Section 5.1

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2.) $D\left(8\sqrt{x}+c\right) = 8\cdot\frac{1}{2}x^{-1/2}+0 = \frac{4}{\sqrt{x}}$

so $\int \frac{4}{\sqrt{x}}\,dx = 8\sqrt{x}+c$

5.) $D\left(\frac{1}{5}\cdot 4x^{3/2}(x-5)+c\right) = D\left(\frac{4}{5}\left(x^{5/2}-5x^{3/2}\right)+c\right)$

$= \frac{4}{5}\cdot\left(\frac{5}{2}x^{3/2}-\frac{15}{2}x^{1/2}\right)+0 = \frac{20}{10}x^{3/2}-\frac{60}{10}x^{1/2}$

$= 2x^{3/2}-6x^{1/2} = 2\sqrt{x}\,(x-3)$ so

$\int 2\sqrt{x}\,(x-3)\,dx = \frac{4}{5}x^{3/2}(x-5)+c$

9.) $\int 6\,dx = 6x+c$

12.) $\int 3t^4\,dt = 3\cdot\frac{1}{5}t^5+c$

13.) $\int 5x^{-3}\,dx = 5\cdot\frac{x^{-2}}{-2}+c$

15.) $\int du = \int 1\,du = u+c$

18.) $\int e^3\,dy = e^3y+c$

22.) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1}+c$

24.) $\int \frac{1}{x^2x^{1/2}}\,dx = \int \frac{1}{x^{5/2}}\,dx = \int x^{-5/2}\,dx$

$= \frac{x^{-3/2}}{-3/2}+c$

25.) $\int x(x^2+3)\,dx = \int (x^3+3x)\,dx = \frac{x^4}{4}+3\cdot\frac{x^2}{2}+c$

34.) $\int (x^2 - 2x + 3)\,dx = \frac{x^3}{3} - x^2 + 3x + C$

36.) $\int \left(x^{1/2} + \frac{1}{2x^{1/2}}\right) dx = \int \left(x^{1/2} + \frac{1}{2} \cdot x^{-1/2}\right) dx$

$= \frac{x^{3/2}}{3/2} + \frac{1}{2} \cdot \frac{x^{1/2}}{1/2} + C$

41.) $\int \frac{2x^3 + 1}{x^3}\,dx = \int \left(\frac{2x^3}{x^3} + \frac{1}{x^3}\right) dx$

$= \int (2 + x^{-3})\,dx = 2x + \frac{x^{-2}}{-2} + C$

44.) $\int x^{1/2}(x+1)\,dx = \int (x^{3/2} + x^{1/2})\,dx = \frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + C$

45.) $\int (x-1)(6x-5)\,dx = \int (6x^2 - 11x + 5)\,dx$

$= 6 \cdot \frac{x^3}{3} - 11 \cdot \frac{x^2}{2} + 5x + C$

49.) $f'(x) = 3x^{1/2} + 3 \rightarrow f(x) = \int (3x^{1/2} + 3)\,dx$

$= 3 \cdot \frac{x^{3/2}}{3/2} + 3x + C = \cancel{3} \cdot \frac{2}{\cancel{3}} x^{3/2} + 3x + C \rightarrow$

$f(x) = 2x^{3/2} + 3x + C$ and $x = 1, y = 4 \rightarrow$

$4 = 2(1)^{3/2} + 3(1) + C \rightarrow 4 = 5 + C \rightarrow C = -1 \rightarrow$

$f(x) = 2x^{3/2} + 3x - 1$

51.) $f'(x) = 6x(x-1) = 6x^2 - 6x \rightarrow$

$f(x) = \int (6x^2 - 6x)\,dx = 2x^3 - 3x^2 + C \rightarrow$

$f(x) = 2x^3 - 3x^2 + C$ and $x = 1, y = -1 \rightarrow$

$-1 = 2(1)^3 - 3(1)^2 + c \rightarrow -1 = -1 + c \rightarrow c = 0 \rightarrow$

$f(x) = 2x^3 - 3x^2$

54.) $f'(x) = \dfrac{x^2 - 5}{x^2} = \dfrac{x^2}{x^2} - \dfrac{5}{x^2} = 1 - 5x^{-2} \rightarrow$

