Section 4.3

Page 279

2.) $y = e^{2x} \xrightarrow{D} y' = 2 \cdot e^{2x}$ so slope at $x=0, y=1$ is $m = y' = 2 \cdot e^0 = 2 \cdot 1 = 2$

6.) $y = e^{1-x} \xrightarrow{D} y' = e^{1-x} \cdot (-1)$

9.) $f(x) = e^{-1/x^2} = e^{-x^{-2}} \xrightarrow{D}$

$f'(x) = e^{-x^{-2}} \cdot 2x^{-3}$

10.) $g(x) = e^{\sqrt{x}} \xrightarrow{D} g'(x) = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

12.) $y = 4x^3 \cdot e^{-x} \xrightarrow{D} y' = 4x^3 \cdot e^{-x} \cdot (-1) + 12x^2 \cdot e^{-x}$

13.) $f(x) = \dfrac{2}{(e^x + e^{-x})^3} \xrightarrow{D}$

$$f'(x) = \frac{(e^x + e^{-x})^3 \cdot (0) - 2 \cdot 3(e^x + e^{-x})^2 \cdot (e^x - e^{-x})}{(e^x + e^{-x})^6}$$

14.) $f(x) = \frac{1}{2}(e^x + e^{-x})^4 \xrightarrow{D} f'(x) = \frac{1}{2} \cdot 4(e^x + e^{-x})^3(e^x - e^{-x})$

16.) $y = x^2e^x - 2xe^x + 2e^x \xrightarrow{D}$

$y' = (x^2 \cdot e^x + 2x \cdot e^x) - (2xe^x + 2e^x) + 2e^x$

17.) $y = e^{-2x+x^2} \xrightarrow{D} y' = e^{-2x+x^2} \cdot (-2 + 2x)$

and $x=0, y=1$ so slope $m = y' = e^0(-2) = -2$

$\rightarrow$ tangent line is $y - 1 = -2(x - 0) \rightarrow$

$y = -2x + 1$

20.) $y = \frac{x}{e^{2x}} \xrightarrow{D} y' = \frac{e^{2x} \cdot (1) - x \cdot 2e^{2x}}{e^{4x}}$ and

$x = 1,\ y = \frac{1}{e^2}$ so slope $m = y' = \frac{e^2 - 2e^2}{e^4}$

$= \frac{-e^2}{e^4} = \frac{-1}{e^2}$ → tangent line is

$y - \frac{1}{e^2} = \frac{-1}{e^2}(x - 1)$

23.) $xe^x + 2ye^x = 0 \xrightarrow{D}$

$xe^x + 1 \cdot e^x + 2y \cdot e^x + 2y' \cdot e^x = 0 \rightarrow$

$2e^x y' = -xe^x - e^x - 2ye^x \rightarrow$

$y' = \frac{-xe^x - e^x - 2ye^x}{2e^x} = \frac{\cancel{e^x}(-x-1-2y)}{2\cancel{e^x}} \rightarrow$

$y' = \frac{-x-1-2y}{2}$

26.) $e^{xy} + x^2 - y^2 = 10 \xrightarrow{D}$

$e^{xy} \cdot (xy' + 1 \cdot y) + 2x - 2yy' = 0 \rightarrow$

$xe^{xy}y' + ye^{xy} + 2x - 2yy' = 0 \rightarrow$

$(xe^{xy} - 2y)y' = -2x - ye^{xy} \rightarrow$

$y' = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}$

27.) $f(x) = 2e^{3x} + 3e^{-2x} \xrightarrow{D}$

$f'(x) = 6e^{3x} - 6e^{-2x} \xrightarrow{D}$

$f''(x) = 18e^{3x} + 12e^{-2x}$

30.) $f(x) = (3+2x)\cdot e^{-3x} \xrightarrow{D}$

$$f'(x) = (3+2x)\cdot e^{-3x}\cdot(-3) + 2\cdot e^{-3x}$$

$$= -9e^{-3x} - 6xe^{-3x} + 2e^{-3x}$$

$$= -7e^{-3x} - 6xe^{-3x} \xrightarrow{D}$$

$$f''(x) = 21e^{-3x} - \left(6x\cdot e^{-3x}\cdot(-3) + 6e^{-3x}\right)$$

$$= 21e^{-3x} + 18xe^{-3x} - 6e^{-3x}$$

$$= 15e^{-3x} + 18xe^{-3x}$$

32.)

