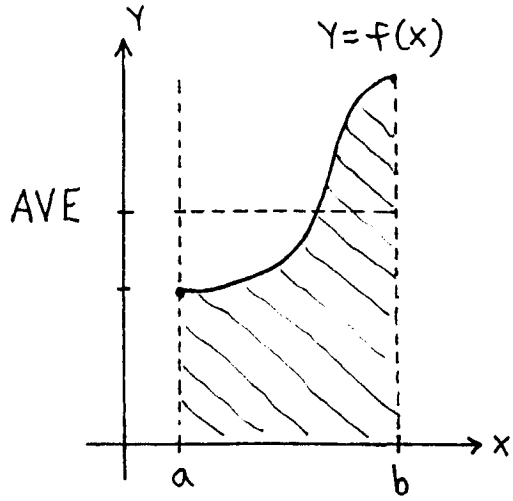
Math 17B
Vogler
Average Value of a Function

DEFINITION : The *average value*, AVE, of a function f on the inteval [a, b] is that y-value which determines the height of the rectangle (See diagram.) which has area exactly equal to the definite integral

$\int_a^b f(x)\,dx$, that is, $\text{AVE}\,(b-a) = \int_a^b f(x)\,dx$, or $\text{AVE} = \frac{1}{b-a}\int_a^b f(x)\,dx$



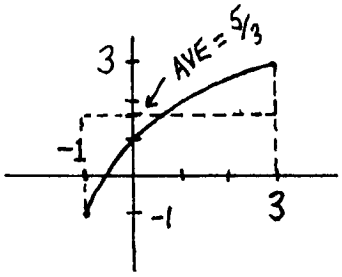
EXAMPLE : Find the average value of $f(x) = -1 + 2\sqrt{x+1}$ on the interval [-1, 3].

$$\text{AVE} = \frac{1}{3-(-1)}\int_{-1}^{3}\left(-1+2\sqrt{x+1}\right)dx$$

$$= \frac{1}{4}\left[-x + 2\cdot\frac{2}{3}(x+1)^{3/2}\right]\Big|_{-1}^{3}$$

$$= \frac{1}{4}\left(-3+\frac{4}{3}(4)^{3/2}\right) - \frac{1}{4}\left(1+\frac{4}{3}(0)^{3/2}\right)$$

$$= \frac{1}{4}\left(-3+\frac{4}{3}(8)\right) - \frac{1}{4} = \frac{20}{12} = \frac{5}{3}$$



EXAMPLE : Money is withdrawn from an account in such a manner that the amount of money A in the account at time t years is given by the equation $A = 800{,}000 / (t+2)^3$ for $t \geq 0$.

a.) What is the initial amount of money in the account ?

b.) How much money is in the account after 10 years ?

c.) What is the average amount of money in the account from t = 0 years to t = 10 years ?

d.) When is the average amount of money in the account equal to the average from part c.) ?