$f(x) = \int (1 - 5x^{-2})\,dx = x - 5 \cdot \dfrac{x^{-1}}{-1} + c \rightarrow$

$f(x) = x + \dfrac{5}{x} + c$ and $x = 1,\ Y = 2 \rightarrow$

$2 = 1 + \dfrac{5}{1} + c \rightarrow 2 = 6 + c \rightarrow c = -4 \rightarrow$

$f(x) = x + \dfrac{5}{x} - 4$

56.) $y' = 2(x-1) = 2x - 2 \rightarrow$

$y = \int (2x - 2)\,dx = x^2 - 2x + C \rightarrow Y = x^2 - 2x + c$

and $x = 3,\ Y = 2 \rightarrow 2 = 3^2 - 2(3) + c \rightarrow$

$2 = 9 - 6 + c \rightarrow 2 = 3 + c \rightarrow c = -1 \rightarrow$

$y = x^2 - 2x - 1$

57.) $f'(x) = 6x^{1/2} - 10 \rightarrow f(x) = \int (6x^{1/2} - 10)\,dx$

$= 6 \cdot \dfrac{x^{3/2}}{3/2} - 10x + c = 6 \cdot \dfrac{2}{3} x^{3/2} - 10x + c \rightarrow$

$f(x) = 4x^{3/2} - 10x + c$ and $x = 4,\ Y = 2 \rightarrow$

$2 = 4 \cdot 4^{3/2} - 10(4) + c \rightarrow 2 = 4 \cdot (8) - 40 + c \rightarrow$

$2 = -8 + c \rightarrow c = 10 \rightarrow$

$f(x) = 4x^{3/2} - 10x + 10$

59.) $\underline{f''(x) = 2} \rightarrow f'(x) = 2x + c$ and
$x = 2,\ f'(2) = 5 \rightarrow 5 = 2(2) + c \rightarrow c = 1$
$\rightarrow \underline{f'(x) = 2x + 1} \rightarrow f(x) = x^2 + x + c$
and $x = 2,\ f(2) = 10 \rightarrow 10 = 2^2 + 2 + c \rightarrow$
$10 = 6 + c \rightarrow c = 4 \rightarrow \underline{f(x) = x^2 + x + 4}$

60.) $\underline{f''(x) = x^2} \rightarrow f'(x) = \frac{1}{3}x^3 + c$ and
$x = 0,\ f'(0) = 6 \rightarrow 6 = \frac{1}{3}(0)^3 + c \rightarrow c = 6 \rightarrow$
$\underline{f'(x) = \frac{1}{3}x^3 + 6} \rightarrow f(x) = \frac{1}{3} \cdot \frac{1}{4}x^4 + 6x + c \rightarrow$
$f(x) = \frac{1}{12}x^4 + 6x + c$ and $x = 0,\ f(0) = 3 \rightarrow$
$3 = \frac{1}{12}(0)^4 + 6(0) + c \rightarrow c = 3 \rightarrow$
$f(x) = \frac{1}{12}x^4 + 6x + 3$

75.) $\underline{\text{acceleration}}$: $a(t) = -32$ ft./sec.^{2} $\rightarrow$

$\underline{\text{velocity}}$: $v(t) = -32t + c$ and $v(0) = 60$ ft./sec.

$\rightarrow 60 = 0 + c \rightarrow c = 60 \rightarrow$ $\boxed{v(t) = -32t + 60}$ $\rightarrow$

$\underline{\text{height}}$: $s(t) = -16t^2 + 60t + c$ and $s(0) = 0$ ft. $\rightarrow$

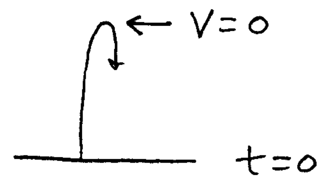
$0 = 0 + c \rightarrow c = 0 \rightarrow$ $\boxed{s(t) = -16t^2 + 60t}$;

highest point $\rightarrow$ $v(t) = 0 \rightarrow$

$0 = -32t + 60 \rightarrow t = \frac{60}{32} = \frac{15}{8}$ sec.

and $s\left(\frac{15}{8}\right) = -16\left(\frac{225}{64}\right) + 60\left(\frac{15}{8}\right)$

$= \frac{225}{4} = 56.25$ ft.



76.) $\underline{\text{acceleration}}$: $a(t) = -32$ ft./sec.^{2} $\rightarrow$

$\underline{\text{velocity}}$: $v(t) = -32t + c$ and $v(0) = 0$ ft./sec.

