Math 17B  

Arc length Example

Fact: One can show that the arc length of a continuous function \( y = f(x) \) from \( x = a \) to \( x = b \) is

\[
ARC = \int_a^b \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx
\]

Ex: Compute the length of the graph of \( y = 1 + \frac{2}{3} x^{3/2} \) on the interval \([0,3]\).

\[
y = 1 + \frac{2}{3} x^{3/2} \quad \Rightarrow \quad y' = x^{1/2}
\]

\[
\Rightarrow \quad ARC = \int_0^3 \sqrt{1 + (y')^2} \, dx = \int_0^3 \sqrt{1 + \left( x^{1/2} \right)^2} \, dx = \int_0^3 \sqrt{1 + x} \, dx
\]

\[
= \left[ \int_0^3 (1+x)^{1/2} \, dx \right] = \left[ \frac{2}{3} (1+x)^{3/2} \right]_0^3 = \frac{2}{3} \left( \left( y \right)^{3/2} - \left( 1 \right)^{3/2} \right)
\]

\[
= \frac{2}{3} \cdot 8 - \frac{2}{3} = \frac{14}{3}
\]

\(
distance = ARC = \frac{14}{3}
\)