

Section 5.2

$$\begin{aligned} 9.) \quad & \int (1+2x)^4 (2) dx \quad (\text{Let } u=1+2x \rightarrow du=2dx) \\ & = \int u^4 du = \frac{1}{5} u^5 + C = \frac{1}{5} (1+2x)^5 + C \end{aligned}$$

$$\begin{aligned} 12.) \quad & \int \sqrt{3-x^3} \cdot (3x^2) dx \quad (\text{Let } u=3-x^3 \rightarrow du=-3x^2 dx \\ & \rightarrow -du=3x^2 dx) \\ & = \int \sqrt{u} \cdot -du \\ & = -\int u^{1/2} du = -\frac{u^{3/2}}{3/2} + C = -\frac{2}{3} (3-x^3)^{3/2} + C \end{aligned}$$

$$\begin{aligned} 15.) \quad & \int x(x^2-1)^7 dx \quad (\text{Let } u=x^2-1 \rightarrow du=2x dx \\ & \rightarrow \frac{1}{2} du = x dx) \\ & = \int u^7 \cdot \frac{1}{2} du \\ & = \frac{1}{2} \cdot \frac{1}{8} u^8 + C = \frac{1}{16} (x^2-1)^8 + C \end{aligned}$$

$$\begin{aligned} 18.) \quad & \int \frac{x^2}{(x^3-1)^2} dx \quad (\text{Let } u=x^3-1 \rightarrow du=3x^2 dx \\ & \rightarrow \frac{1}{3} du = x^2 dx) \\ & = \int \frac{\frac{1}{3}}{u^2} du \\ & = \frac{1}{3} \int u^{-2} du = \frac{1}{3} \cdot \frac{u^{-1}}{-1} + C = -\frac{1}{3} (x^3-1)^{-1} + C \end{aligned}$$

$$\begin{aligned} 21.) \quad & \int \frac{x-2}{\sqrt{x^2-4x+3}} dx \quad (\text{Let } u=x^2-4x+3 \rightarrow \\ & du=(2x-4) dx = 2(x-2) dx \rightarrow \\ & \frac{1}{2} du = (x-2) dx) \\ & = \int \frac{\frac{1}{2}}{\sqrt{u}} du \\ & = \frac{1}{2} \int u^{-1/2} du = \frac{1}{2} \cdot \frac{u^{1/2}}{\frac{1}{2}} + C = (x^2-4x+3)^{1/2} + C \end{aligned}$$

$$\begin{aligned} 24.) \quad & \int u^3 \sqrt{u^4+2} du \quad (\text{Let } x=u^4+2 \rightarrow dx=4u^3 du \\ & \rightarrow \frac{1}{4} dx = u^3 du) \\ & = \int \sqrt{x} \cdot \frac{1}{4} dx \\ & = \frac{1}{4} \cdot \frac{x^{3/2}}{3/2} + C = \frac{1}{6} \cdot (u^4+2)^{3/2} + C \end{aligned}$$

$$\begin{aligned} 27.) \quad & \int \frac{-3}{\sqrt{2x+3}} dx \quad (\text{Let } u=2x+3 \rightarrow du=2 dx \rightarrow \\ & \frac{1}{2} du = dx) \end{aligned}$$

$$= \int \frac{-3}{\sqrt{u}} \cdot \frac{1}{2} du = -\frac{3}{2} \int u^{-1/2} du = -\frac{3}{2} \cdot \frac{u^{1/2}}{1/2} + C$$

$$= -3(2x+3)^{1/2} + C$$

32.) $\int (1 + \frac{1}{t})^3 (\frac{1}{t^2}) dt$ (Let $u = 1 + \frac{1}{t} \rightarrow du = -\frac{1}{t^2} dt$
 $\rightarrow -du = \frac{1}{t^2} dt$)

$$= \int u^3 \cdot -du$$

$$= -\frac{1}{4} u^4 + C = -\frac{1}{4} (1 + \frac{1}{t})^4 + C$$

33.) $\int (x^3 + 3x)(x^2 + 1) dx$ (Let $u = x^3 + 3x \rightarrow$
 $du = (3x^2 + 3) dx = 3(x^2 + 1) dx \rightarrow$
 $\frac{1}{3} du = (x^2 + 1) dx$)

$$= \int u \cdot \frac{1}{3} du$$

$$= \frac{1}{3} \int u du$$

$$= \frac{1}{3} \cdot \frac{u^2}{2} + C = \frac{1}{6} (x^3 + 3x)^2 + C$$

41.) $\int \frac{x^2 + 1}{\sqrt{x^3 + 3x + 4}} dx$ (Let $u = x^3 + 3x + 4 \rightarrow$
 $du = (3x^2 + 3) dx = 3(x^2 + 1) dx \rightarrow$
 $\frac{1}{3} du = (x^2 + 1) dx$)

$$= \int \frac{1}{3} \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{3} \int u^{-1/2} du = \frac{1}{3} \cdot \frac{u^{1/2}}{1/2} + C = \frac{2}{3} (x^3 + 3x + 4)^{1/2} + C$$