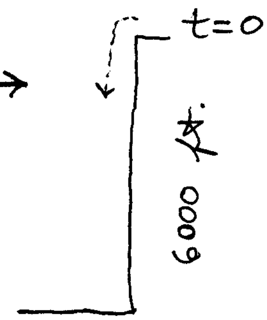
$\rightarrow 0 = 0 + c \rightarrow c = 0 \rightarrow$ $v(t) = -32t$ $\rightarrow$

$\underline{\text{height}}$: $s(t) = -16t^2 + c$ and $s(0) = 6000 \rightarrow$

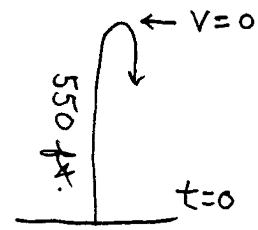
$6000 = 0 + c \rightarrow c = 6000 \rightarrow$ $s(t) = -16t^2 + 6000$;

hit floor $\rightarrow$ $s(t) = 0 \rightarrow 0 = -16t^2 + 6000 \rightarrow$

$t^2 = \frac{6000}{16} \rightarrow$ $t \approx 19.36$ sec.



77.) Let A be initial velocity so $v(0) = A$ ft./sec. :



acceleration: $a(t) = -32$ ft./sec.^{2} $\rightarrow$

velocity: $v(t) = -32t + c$ and $v(0) = A \rightarrow$

$A = 0 + c \rightarrow c = A \rightarrow v(t) = -32t + A \rightarrow$

height: $s(t) = -16t^2 + At + c$ and $s(0) = 0 \rightarrow$

$0 = 0 + c \rightarrow c = 0 \rightarrow s(t) = -16t^2 + At$;

at highest point $v(t) = 0 \rightarrow 0 = -32t + A \rightarrow$

$t = \frac{A}{32}$ sec. ; if highest point is 550 ft.

then $s\left(\frac{A}{32}\right) = 550 \rightarrow -16\left(\frac{A}{32}\right)^2 + A\left(\frac{A}{32}\right) = 550 \rightarrow$

$\frac{A^2}{64} = 550 \rightarrow A^2 = 35{,}200 \rightarrow A \approx 187.62$ ft./sec.

78.) acceleration: $a(t) = -32$ ft./sec.^{2} $\rightarrow$

velocity: $v(t) = -32t + c$ and $v(0) = 16$ ft./sec. $\rightarrow$

$16 = 0 + c \rightarrow c = 16 \rightarrow v(t) = -32t + 16 \rightarrow$

height: $s(t) = -16t^2 + 16t + c$ and $s(0) = 64$ ft. $\rightarrow$

$64 = 0 + c \rightarrow c = 64 \rightarrow s(t) = -16t^2 + 16t + 64$;

a.) hit ground : $s(t) = 0 \rightarrow -16t^2 + 16t + 64 = 0 \rightarrow$

$-16(t^2 - t - 4) = 0 \rightarrow t = \frac{1 \pm \sqrt{1+16}}{2}$

$\rightarrow t = \frac{1}{2}(1 + \sqrt{17})$ sec. ≈ 2.56 sec.

b.) strike velocity :

$v\left(\frac{1}{2}(1+\sqrt{17})\right) = -32\left(\frac{1}{2}(1+\sqrt{17})\right) + 16 = -16\sqrt{17}$ ft./sec. ≈ -66 ft./sec.

Handout 4

1.) $D\left(\frac{1}{2}\sin 2x + 30\right) = \frac{1}{2}\cdot\cos 2x\cdot 2 = \cos 2x$

2.) $D\left(\frac{1}{8}\tan 8x + 4\sec 5x - 3\right) = \frac{1}{8}\sec^2 8x\cdot 8 + 4\sec 5x\tan 5x\cdot 5$

$= \sec^2 8x + 20\sec 5x\tan 5x$

3.) $\int \sin 9x\,dx = \frac{-1}{9}\cos 9x + c$

check: $D\left(\frac{-1}{9}\cos 9x + c\right) = \frac{-1}{9}\cdot -\sin 9x\cdot 9 = \sin 9x$

4.) $\int \sin kx\,dx = \frac{-1}{k}\cos kx + c$

5.) $D(e^x\cos x) = e^x\cdot -\sin x + e^x\cos x = e^x\cos x - e^x\sin x$

6.) $x^2\sin x$ is an antiderivative since

$D(x^2\sin x) = x^2\cos x + 2x\sin x$

$\boxed{SA5}$: a) Rate $r = k\cdot p(A-p)$, where

k is a proportionality constant (+).

b.) Maximize rate $r \rightarrow$

$r = kp(A-p) \xrightarrow{D} r' = kp(-1) + k(A-p) \rightarrow$

$r' = -kp + kA - kp = kA - 2kp = k(A-2p) = 0$

+	0	−	
	$p = \frac{A}{2}$		r'

so $p = \frac{A}{2}$ determines a maximum rate $r = \frac{kA^2}{4}$.

