Math 25
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Some Proof Templates

Let P, Q, M, L be propositions and S a set.

Direct Proof of $P \Rightarrow Q$

Suppose P.
$\vdots$ } Deductive Reasoning
Therefore, Q.
Thus, $P \Rightarrow Q$.

Contrapositive Proof of $P \Rightarrow Q$

Suppose $\sim Q$.
$\vdots$ } Deductive Reasoning
Therefore, $\sim P$.
Thus, $\sim Q \Rightarrow \sim P$ and we conclude $P \Rightarrow Q$.

-Note: To show $(P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P)$ one must use true tables. This is covered in Math 108.

Proof by Contradiction of M

Find a proposition L for which one can do the following:
Suppose $\sim M$.
$\vdots$ } Deductive Reasoning
Therefore, L.
$\vdots$ } More Deductive Reasoning
Therefore, $\sim L$.
Hence, $\sim M \Rightarrow L$ and $\sim L$, which is a contradiction.
Thus M.

Proof of $(\forall x \in S) P(x)$

Let $x \in S$ (This x is arbitrary, without restrictions)

$\left.\begin{array}{c}\vdots\end{array}\right\}$ Deductive Reasoning

Hence, $P(x)$ is true.

Since x is arbitrary, $(\forall x \in S) P(x)$ is true.

Note: One can also prove $(\forall x \in S) P(x)$ by contradiction, using $\sim(\forall x \in S) P(x) \iff (\exists x \in S)(\sim P(x))$, which is discussed in more detail in Math 108.

Proof of $(\exists x \in S) P(x)$

Find a $c \in S$ for which $P(c)$ is true and show, using deductive reasoning, that $P(c)$ is true.

Note: One can also prove $(\exists x \in S) P(x)$ by contradiction, using $\sim(\exists x \in S) P(x) \iff (\forall x \in S)(\sim P(x))$, which is discussed in more detail in Math 108.

Proof of $(\exists! x \in S) P(x)$

(i) Prove $(\exists x) P(x)$ is true (See above)

(ii) Assume $t_1 \in S$ and $t_2 \in S$ such that $P(t_1)$ and $P(t_2)$ are true.

$\left.\begin{array}{c}\vdots\end{array}\right\}$ Deductive Reasoning

Therefore, $t_1 = t_2$.

Thus, we conclude $(\exists! x) P(x)$.

Proof by Induction of $P(n)$ is true $\forall n \in \mathbb{N}$

(i) Prove $P(1)$ is true (Base Step)

(ii) Prove if $P(n)$ is true $\implies P(n+1)$ is true (Inductive Step)

Thus, by the Principle of Mathematical Induction (PMI), $P(n)$ is true $\forall n \in \mathbb{N}$