Section 9.2

6.) $f(x) = \frac{1}{36}x(6-x) \geq 0$ on $[0,6]$ and

$$\int_0^6 \frac{1}{36}x(6-x)\,dx = \frac{1}{36}\int_0^6 (6x - x^2)\,dx = \frac{1}{36}\left(3x^2 - \frac{x^3}{3}\right)\Big|_0^6$$

$$= \frac{1}{36}(36) - \frac{1}{36}(0) = 1$$

9.) $f(x) = \frac{3}{8}x\sqrt{4-x^2} \geq 0$ on $[0,2]$ and

$$\int_0^2 \frac{3}{8}x\sqrt{4-x^2}\,dx = \frac{3}{8}\cdot\frac{2}{3}\cdot\frac{-1}{2}(4-x^2)^{3/2}\Big|_0^2$$

$$= \frac{-1}{8}(0) - \frac{-1}{8}(4)^{3/2} = 1$$

10.) $f(x) = 12x^2(1-x) \geq 0$ on $[0,1]$ and

$$\int_0^1 12x^2(1-x)\,dx = \int_0^1 (12x^2 - 12x^3)\,dx$$

$$= (4x^3 - 3x^4)\Big|_0^1 = 4 - 3 = 1$$

13.) $f(x) = \frac{1}{3}e^{-x/3} \geq 0$ on $[0,\infty)$ and

$$\int_0^\infty \frac{1}{3}e^{-x/3}\,dx = \lim_{b\to\infty}\int_0^b \frac{1}{3}e^{-x/3}\,dx$$

$$= \lim_{b\to\infty} -e^{-x/3}\Big|_0^b = \lim_{b\to\infty}\left(\frac{-1}{e^{b/3}} - {}^-e^0\right) = 0 + 1 = 1$$

14.) $f(x) = \frac{1}{4} \geq 0$ on $[8,12]$ and

$$\int_8^{12} \frac{1}{4}\,dx = \frac{1}{4}x\Big|_8^{12} = 3 - 2 = 1$$

16.) $$\int_0^4 kx^3\,dx = \frac{1}{4}kx^4\Big|_0^4 = 64k = 1 \rightarrow$$

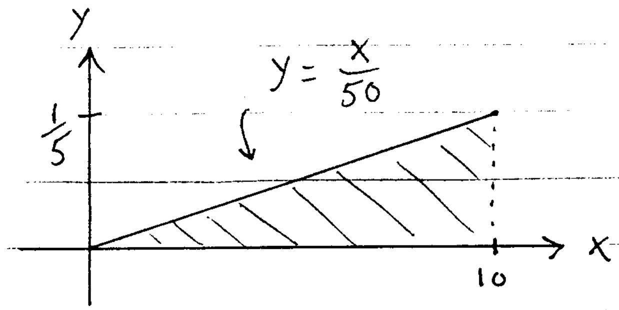
$$k = 1/64$$

19.) $\int_0^\infty ke^{-x/2}\,dx = \lim_{b\to\infty} \int_0^b ke^{-x/2}\,dx$

$= \lim_{b\to\infty} -2ke^{-\frac{x}{2}}\Big|_0^b = \lim_{b\to\infty} \left(-2k\cdot\frac{1}{e^{b/2}} - -2ke^0\right)$

$= 2k = 1 \longrightarrow k = 1/2$

22.)



a.) $P(0 < x < 6)$

$= \int_0^6 \frac{x}{50}\,dx$

$= \frac{1}{100}x^2\Big|_0^6 = .36$

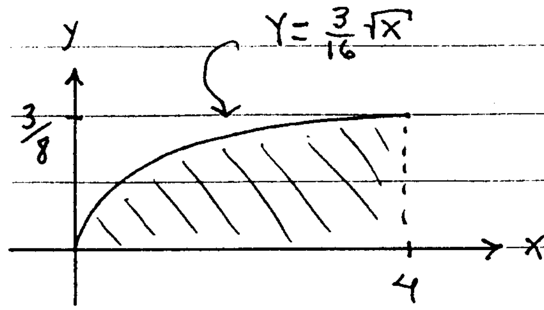
b.) $P(4 < x < 6) = \int_4^6 \frac{x}{50}\,dx = \frac{1}{100}x^2\Big|_4^6 = .2$

c.) $P(8 < x < 10) = 1 - (0.64) = 0.36$

d.) $P(x \geq 2) = \int_2^\infty \frac{x}{50}\,dx = \int_2^{10} \frac{x}{50}\,dx + \underbrace{\int_{10}^\infty 0\,dx}_{0}$