Gravity Problems

1.) acceleration : $s''(t) = -32 \rightarrow$

velocity : $s'(t) = -32t + c$

(assume $s'(0) = v_0$.)

$\rightarrow v_0 = -32(0) + c \rightarrow c = v_0$

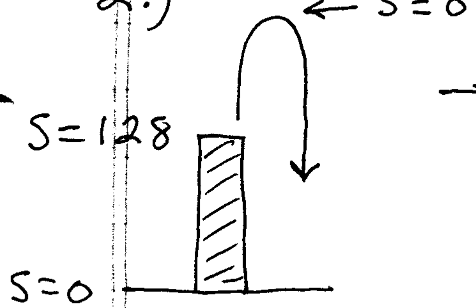
$\rightarrow \quad s'(t) = -32t + v_0 \quad \rightarrow$

height : $s(t) = -16t^2 + v_0 t + c$

(assume $s(0) = s_0$.)

$\rightarrow s(t) = -16t^2 + v_0 t + s_0$.

2.)



assume $s(t) = -16t^2 + 112t + 128$

$\rightarrow$ vel. $\boxed{s'(t) = -32t + 112}$

a.) highest pt. : $s'(t) = 0 \rightarrow$

$-32t + 112 = 0 \rightarrow t = \frac{112}{32} = 3.5$ sec.

and $s(3.5) = -16(3.5)^2 + 112(3.5) + 128 = \boxed{324 \text{ ft.}}$

b.) strike ground : $s(t) = 0 \rightarrow$

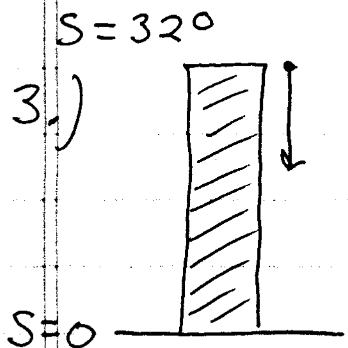
$-16t^2 + 112t + 128 = -16(t^2 - 7t - 8)$

$= -16(t-8)(t+1) = 0 \rightarrow \boxed{t = 8 \text{ sec.}}$

c.) $s'(3) = 16$ ft./sec.,

$s'(4) = -16$ ft./sec.,

$s'(8) = -144$ ft./sec.

3.) 

assume $s(t) = -16t^2 - 16t + 320$

$\rightarrow$ vel. $\boxed{s'(t) = -32t - 16}$

a.) strike ground : $s(t) = 0 \rightarrow$

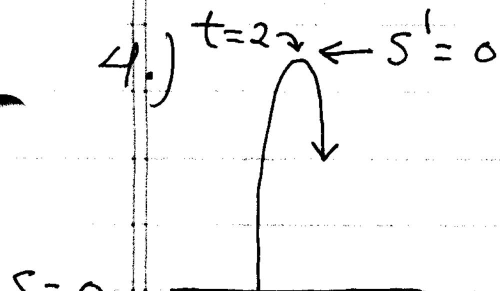
$-16t^2 - 16t + 320 = -16(t^2 + t - 20)$

$= -16(t-4)(t+5) = 0 \rightarrow \boxed{t = 4 \text{ sec.}}$

b.) $s'(1) = -48$ ft./sec.,

$s'(2) = -80$ ft./sec.,

$s'(4) = -144$ ft./sec.