42.) $\int \sqrt{x} (4 - x^{3/2})^2 dx$ (Let $u = 4 - x^{3/2} \rightarrow du = -\frac{3}{2} x^{1/2} dx$
 $\rightarrow -\frac{2}{3} du = \sqrt{x} dx$)

$$= \int u^2 \cdot -\frac{2}{3} du$$

$$= -\frac{2}{3} \cdot \frac{u^3}{3} + C = -\frac{2}{9} (4 - x^{3/2})^3 + C$$

47.) $f'(x) = x\sqrt{1-x^2} \rightarrow f(x) = \int x\sqrt{1-x^2} dx \rightarrow$
(Let $u = 1 - x^2 \rightarrow du = -2x dx \rightarrow -\frac{1}{2} du = x dx$)

$$f(x) = \int \sqrt{u} \cdot -\frac{1}{2} du = -\frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C = -\frac{1}{3} (1 - x^2)^{3/2} + C$$

and

$$f(0) = \frac{4}{3} \rightarrow \frac{4}{3} = -\frac{1}{3} (1)^{3/2} + C \rightarrow C = \frac{5}{3} \rightarrow$$

$$f(x) = -\frac{1}{3} (1 - x^2)^{3/2} + \frac{5}{3}$$

$$\begin{aligned}
 55.) \quad \frac{dh}{dt} &= \frac{17.6t}{\sqrt{17.6t^2+1}} \rightarrow h = \int \frac{17.6t}{\sqrt{17.6t^2+1}} dt \\
 &= \int \frac{17.6}{\sqrt{u}} \cdot \frac{1}{35.2} du \quad (\text{Let } u = 17.6t^2 + 1 \rightarrow \\
 &= \frac{1}{2} \int u^{-1/2} du \quad du = 35.2t dt \rightarrow \frac{1}{35.2} du = t dt) \\
 &= \frac{1}{2} \cdot \frac{u^{1/2}}{1/2} + C = \sqrt{17.6t^2+1} + C \rightarrow
 \end{aligned}$$

$$\begin{aligned}
 h &= \sqrt{17.6t^2+1} + C \text{ and } t=0, h=6 \rightarrow \\
 6 &= \sqrt{17.6(0)^2+1} + C \rightarrow 6 = 1 + C \rightarrow C = 5 \rightarrow
 \end{aligned}$$

$$a.) \quad h = \sqrt{17.6t^2+1} + 5$$

$$b.) \quad t=5 \rightarrow h = \sqrt{17.6(5)^2+1} + 5 = 26 \text{ in.}$$

$$\begin{aligned}
 59.) \quad \int \frac{1}{\sqrt{x}+\sqrt{x+1}} dx &= \int \frac{1}{\sqrt{x}+\sqrt{x+1}} \cdot \frac{\sqrt{x}-\sqrt{x+1}}{\sqrt{x}-\sqrt{x+1}} dx \\
 &= \int \frac{\sqrt{x}-\sqrt{x+1}}{x-(x+1)} dx = \int (\sqrt{x+1}-\sqrt{x}) dx = \frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + C
 \end{aligned}$$

$$\begin{aligned}
 60.) \quad \int \frac{x}{\sqrt{3x+2}} dx & \quad (\text{Let } u = 3x+2 \rightarrow du = 3 dx \rightarrow \\
 &= \int \frac{\frac{1}{3}(u-2)}{\sqrt{u}} \cdot \frac{1}{3} du \quad \frac{1}{3} du = dx \text{ and } x = \frac{1}{3}(u-2)) \\
 &= \frac{1}{9} \int \frac{u-2}{u^{1/2}} du = \frac{1}{9} \int (u^{1/2} - 2u^{-1/2}) du \\
 &= \frac{1}{9} \left(\frac{u^{3/2}}{3/2} - 2 \cdot \frac{u^{1/2}}{1/2} \right) + C = \frac{1}{9} \left(\frac{2}{3}(3x+2)^{3/2} - 4(3x+2)^{1/2} \right) + C
 \end{aligned}$$

SA6: a.) $Y = 7e^{x^3+1} - 2$ (switch x, Y) $\rightarrow$

$$x = 7e^{Y^3+1} - 2 \quad (\text{solve for } Y) \rightarrow$$

$$x+2 = 7e^{Y^3+1} \rightarrow \ln\left(\frac{x+2}{7}\right) = \ln e^{Y^3+1} \rightarrow$$

$$\ln\left(\frac{x+2}{7}\right) = Y^3+1 \rightarrow Y^3 = \ln\left(\frac{x+2}{7}\right) - 1 \rightarrow$$

$$Y = \left[\ln\left(\frac{x+2}{7}\right) - 1\right]^{1/3} \text{ is inverse function.}$$

b.) $Y = 3 \log_7(4X-5)$ (switch x, Y) $\rightarrow$

$$X = 3 \log_7(4Y-5) \quad (\text{solve for } Y) \rightarrow$$

$$\frac{X}{3} = \log_7(4Y-5) \rightarrow 4Y-5 = 7^{X/3} \rightarrow$$

$$4Y = 5 + 7^{X/3} \rightarrow Y = \frac{1}{4}(5 + 7^{X/3}) \text{ is inverse function.}$$

c.) $Y = \ln(x+1) - \ln(x+2)$ (switch x, Y) $\rightarrow$

$$X = \ln(Y+1) - \ln(Y+2) \quad (\text{solve for } Y) \rightarrow$$

$$X = \ln \frac{Y+1}{Y+2} \rightarrow \frac{Y+1}{Y+2} = e^X \rightarrow Y+1 = Ye^X + 2e^X \rightarrow$$

$$Y - Ye^X = 2e^X - 1 \rightarrow Y(1 - e^X) = 2e^X - 1 \rightarrow$$

$$Y = \frac{2e^X - 1}{1 - e^X} \text{ is inverse function.}$$