Math 25

Some Proof Templates

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Let \( P, Q, M, L \) be propositions and \( S \) a set.

Direct Proof of \( P \Rightarrow Q \)

Suppose \( P \).

\[ \Rightarrow \text{ Deductive} \]
\[ \Rightarrow \text{ Reasoning} \]

Therefore, \( Q \).

Thus, \( P \Rightarrow Q \).

Contrapositive Proof of \( P \Rightarrow Q \)

Suppose \( \sim Q \).

\[ \Rightarrow \text{ Deductive} \]
\[ \Rightarrow \text{ Reasoning} \]

Therefore, \( \sim P \).

Thus, \( \sim Q \Rightarrow \sim P \) and we conclude \( P \Rightarrow Q \).

Note: To show \( (P \Rightarrow Q) \Rightarrow (\sim Q \Rightarrow \sim P) \) one must use true tables. This is covered in Math 108.

Proof by Contradiction of \( M \)

Find a proposition \( L \) for which one can do the following:

Suppose \( \sim M \).

\[ \Rightarrow \text{ Deductive} \]
\[ \Rightarrow \text{ Reasoning} \]

Therefore, \( L \).

\[ \Rightarrow \text{ More Deductive} \]
\[ \Rightarrow \text{ Reasoning} \]

Therefore, \( \sim L \).

Hence, \( \sim M \Rightarrow L \) and \( \sim L \), which is a contradiction.

Thus \( M \).
Proof of \((\forall x \in S) P(x)\)

Let \(x \in S\) (This \(x\) is arbitrary, without restrictions)

\[\exists\] Deductive Reasoning

Hence, \(P(x)\) is true.

Since \(x\) is arbitrary, \((\forall x \in S) P(x)\) is true.

Note: One can also prove \((\forall x \in S) P(x)\) by contradiction, using \(\neg (\forall x \in S) P(x) \iff (\exists x \in S) (\neg P(x))\), which is discussed in more detail in Math 108.

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Proof of \((\exists x \in S) P(x)\)

Find a \(c \in S\) for which \(P(c)\) is true and show, using deductive reasoning, that \(P(c)\) is true.

Note: One can also prove \((\exists x \in S) P(x)\) by contradiction, using \(\neg (\exists x \in S) P(x) \iff (\forall x \in S) (\neg P(x))\), which is discussed in more detail in Math 108.

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Proof of \((\exists! x \in S) P(x)\)

(i) Prove \((\exists x) P(x)\) is true (See above)

(ii) Assume \(t, c \in S\) and \(t_2 \in S\) such that \(P(t)\) and \(P(t_2)\) are true.

\[\exists\] Deductive Reasoning

Therefore, \(t = t_2\).

Thus, we conclude \((\exists! x) P(x)\).

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Proof by Induction of \(P(n)\) is true \(\forall n \in \mathbb{N}\)

(i) Prove \(P(1)\) is true (Base Step)

(ii) Prove if \(P(n)\) is true \(\Rightarrow P(n+1)\) is true (Inductive Step)

Thus, by the Principle of Mathematical Induction (PMI), \(P(n)\) is true \(\forall n \in \mathbb{N}\).