4.) 

assume $s(t) = -16t^2 + V_0 t + (0)$

$\rightarrow s(t) = -16t^2 + V_0 t$

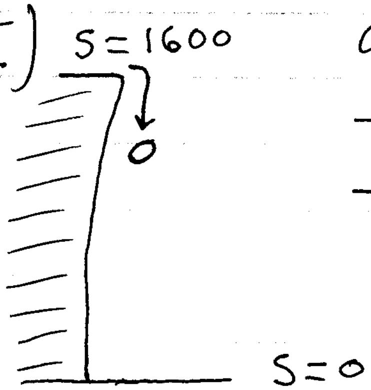
$\rightarrow$ vel. $\boxed{s'(t) = -32t + V_0}$

a.) $s'(2) = -32(2) + V_0 = 0 \rightarrow$

$V_0 = 64 \rightarrow \quad s(t) = -16t^2 + 64t \rightarrow$

$\boxed{s(2) = 64 \text{ ft.}}$

b.) $\boxed{V_0 = 64 \text{ ft./sec.}}$

5.) 

assume $s(t) = -16t^2 + (0)t + 1600$

$\rightarrow s(t) = -16t^2 + 1600$

$\rightarrow$ vel. $s'(t) = -32t$

a.) strikes ground : $s(t) = 0 \rightarrow$

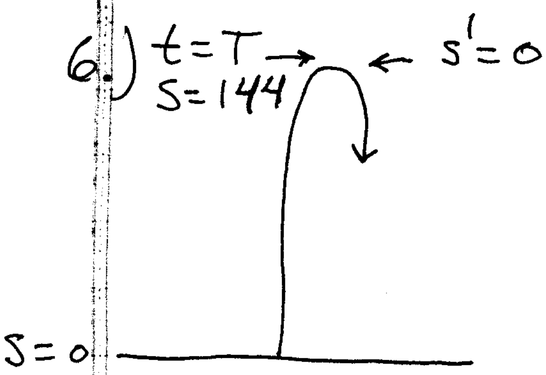
$$-16t^2+1600 = -16(t^2-100)$$
$$= -16(t-10)(t+10) = 0 \rightarrow \boxed{t = 10 \text{ sec.}}$$

b.) $s'(5) = -160$ ft./sec.,
$s'(10) = \boxed{-320 \text{ ft/sec.}}$

$$= \frac{-320 \text{ ft.}}{\text{sec.}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft.}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}}$$

$$\approx \boxed{-218.2 \text{ mph}}$$

6.)



assume $s(t) = -16t^2 + v_0 t$
$\rightarrow$ vel. $s'(t) = -32t + v_0$

a.) $s'(T) = \boxed{-32T + v_0 = 0}$

and $S(T) = \boxed{-16T^2 + v_0 T = 144}$;

(Solve 2 equations, 2 unknowns)

$$\begin{cases} v_0 = 32T \\ -16T^2 + v_0 T = 144 \rightarrow -16T^2 + (32T)T = 144 \end{cases}$$

$\rightarrow 16T^2 = 144 \rightarrow T^2 = 9 \rightarrow T = 3$

so $\boxed{T = 3 \text{ sec.}}$ to reach highest point ;

and $v_0 = 32(3) \rightarrow \boxed{v_0 = 96 \text{ ft./sec.}}$.

b.) strike ground : $s(t) = 0 \rightarrow$

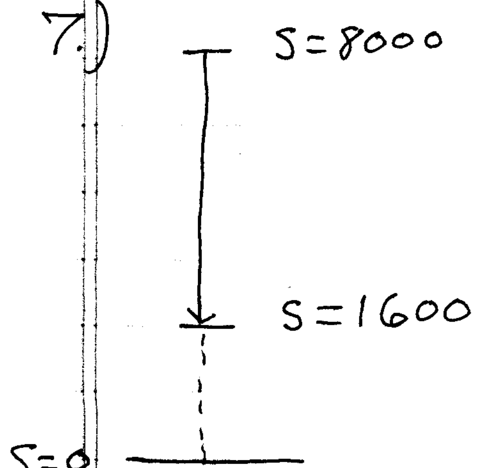
$s(t) = -16t^2 + 96t = 0 \rightarrow$

$-16t(t-6) = 0 \rightarrow \boxed{t = 6 \text{ sec.}}$

c.) $s'(0) = -32(0) + V_0 = \boxed{V_0 = 96 \text{ ft./sec.}}$

d.) You know.

