Section 8.5

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1.) $\int (2\sin x + 3\cos x)\,dx = -2\cos x + 3\sin x + c$

5.) $\int (\csc^2\theta - \cos\theta)\,d\theta = -\cot\theta - \sin\theta + c$

6.) $\int (\sec y \tan y - \sec^2 y)\,dy = \sec y - \tan y + c$

7.) $\int \sin 2x\,dx$ (Let $u = 2x \rightarrow du = 2\,dx \rightarrow \frac{1}{2}du = dx$)

$= \frac{1}{2}\int \sin u\,du$

$= \frac{-1}{2}\cos u + c = \frac{-1}{2}\cos 2x + c$

10.) $\int x \sin x^2\,dx$ (Let $u = x^2 \rightarrow du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\int \sin u\,du$

$= \frac{-1}{2}\cos u + c = \frac{-1}{2}\cos x^2 + c$

11.) $\int \sec^2\left(\frac{x}{2}\right)dx$ (Let $u = \frac{x}{2} \rightarrow du = \frac{1}{2}dx \rightarrow 2\,du = dx$)

$= 2\int \sec^2 u\,du = 2\tan u + c = 2\tan\left(\frac{x}{2}\right) + c$

12.) $\int \csc^2\left(\frac{x}{2}\right)dx$ (Let $u = \frac{x}{2} \rightarrow du = \frac{1}{2}dx \rightarrow 2\,du = dx$)

$= 2\int \csc^2 u\,du = -2\cdot\cot u + c = -2\cot\left(\frac{x}{2}\right) + c$

14.) $\int \csc 2x \cdot \cot 2x\,dx$ (Let $u = 2x \rightarrow du = 2\,dx \rightarrow \frac{1}{2}du = dx$)

$= \frac{1}{2}\int \csc u \cot u\,du$

$= \frac{-1}{2}\csc u + c = \frac{-1}{2}\csc 2x + c$

16.) $\int \sqrt{\cot x} \cdot \csc^2 x \, dx$ (Let $u = \cot x \rightarrow$
$du = -\csc^2 x \, dx \rightarrow -du = \csc^2 x \, dx$)

$= -\int \sqrt{u} \, du = -\dfrac{u^{3/2}}{3/2} + c = -\dfrac{2}{3}(\cot x)^{3/2} + c$

22.) $\int \dfrac{\sin x}{\cos^2 x} dx$ (Let $u = \cos x \rightarrow du = -\sin x \, dx$
$\rightarrow -du = \sin x \, dx$)

$= -\int \dfrac{1}{u^2} du = -\int u^{-2} du = -(-u^{-1}) + c$

$= \dfrac{1}{u} + c = \dfrac{1}{\cos x} + c$

26.) $\int \dfrac{\sin \sqrt{x}}{\sqrt{x}} dx$ (Let $u = \sqrt{x} \rightarrow$
$du = \frac{1}{2} x^{-1/2} dx = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} dx \rightarrow$

$= 2\int \sin u \, du$ $\quad 2\,du = \frac{1}{\sqrt{x}} dx$)

$= -2\cos u + c = -2\cos\sqrt{x} + c$

32.) $\int e^{\sec x} \cdot \sec x \tan x \, dx$ (Let $u = \sec x \rightarrow$

$= \int e^u du$ $\quad du = \sec x \tan x \, dx$)

$= e^u + c = e^{\sec x} + c$

33.) $\int (\sin 2x + \cos 2x)^2 dx$

$= \int (\sin^2 2x + 2\sin 2x \cos 2x + \cos^2 2x) \, dx$

$= \int [(\sin^2 2x + \cos^2 2x) + 2\sin 2x \cos 2x] \, dx$

$= \int [1 + 2\sin 2x \cos 2x] \, dx = x + 2\int \sin 2x \cos 2x \, dx$

(Let $u = \sin 2x \rightarrow du = 2\cos 2x \, dx \rightarrow$

$$\tfrac{1}{2}\,du = \cos 2x\,dx\ )$$

$$= x + \cancel{2}\cdot \tfrac{1}{\cancel{2}} \int u\,du = x + \tfrac{1}{2}u^2 + c$$

$$= x + \tfrac{1}{2}(\sin 2x)^2 + c$$

34.) $\int (\csc 2\theta - \cot 2\theta)^2\,d\theta$

$$= \int (\csc^2 2\theta - 2\csc 2\theta \cot 2\theta + \cot^2 2\theta)\,d\theta$$

$$= \int (\underline{\csc^2 2\theta} - 2\csc 2\theta \cot 2\theta + \underline{(\csc^2 2\theta - 1)})\,d\theta$$

$$= \int (2\csc^2 2\theta - 1 - 2\csc 2\theta \cot 2\theta)\,d\theta$$

$$(\text{Let } u = 2\theta \rightarrow du = 2\,d\theta \rightarrow \tfrac{1}{2}du = d\theta)$$

