Math 25

Problem Sheet 1

1) Prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = n(n+1)(n+2)/3$ for all n in N.

2) Prove that $(1+x)^n \geq 1+nx$ for every n in N if $x > -1$. (*Bernoulli's inequality*)

3) Let m be in Z. Prove that if m^3 is even, then m is even.

4) Prove that if x is an irrational number and r is a rational number, then x+r is irrational.

5) Give a proof by contradiction that $\sqrt{5}$ is irrational.

6) Prove that if a, b, and c are integers with $a^2+b^2=c^2$, then a or b is even.

7) Use the Well-Ordering Principle to show that every integer $n>1$ has a prime factor.

8) Prove the following statement, or else show that it is false by giving a counterexample:
If a and b are integers such that ab is divisible by 6, then a or b is divisible by 6.

9) Give a proof by contradiction that $\sqrt{2} + \sqrt[3]{5}$ is irrational.