$= \frac{1}{100}x^2\Big|_2^{10} = .96$

23.)



a.) $P(0 < x < 2)$

$= \int_0^2 \frac{3}{16}\sqrt{x}\,dx$

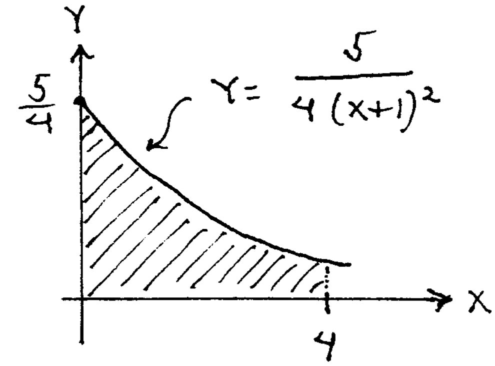
$= \frac{1}{8}x^{3/2}\Big|_0^2 \approx .354$

b.) $P(2 < x < 4) = \int_2^4 \frac{3}{16}\sqrt{x}\,dx = \frac{1}{8}x^{3/2}\Big|_2^4 \approx .646$

c.) $P(1 < x < 3) = \int_1^3 \frac{3}{16}\sqrt{x}\,dx = \frac{1}{8}x^{3/2}\Big|_1^3 = .525$

d.) $P(x \le 3) = \int_0^3 \frac{3}{16}\sqrt{x}\,dx = \frac{1}{8}x^{3/2}\Big|_0^3 = .650$

24.) a.)



$P(0<x<2) = \int_0^2 \frac{5}{4}(x+1)^{-2}dx$

$= -\frac{5}{4}(x+1)^{-1}\Big|_0^2$

$= -\frac{5}{4}\left(\frac{1}{3}-1\right) = \frac{5}{6} = .833$

b.) $P(2<x<4) = \int_2^4 \frac{5}{4}(x+1)^{-2}dx$

$= -\frac{5}{4}(x+1)^{-1}\Big|_2^4 = -\frac{5}{4}\left(\frac{1}{5}-\frac{1}{3}\right) = \frac{1}{6} = .167$

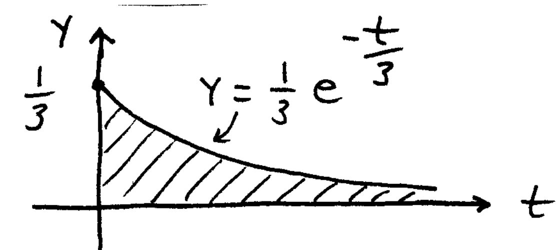
c.) $P(1<x<3) = \int_1^3 \frac{5}{4}(x+1)^{-2}dx$

$= -\frac{5}{4}(x+1)^{-1}\Big|_1^3 = -\frac{5}{4}\left(\frac{1}{4}-\frac{1}{2}\right) = \frac{5}{16} = .313$

d.) $P(x \le 3) = \int_0^3 \frac{5}{4}(x+1)^{-2}dx$

$= -\frac{5}{4}(x+1)^{-1}\Big|_0^3 = -\frac{5}{4}\left(\frac{1}{4}-1\right) = \frac{15}{16} = .938$

25.) a.) $P(t<2) = P(0<t<2)$



$= \int_0^2 \frac{1}{3}e^{-\frac{t}{3}}dt$

$= -e^{-\frac{t}{3}}\Big|_0^2 = \frac{-1}{e^{2/3}} - \frac{-1}{e^0} \approx 0.487$

b.) $P(t \ge 2) = \int_2^\infty \frac{1}{3}e^{-t/3}dt = \lim_{b\to\infty} -e^{-t/3}\Big|_2^b$