$$= \tfrac{1}{2}\int (2\csc^2 u - 1 - 2\csc u \cot u)\,du$$

$$= \tfrac{1}{2}\left[-2\cot u - u - 2\cdot(-\csc u)\right] + c$$

$$= -\cot u - \tfrac{1}{2}u + \csc u + c$$

$$= -\cot 2\theta - \tfrac{1}{2}2\theta + \csc 2\theta + c$$

$$= -\cot 2\theta - \theta + \csc 2\theta + c$$

Handout 5

1.) $\int 5(\sin x)^4 \cos x \, dx = (\sin x)^5 + c$

or

let $u = \sin x \rightarrow du = \cos x \, dx \rightarrow$

$\int 5(\sin x)^4 \cos x \, dx = \int 5u^4 \, du = u^5 + c = (\sin x)^5 + c$

2.) $\int (\sin x)^{99} \cos x \, dx = \frac{1}{100}(\sin x)^{100} + c$

or

let $u = \sin x \rightarrow du = \cos x \, dx \rightarrow$

$\int (\sin x)^{99} \cos x \, dx = \int u^{99} \, du = \frac{1}{100} u^{100} + c = \frac{1}{100}(\sin x)^{100} + c$

3.) $\int \sin x \cos x = \frac{1}{2}(\sin x)^2 + c$

or

let $u = \sin x \rightarrow du = \cos x \, dx \rightarrow$

$\int \sin x \cos x \, dx = \int u \, du = \frac{1}{2}u^2 + c = \frac{1}{2}(\sin x)^2 + c$

4.) $\int (\sin x + \cos x)^2 dx = \int (\sin^2 x + 2\sin x \cos x + \cos^2 x) \, dx$

$= \int (1 + 2\sin x \cos x) \, dx = x + (\sin x)^2 + c$

5.) $\int \sin^2 x \, dx \quad \left(\cos 2\theta = 1 - 2\sin^2\theta \rightarrow \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)\right)$

$= \int \frac{1}{2}(1 - \cos 2x) \, dx = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + c$

6.) $\int \sec x \tan x \, dx = \sec x + c$

7.) $\int \sec^2 x \cdot \tan x \, dx = \frac{1}{2}(\tan x)^2 + c$

or

let $u = \tan x \rightarrow du = \sec^2 x \, dx \rightarrow$

$\int \sec^2 x \tan x \, dx = \int u \, du = \frac{1}{2}u^2 + c = \frac{1}{2}(\tan x)^2 + c$

8.) $\int \sec^2 x \tan^2 x \, dx = \frac{1}{3}\tan^3 x + c$

or

let $u = \tan x \rightarrow du = \sec^2 x \, dx \rightarrow$

$\int \sec^2 x \tan^2 x \, dx = \int u^2 du = \frac{1}{3}u^3 + c = \frac{1}{3}(\tan x)^3 + c$

9.) $\int \sec^2 x \, dx = \tan x + c$

10.) $\int \tan^2 x \, dx \quad (1 + \tan^2 x = \sec^2 x \rightarrow \tan^2 x = \sec^2 x - 1)$

$= \int (\sec^2 x - 1) \, dx = \tan x - x + c$

11.) $\int \sec^3 x \tan x \, dx = \int \sec^2 x \cdot \sec x \tan x \, dx$

$= \frac{1}{3}(\sec x)^3 + c$ or

let $u = \sec x \rightarrow du = \sec x \tan x \, dx \rightarrow$

$= \int u^2 du = \frac{1}{3}u^3 + c = \frac{1}{3}(\sec x)^3 + c$

12.) $\int \csc^2 x \, dx = -\int -\csc^2 x \, dx = -\cot x + c$

13.) $\int \csc x \cot x \, dx = -\int -\csc x \cot x \, dx$

$= -\csc x + c$

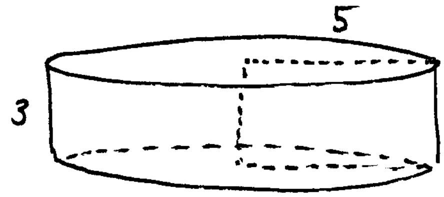
14.) $\int \cot^2 x \, dx \quad (1 + \cot^2 x = \csc^2 x \rightarrow \cot^2 x = \csc^2 x - 1)$

$= \int (\csc^2 x - 1) \, dx = -\cot x - x + c$

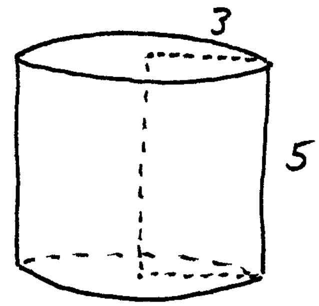
15.) $\int (x \cdot \sec^2 x + \tan x) \, dx = x \tan x + c$

product rule

$\boxed{SA9}$ a.)



$V = \pi(5)^2(3)$
$= 75\pi$



b.) $V = \pi(3)^2(5) = 45\pi$