1.) Compute the Midpoint Estimate, $M_6$, for \( \int_0^1 \frac{1}{x^2 + 1} \, dx \). Compare your answer with the exact value of the integral.

2.) Compute the Trapezoidal Estimate, $M_5$, for \( \int_{-1}^1 \sqrt{1-x} \, dx \). Compare your answer with the exact value of the integral.

3.) Determine the value of $n$ so that the Trapezoidal Estimate, $T_n$, estimates the exact value of \( \int_0^{1/2} e^{-2x^2} \, dx \) with absolute error at most 0.00001.

4.) Determine the value of $n$ so that the Midpoint Estimate, $T_n$, estimates the exact value of \( \int_0^3 \frac{x+1}{x+5} \, dx \) with absolute error at most 0.00001.

5.) Compute the following improper integrals.

   a.) \( \int_0^4 \frac{1}{\sqrt{x}} \, dx \)  
   b.) \( \int_1^\infty \frac{3}{x^2} \, dx \)  
   c.) \( \int_0^1 \frac{3}{x^2} \, dx \)  
   d.) \( \int_{\sqrt{3}}^\infty \frac{1}{1+x^2} \, dx \)  
   e.) \( \int_e^\infty \frac{1}{x \ln x} \, dx \)  
   f.) \( \int_{-\infty}^0 \frac{e^x}{1+e^{2x}} \, dx \)  
   g.) \( \int_1^x \frac{1}{x(x+4)} \, dx \)  
   h.) \( \int_0^1 e^{3x} \, dx \)  
   i.) \( \int_{-1}^\infty \frac{1}{\sqrt{x}+1} \, dx \)  
   j.) \( \int_{-\infty}^{\sqrt{3}} \frac{1}{x^2+9} \, dx \)  
   k.) \( \int_{-\infty}^1 e^{x+1} \, dx \)  
   l.) \( \int_{x-1}^{7} \frac{7}{x-1} \, dx \)

6.) Consider the region $R$ (in the first quadrant) bounded by the graphs of $y = \frac{1}{x}$, $x = 1$, and $y = 0$.

   a.) Determine if $R$ has finite or infinite area.
   b.) Form a solid by revolving $R$ about the x-axis. Determine if the resulting volume is finite or infinite.

7.) Find the following Taylor polynomials of degree $n$ about $a = 0$, $P_n(x)$, for the indicated functions.

   a.) $f(x) = x^4 + x^3 - x^2 + 3x - 5$, $n = 2$  
   b.) $f(x) = x^4 + x^3 - x^2 + 3x - 5$, $n = 4$  
   c.) $f(x) = xe^x$, $n = 3$  
   d.) $f(x) = \sqrt{x+4}$, $n = 2$  
   d.) $f(x) = \ln(x+1)$, $n = 3$

8.) Find the following Taylor polynomials about $a = 0$ for the function $f(x) = \frac{x-2}{x+1}$:
$P_0(x), P_1(x), P_2(x), P_3(x)$. Compare the values of the function and its Taylor polynomials at $x = 0.1$ and $x = 2$. What conclusion do you draw?

9.) It is well known that the integral $\int_0^1 e^{x^2} \, dx$ has no closed-form anti-derivative. Replace $f(x) = e^{x^2}$ with $P_3(x)$, its fourth-degree Taylor Polynomial centered at $x = 0$, to get an estimate for this definite integral. Compare this value with one obtained by a calculator which computes definite integrals and determine the absolute percentage error in your estimate.

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THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

10.) A nonnegative integer $I$ is a perfect square, triangular (PST) number if $I$ is equal to the square of a nonnegative integer AND is also equal to one-half the product of consecutive nonnegative integers. Find the first four PST numbers.