Math 25 Problem Sheet 4

1) Show that if $\lim\limits_{n\to\infty} s_n = s$, then $\lim\limits_{n\to\infty} |s_n| = |s|$.

2) Recall that a function f is continuous at c iff for every $\epsilon > 0$, there is a $\delta > 0$ such that if $|x - c| < \delta$, then $|f(x) - f(c)| < \epsilon$.

Prove that if $\lim\limits_{n\to\infty} s_n = L$ and f is continuous at L, then $\lim\limits_{n\to\infty} f(s_n) = f(L)$.

3) Prove that if $\lim\limits_{n\to\infty} s_n = s$ and $\lim\limits_{n\to\infty} t_n = t$, then $\lim\limits_{n\to\infty} s_n t_n = st$ by citing the justification for each of the following steps:

a) $\lim\limits_{n\to\infty} (s_n - s) = 0$ and $\lim\limits_{n\to\infty} (t_n - t) = 0$.

b) $\lim\limits_{n\to\infty} (s_n - s)(t_n - t) = 0$, and so $\lim\limits_{n\to\infty} (s_n t_n - s t_n - s_n t + st) = 0$.

c) $\lim\limits_{n\to\infty} s_n t_n = \lim\limits_{n\to\infty} [(s_n t_n - s t_n - s_n t + st) + s t_n + s_n t - st] =$

$\lim\limits_{n\to\infty} (s_n t_n - s t_n - s_n t + st) + \lim\limits_{n\to\infty} s t_n + \lim\limits_{n\to\infty} s_n t - \lim\limits_{n\to\infty} st.$

d) $\lim\limits_{n\to\infty} s_n t_n = 0 + st + st - st = st.$

4) If (s_n) is bounded, show that $\lim\limits_{n\to\infty} \frac{s_n}{n} = 0$.

5) Use the Squeeze Theorem to find $\lim\limits_{n\to\infty} (3^n + 2^n)^{\frac{1}{n}}$.

6) Prove that $\lim\limits_{n\to\infty} \dfrac{n^2 + 5}{n + 3} = \infty$.