$= \lim_{b\to\infty}\left(\frac{-1}{e^{b/3}} - \frac{-1}{e^{2/3}}\right) = \text{"}\frac{-1}{\infty}\text{"} + 0.513 = 0.513$

OR $P(t \ge 2) = 1 - P(t<2) = 1 - 0.487 = 0.513$

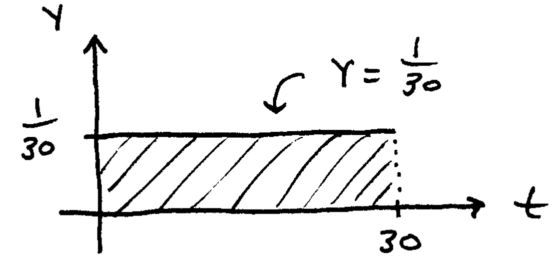
c.) $P(1<t<4) = \int_1^4 \frac{1}{3} e^{-\frac{t}{3}} dt = -e^{-\frac{t}{3}} \Big|_1^4$

$= \frac{-1}{e^{4/3}} - \frac{-1}{e^{1/3}} \approx 0.453$

d.) $P(t=3) = \int_3^3 \frac{1}{3} e^{-t/3} dt = 0$

27.) a.) $P(t \le 5) = P(0 \le t \le 5)$

$= \int_0^5 \frac{1}{30} dt = 5\left(\frac{1}{30}\right) = \frac{1}{6} \approx 0.167$



b.) $P(t \ge 18) = \int_{18}^{30} \frac{1}{30} dt = 12\left(\frac{1}{30}\right) = 0.4$

28.) a.) $P(0 \le x < 3) = \int_0^3 \frac{5}{324} t\sqrt{9-t}\, dt$

(Let $u = 9-t \rightarrow du = -dt \rightarrow -du = dt$
and $t = 9-u$, $t: 0 \rightarrow 3$ so $u: 9 \rightarrow 6$)

$= -\int_9^6 \frac{5}{324}(9-u)u^{1/2} du = \frac{-5}{324}\int_9^6 (9u^{1/2} - u^{3/2}) du$

$= \frac{-5}{324}\left(6u^{3/2} - \frac{2}{5}u^{5/2}\right)\Big|_9^6 = \frac{-5}{324}\left(6 \cdot 6^{3/2} - \frac{2}{5} 6^{5/2}\right)$

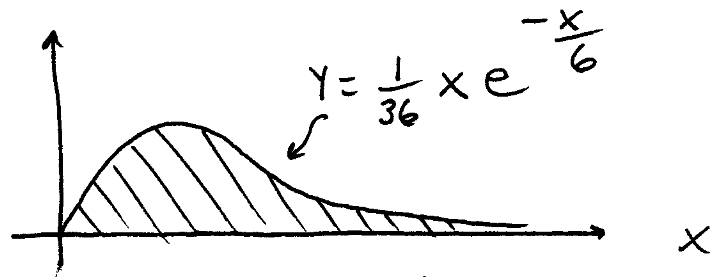
$- \frac{-5}{324}\left(6 \cdot 9^{3/2} - \frac{2}{5} 9^{5/2}\right) = .1835$

b.) $P(4 \le x \le 8) = \int_4^8 \frac{5}{324} t\sqrt{9-t}\, dt$ ($t: 4 \rightarrow 8$ so $u = 9-t$, $u: 5 \rightarrow 1$)

$= \cdots = \frac{-5}{324}\left(6u^{3/2} - \frac{2}{5}u^{5/2}\right)\Big|_5^1$

$= \frac{-5}{324}\left(6 - \frac{2}{5}\right) - \frac{-5}{324}\left(6(5)^{3/2} - \frac{2}{5}(5)^{5/2}\right) = .6037$

33.)



