x-3) = 1$$

$$\text{Let } \underline{x=3}: 6A = 1 \rightarrow A = \frac{1}{6}$$

$$\text{Let } \underline{x=-3}: -6B = 1 \rightarrow B = -\frac{1}{6})$$

$$= \lim_{C \rightarrow \infty} \int_5^C \left[\frac{1/6}{x-3} + \frac{-1/6}{x+3} \right] dx$$

$$= \lim_{C \rightarrow \infty} \left[\frac{1}{6} \ln|x-3| - \frac{1}{6} \ln|x+3| \right] \Big|_5^C$$

$$= \lim_{C \rightarrow \infty} \left[\left(\frac{1}{6} \ln|C-3| - \frac{1}{6} \ln|C+3| \right) - \left(\frac{1}{6} \ln 2 - \frac{1}{6} \ln 8 \right) \right]$$

indeterminate form

$$= \lim_{C \rightarrow \infty} \frac{1}{6} \ln \left| \frac{C-3}{C+3} \right| - \frac{1}{6} \ln 2 + \frac{1}{6} \ln 8$$

indeterminate form

$$= \lim_{C \rightarrow \infty} \frac{1}{6} \ln \left| \frac{C-3}{C+3} \cdot \frac{1/C}{1/C} \right| + \frac{1}{6} (\ln 8 - \ln 2)$$

$$= \lim_{C \rightarrow \infty} \frac{1}{6} \ln \left| \frac{1-3/C}{1+3/C} \right| + \frac{1}{6} \ln \left(\frac{8}{2} \right)$$

$$= \frac{1}{6} \ln \left| \frac{1-0}{1+0} \right| + \frac{1}{6} \ln 4$$

$$= \frac{1}{6} \ln 1 + \frac{1}{6} \ln 4 = \frac{1}{6} \ln 4$$

$$8.) \int_3^{\infty} \frac{x^3 - x + 1}{x^2 - 4} dx$$

$$= \int_3^{\infty} \left[x + \frac{3x+1}{x^2-4} \right] dx$$

$\begin{array}{c} x \\ \hline x^2 - 4 \overline{) x^3 - x + 1} \\ \underline{-(x^3 - 4x)} \\ 3x + 1 \end{array}$

$$= \lim_{A \rightarrow \infty} \int_3^A \left[x + \frac{3x+1}{x^2-4} \right] dx$$

$$= \lim_{A \rightarrow \infty} \left(\frac{x^2}{2} \Big|_3^A + \int_3^A \frac{3x+1}{(x-2)(x+2)} dx \right)$$

$$= \lim_{A \rightarrow \infty} \left(\frac{A^2}{2} - \frac{9}{2} + \int_3^A \left[\frac{B}{x-2} + \frac{C}{x+2} \right] dx \right)$$

$$(B(x+2) + C(x-2) = 3x+1$$

$$\text{Let } x=2: 4B=7 \rightarrow B=7/4$$

$$\text{Let } x=-2: -4C=-5 \rightarrow C=5/4$$

$$= \lim_{A \rightarrow \infty} \left(\frac{A^2}{2} - \frac{9}{2} + \int_3^A \left[\frac{7/4}{x-2} + \frac{5/4}{x+2} \right] dx \right)$$

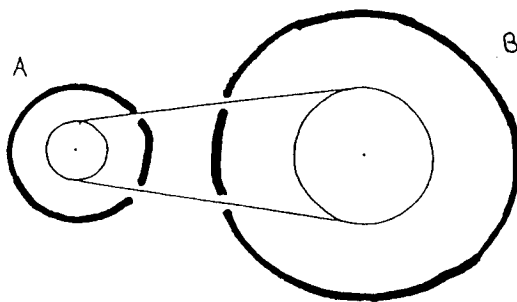
$$= \lim_{A \rightarrow \infty} \left(\frac{A^2}{2} - \frac{9}{2} + \left(\frac{7}{4} \ln|x-2| + \frac{5}{4} \ln|x+2| \right) \Big|_3^A \right)$$

$$= \lim_{A \rightarrow \infty} \left(\frac{A^2}{2} - \frac{9}{2} + \frac{7}{4} \ln|A-2| + \frac{5}{4} \ln|A+2| \right. \\ \left. - \left(\frac{7}{4} \ln 1 + \frac{5}{4} \ln 5 \right) \right)$$

$$= \infty - \frac{9}{2} + \infty + \infty - \frac{5}{4} \ln 5$$

$$= \infty \quad (\text{Diverges})$$

SA13 :



a.) sprocket A: $C = 2\pi r = 2\pi(2) = 4\pi$ in. ,
sprocket B: $C = 2\pi r = 2\pi(6) = 12\pi$ in. , so
3 rotations of sprocket A is $3(4\pi) = 12\pi$ in.
which is (1) rotation of sprocket B.

b.) 3 rotations of sprocket B is $3(12\pi) = 36\pi$ in.
which is $\frac{36\pi}{4\pi} = (9)$ rotations of sprocket A.

c.) If sprocket A rotates 3 times/sec. then
sprocket B and wheel B rotate 1 time/sec.

i.) a pt. on sprocket A is traveling
at $3(4\pi) = 12\pi \approx (37.7 \text{ in./sec.})$

or $\frac{37.7 \text{ in.}}{\text{sec.}} \times \frac{3600 \text{ sec.}}{1 \text{ hr.}} \times \frac{1 \text{ ft.}}{12 \text{ in.}} \times \frac{1 \text{ mi.}}{5280 \text{ ft.}} \approx (2.14 \text{ mph.})$

ii.) a pt. on wheel B ($C = 2\pi r = 2\pi(30) = 60\pi$ in.)
is traveling at $60\pi \approx (188.5 \text{ in./sec.})$

or $\frac{188.5 \text{ in.}}{\text{sec.}} \times \frac{3600 \text{ sec.}}{\text{hr.}} \times \frac{1 \text{ ft.}}{12 \text{ in.}} \times \frac{1 \text{ mi.}}{5280 \text{ ft.}} \approx (10.71 \text{ mph.})$

d.) If sprocket B rotates 3 times/sec. then
sprocket A and wheel A rotate 9 times/sec.

i.) a pt. on sprocket B is traveling at
 $3(12\pi) = 36\pi \approx (113.1 \text{ in./sec.})$ or (6.43 mph.)

ii.) a pt. on wheel A is traveling at
 $9(2\pi r) = 9(2\pi(6)) = 108\pi \approx (339.3 \text{ in./sec.})$

or (19.28 mph.)