$$Y = \tfrac{1}{2}(e^{x} - e^{-x}) \rightarrow Y' = \tfrac{1}{2}(e^{x} + e^{-x})$$

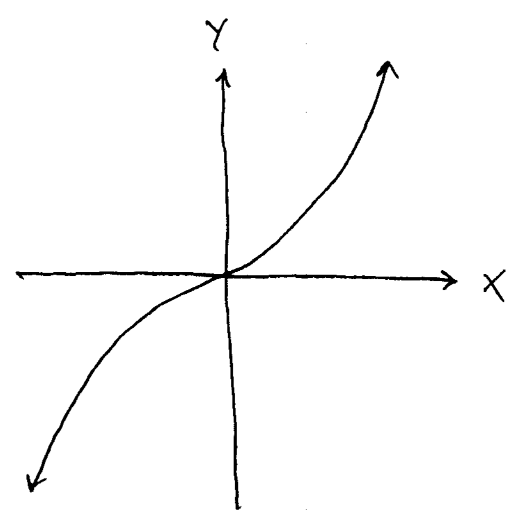
$+ \quad + \quad +$ (Y')

$$\rightarrow Y'' = \tfrac{1}{2}(e^{x} - e^{-x}) = 0$$

$$\rightarrow e^{x} - e^{-x} = 0 \rightarrow e^{x} = e^{-x} \rightarrow x = -x \rightarrow 2x = 0 \rightarrow x = 0$$

$- \quad 0 \quad +$ (Y'')

infl. pt. $\begin{cases} x = 0 \\ Y = 0 \end{cases}$



Y is ↑ for all x-values,

Y is ∪ for $x > 0$,

Y is ∩ for $x < 0$

$x = 0 \rightarrow Y = 0$

$Y = 0 \rightarrow x = 0$

$\lim_{x\to\infty} Y = \infty$, $\lim_{x\to -\infty} Y = -\infty$

34.) $Y = xe^{-x} \rightarrow Y' = -xe^{-x} + e^{-x} = e^{-x}(1-x) = 0$

$\underline{\;+ \quad 0 \quad -\;}\; Y'$ $\rightarrow Y'' = -e^{-x} - e^{-x}(1-x)$

$= -e^{-x}(1+1-x) = e^{-x}(x-2) = 0$

abs. max. $\begin{cases} x = 1 \\ Y = \frac{1}{e} \end{cases}$

$\lim_{x \to \infty} Y = 0$, $\lim_{x \to -\infty} Y = -\infty$

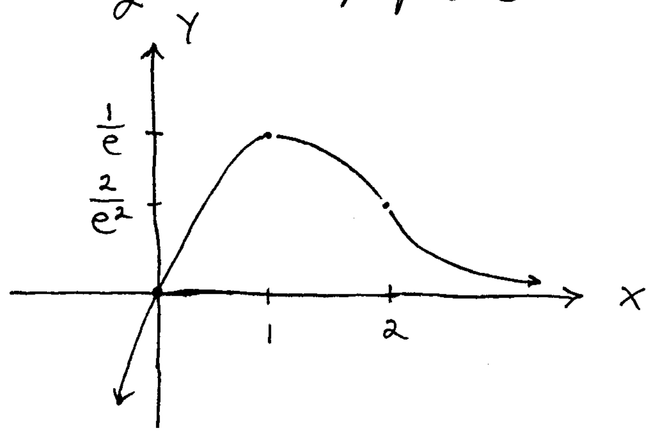
$\underline{\;- \quad 0 \quad +\;}\; Y''$

horizontal asymptote : $Y = 0$

infl. pt. $\begin{cases} x = 2 \\ Y = \frac{2}{e^2} \end{cases}$

Y is $\uparrow$ for $x < 1$,
Y is $\downarrow$ for $x > 1$,
Y is $\cup$ for $x > 2$,
Y is $\cap$ for $x < 2$