a.) $P(x<6) = \int_0^6 \frac{1}{36} x e^{-x/6}\, dx$

$\left(\text{Let } u = \frac{1}{36}x \quad , \quad dv = e^{-x/6} dx \right.$

$\left. du = \frac{1}{36} dx \quad , \quad v = -6e^{-x/6} \right)$

$= -\frac{1}{6} x e^{-x/6} \Big|_0^6 + \frac{1}{6} \int_0^6 e^{-x/6}\, dx$

$= (-e^{-1} - 0) + \frac{1}{6} \cdot -6e^{-x/6} \Big|_0^6$

$= \frac{-1}{e} - \left(\frac{1}{e} - \frac{1}{e^0} \right) = 1 - \frac{2}{e} \approx 0.264$

b.) $P(6<x<12) = \int_6^{12} \frac{1}{36} x e^{-x/6}\, dx$

$= \left(-\frac{1}{6} x e^{-\frac{x}{6}} - e^{-x/6} \right) \Big|_6^{12} = \left(\frac{-2}{e^2} - \frac{1}{e^2} \right) - \left(\frac{-1}{e} - \frac{1}{e} \right)$

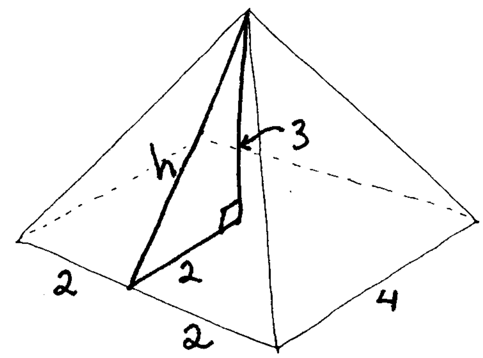
$= \frac{-3}{e^2} + \frac{2}{e} \approx 0.33$

c.) $P(x>12) = 1 - P(x<12)$

$= 1 - (0.264 + 0.330)$

$= 0.406$

$\boxed{SA16}$: a.) By Pythagorean Theorem



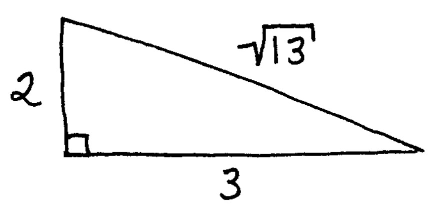
$2^2 + 3^2 = h^2 \rightarrow h = \sqrt{13}$,

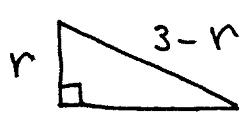
then area of triangle is $A = \frac{1}{2} bh = \frac{1}{2}(4)\sqrt{13} = 2\sqrt{13}$

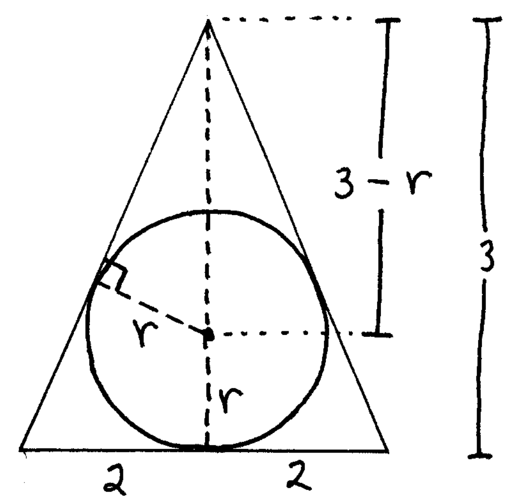
so total surface area of pyramid is

$(4)^2 + 4(2\sqrt{13}) = 16 + 8\sqrt{13}$;

b.) SIDE VIEW :







By similar triangles

$$\frac{r}{3-r} = \frac{2}{\sqrt{13}} \rightarrow \sqrt{13}\, r = 6 - 2r \rightarrow$$

$$(\sqrt{13} + 2) r = 6 \rightarrow r = \frac{6}{\sqrt{13} + 2}$$

is radius of sphere