7.)



assume $s(t) = -16t^2 + (0)t + 8000$

$\rightarrow s(t) = -16t^2 + 8000$

$\rightarrow$ vel. $s'(t) = -32t$

a.) $s(t) = 1600 \rightarrow$

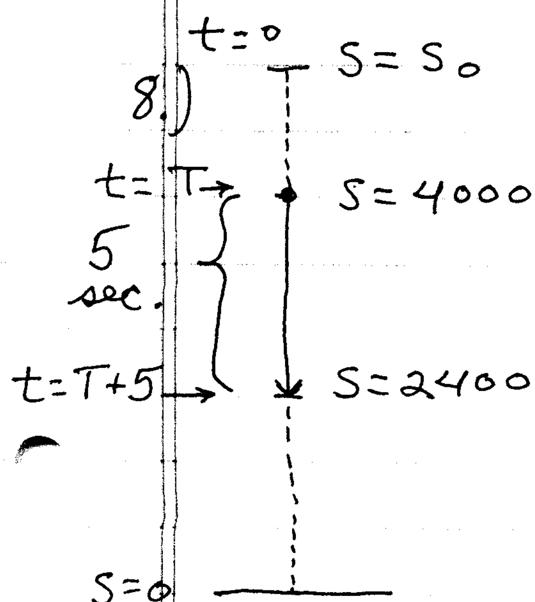
$-16t^2 + 8000 = 1600 \rightarrow$

$6400 = 16t^2 \rightarrow$

$t^2 = 400 \rightarrow \boxed{t = 20 \text{ sec.}}$

b.) $s'(20) = \boxed{-640 \text{ ft./sec.}}$

8.)



assume $s(t) = -16t^2 + V_0 t + S_0$

$\rightarrow$ vel. $s'(t) = -32t + V_0$;

a.) Given $s'(10) = -400 \rightarrow$

$-32(10) + V_0 = -400 \rightarrow$

$\boxed{V_0 = -80 \text{ ft./sec.}}$

$\rightarrow \boxed{s(t) = -16t^2 - 80t + s_0}$;

$S(T) = 4000 \rightarrow \begin{cases} -16T^2 - 80T + S_0 = 4000 \\ -16(T+5)^2 - 80(T+5) + S_0 = 2400 \end{cases}$
$S(T+5) = 2400 \rightarrow$

$\rightarrow \begin{cases} S_0 = 4000 + 16T^2 + 80T \\ -16(T^2 + 10T + 25) - 80T - 400 + S_0 = 2400 \end{cases}$ (SUB)

$\rightarrow \cancel{-16T^2} - 160T - 400 - \cancel{80T}$
$- 400 + (4000 + \cancel{16T^2} + \cancel{80T}) = 2400$

$\rightarrow -160T + 3200 = 2400$
$\rightarrow 800 = 160T \rightarrow T = 5$ sec. $\rightarrow$

b.) $S_0 = 4000 + 16(5)^2 + 80(5) \rightarrow$

$\boxed{S_0 = 4800 \text{ ft.}}$; then

$\boxed{s(t) = -16t^2 - 80t + 4800}$

c.) strike ground : $s(t) = 0 \rightarrow$
$-16t^2 - 80t + 4800 = -16(t^2 + 5t - 300)$
$= -16(t-15)(t+20) = 0 \rightarrow \boxed{t = 15 \text{ sec}}$;

d.) peach tea, raspberry tea

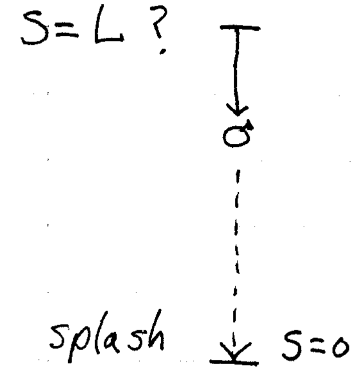
9.) $s''(t) = -32 \rightarrow$

$s'(t) = -32t + C$

$(t=0,\ s'=0 \rightarrow 0 = -32(0) + C$

$\rightarrow C = 0) \rightarrow$

$\boxed{s'(t) = -32t} \rightarrow$



$s(t) = -16t^2 + C \quad (t=0,\ s=L \rightarrow$

$L = -16(0)^2 + C \rightarrow C = L) \rightarrow$

$\boxed{s(t) = -16t^2 + L}$;

Given $s(5) = 0 \rightarrow -16(5)^2 + L = 0 \rightarrow$

$L = 400 \rightarrow \boxed{s(t) = -16t^2 + 400}$

a.) $s(0) = -16(0)^2 + 400 = 400$ ft.

b.) $s'(1) = -32(1) = -32$ ft./sec. ,

$s'(3) = -32(3) = -96$ ft./sec.

c.) $s'(5) = -32(5) = -160$ ft./sec. ;

$$\frac{-160 \text{ ft.}}{\text{sec.}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{3600 \text{ sec.}}{1 \text{ hr.}} \approx -109.1 \text{ mph}$$