Math 17B
Vogler
Even and Odd Functions
Knowing if a function is even or odd can sometimes lead to a relatively easy solution to a definite integral.

DEFINITIONS : Function $f$ is even if $f(\mathrm{x})=f(-\mathrm{x})$. Function $f$ is odd if $f(\mathrm{x})=-f(-\mathrm{x})$. EXAMPLE:


## REMARKS:

I. If $f$ is even then $\int_{0}^{a} f(x) d x=\int_{-a}^{0} f(x) d x$ so that $\int_{-a}^{a} f(x) d x=2 \int_{0}^{a} f(x) d x$.
II. If $f$ is odd then $\int_{0}^{\mathrm{a}} f(\mathrm{x}) \mathrm{dx}=-\int_{-\mathrm{a}}^{0} f(\mathrm{x}) \mathrm{dx}$ so that $\int_{-\mathrm{a}}^{\mathrm{a}} f(\mathrm{x}) \mathrm{dx}=0$.

PROBLEM: Show that $f(\mathrm{x})=\mathrm{x} \sqrt{\mathrm{x}^{2}+\cos \mathrm{x}}$ is an odd function, then evaluate the definite integral $\int_{-5}^{5} x \sqrt{x^{2}+\cos x} d x$.

