Homework 6 Solutions

7.3) (a) $\lim_{n\to\infty} \dfrac{n}{n+1} = 1$

(f) $\lim_{n\to\infty} 2^{\frac{1}{n}} = 2^0 = 1$

(i) $\lim_{n\to\infty} \dfrac{(-1)^n}{n} = 0$

k) $\lim_{n\to\infty} \dfrac{9n^2-18}{6n+18}$ does not exist, so the sequence diverges.

p) $\lim_{n\to\infty} \dfrac{2^{n+1}+5}{2^n-7} = 2$

q) $\lim_{n\to\infty} \dfrac{3^n}{n!} = 0$

r) $\lim_{n\to\infty} \left(1+\dfrac{1}{n}\right)^2 = 1^2 = 1$

s) $\lim_{n\to\infty} \dfrac{4n^2+3}{3n^2-2} = \dfrac{4}{3}$

t) $\lim_{n\to\infty} \dfrac{6n+4}{9n^2+7} = 0$

For the next problem, we need the following fact (which is an optional homework assignment):

Proposition 1 *If $x \in \mathbb{I}$ and $r \in \mathbb{Q}$, then $r \cdot x \in \mathbb{I}$.*

7.4) (a) Let $x_n = \dfrac{\sqrt{2}}{n}$. Then $x_n = \dfrac{1}{n} \cdot \sqrt{2} \in \mathbb{I}\ \ \forall n \in \mathbb{N}$ since $\dfrac{1}{n} \in \mathbb{Q}$, $\dfrac{1}{n} \neq 0$, and $\sqrt{2} \in \mathbb{I}$. Moreover, we have

$$\lim_{n\to\infty} x_n = 0$$