$x = 0 \rightarrow Y = 0$
$Y = 0 \rightarrow x = 0$



39.) $p = \dfrac{300}{3 + 17e^{-1.57x}}$

b.) 2000 egg masses $\rightarrow x = 2 \rightarrow$

$p = \dfrac{300}{3 + 17e^{-1.57(2)}} = 80.3$ so 80.3 % defoliation

c.) $p = 300(3 + 17e^{-1.57x})^{-1} \xrightarrow{D}$

$p' = -300(3 + 17e^{-1.57x})^{-2} \cdot (17)(-1.57)e^{-1.57x}$

$= \dfrac{8007 \cdot e^{-1.57x}}{(3 + 17e^{-1.57x})^2} \xrightarrow{D}$

$$p'' = \frac{(3+17e^{-1.57x})^2 \cdot (8007)(-1.57)e^{-1.57x} - 8007 \cdot e^{-1.57x} \cdot 2(3+17e^{-1.57x}) \cdot (17)(-1.57)e^{-1.57x}}{(3+17e^{-1.57x})^4}$$

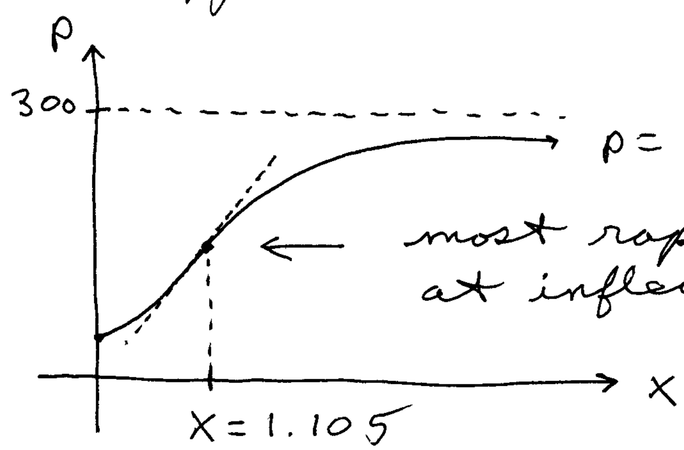
$$= \frac{(8007)(-1.57)e^{-1.57x}\cdot(3+17e^{-1.57x})\cdot\left[(3+17e^{-1.57x})-34e^{-1.57x}\right]}{(3+17e^{-1.57x})^4} = 0$$

$$\rightarrow 3-17e^{-1.57x} = 0 \rightarrow 3 = 17e^{-1.57x} \rightarrow$$

$$e^{-1.57x} = \frac{3}{17} \rightarrow \ln e^{-1.57x} = \ln(3/17) \rightarrow$$

$$-1.57x = \ln(3/17) \rightarrow x = \frac{\ln(3/17)}{-1.57} \approx 1.105 \rightarrow$$

1105 egg masses



$$p = \frac{300}{3+17e^{-1.57x}}$$

← most rapid ↑ occurs at inflection point

40.) $N = \dfrac{95}{1+8.5e^{-0.12t}} = 95(1+8.5e^{-0.12t})^{-1}$

$$\xrightarrow{D} \frac{dN}{dt} = -95(1+8.5e^{-0.12t})^{-2}\cdot(8.5)(-0.12)e^{-0.12t}$$

$$\rightarrow \frac{dN}{dt} = \frac{96.9e^{-0.12t}}{(1+8.5e^{-0.12t})^2}$$

a.) $t=5 \rightarrow \dfrac{dN}{dt} \approx 1.66$ $\dfrac{\text{words per min.}}{\text{week}}$

b.) $t=10 \rightarrow \dfrac{dN}{dt} \approx 2.30$ $\dfrac{\text{words per min.}}{\text{week}}$

c.) $t = 30 \longrightarrow \frac{dN}{dt} \approx 1.74 \frac{\text{words per minute}}{\text{week}}$

42.) $p = 80e^{-0.5t} + 20 \xrightarrow{D}$

$$\frac{dp}{dt} = 80(-0.5)e^{-0.5t} = -40e^{-0.5t}$$

a.) $t = 1 \rightarrow \frac{dp}{dt} \approx -24.26$ %/wk.

b.) $t = 3 \rightarrow \frac{dp}{dt} \approx -8.93$ %/wk.

Handout 2

1.) a.) $f'(x) = e^{\sin x} \cdot \cos x - \sin(e^x) \cdot e^x$

b.) $f'(x) = 5\left(e^{3x} + e^{-3x}\right)^4 \cdot \left(3e^{3x} - 3e^{-3x}\right)$

c.) $g'(x) = e^x \cdot \sec(3x)\tan(3x) \cdot 3 + e^x \sec(3x)$

d.) $y' = 7e^x + \sec^2(2x) \cdot 2$

e.) $y' = 1 - e^{5 + x\cot x} \cdot (-x\csc^2 x + \cot x)$

f.) $y' = e x^{e-1} + e^x + e^{e^{-x}} \cdot e^{-x} \cdot (-1)$

g.) $f'(x) = e^{e^{e^{7x}}} \cdot e^{e^{7x}} \cdot e^{7x} \cdot 7$

2.) a.) $y = x^2 e^{-x} - xe^{-x} \rightarrow y' = -x^2e^{-x} + 2xe^{-x} - (-xe^{-x} + e^{-x})$

$\rightarrow y' = -e^{-x}(x^2 - 3x + 1) = 0 \rightarrow x = \dfrac{3 \pm \sqrt{9-4}}{2} = \dfrac{3 \pm \sqrt{5}}{2}$;

$\rightarrow y'' = -e^{-x}(2x - 3) + e^{-x}(x^2 - 3x + 1) = e^{-x}(x^2 - 5x + 4)$

$= e^{-x}(x-1)(x-4) = 0 \rightarrow x = 1,\ x = 4$

b.) $y = e^{-(x-1)^2} \longrightarrow y' = -2(x-1)e^{-(x-1)^2} = 0 \rightarrow x = 1$;

$\rightarrow y'' = 4(x-1)^2 e^{-(x-1)^2} - 2e^{-(x-1)^2} = 2e^{-(x-1)^2}(2x^2 - 4x + 1) = 0.$

$\rightarrow x = \dfrac{4 \pm \sqrt{16-8}}{4} = \dfrac{4 \pm 2\sqrt{2}}{4} = 1 \pm \frac{1}{2}\sqrt{2}$

3.) $y = 5 + \cos 2x - 4e^{5\tan 3x} \rightarrow$

$y' = -2\sin 2x - 4e^{5\tan 3x} \cdot 5\sec^2 3x \cdot 3$, at $x = \frac{\pi}{3}$

$y' = -2\sin\frac{2\pi}{3} - 60\, e^{5\overset{0}{\cancel{\tan\pi}}} \cdot \overset{1}{\cancel{\sec^2\pi}} = -2\cdot\frac{\sqrt{3}}{2} - 60 = -\sqrt{3} - 60$

so slope $m = \dfrac{1}{\sqrt{3} + 60}$; at $x = \frac{\pi}{3} \rightarrow y = 5 - \frac{1}{2} - 4 = \frac{1}{2}$

so line is $y - \frac{1}{2} = \dfrac{1}{\sqrt{3} + 60}\left(x - \frac{\pi}{3}\right)$.