Defn Consider the autonomous Differential Equation
\[ \frac{dy}{dx} = g(y), \] (*)
and assume that \( \hat{y} = k \) is a constant. If \( g(\hat{y}) = 0 \), then we say that the constant function \( \hat{y} \), as a solution to the D.E. (*), is an equilibrium of (*).

Note: To find equilibria of D.E.s of the form (*), you just solve the equation \( g(y) = 0 \) for \( y \) algebraically.

Defn Let \( \hat{y} = k \) be an equilibrium for \( \frac{dy}{dx} = g(y) \), i.e \( g(\hat{y}) = 0 \).

1) \( \hat{y} \) is stable if solutions to D.E. (*) with nearby initial values \( (t=0) \) "converge to" \( \hat{y} \) as \( t \to \infty \).
2) \( \hat{y} \) is unstable if solutions to D.E. with nearby initial values \( (t=0) \) "diverge away from" \( \hat{y} \) as \( t \to \infty \).

Classifying Stability (2 Methods)

Let \( \hat{y} = k \) be an equilibrium for \( \frac{dy}{dx} = g(y) \), so \( g(\hat{y}) = 0 \).

I) Graphical Approach (Using Sign charts)

a) \[ \begin{array}{c|ccc} + & 0 & - \\ \hline y & \hat{y} & (\text{Stable}) \end{array} \]
b) \[ \begin{array}{c|ccc} - & 0 & + \\ \hline y & \hat{y} & (\text{Unstable}) \end{array} \]
c) \[ \begin{array}{c|ccc} + & 0 & + \\ \hline y & \hat{y} & (\text{semi-stable}) \end{array} \]
d) \[ \begin{array}{c|ccc} - & 0 & - \\ \hline y & \hat{y} & (\text{semi-stable}) \end{array} \]
II) Analytical/Eigenvalue Approach (Using derivative)
Let \( \lambda = g'(\hat{y}) \), which is called the eigenvalue for \( y = \hat{y} \).
Then,
   a) If \( \lambda < 0 \), then \( \hat{y} \) is stable.
   b) If \( \lambda > 0 \), then \( \hat{y} \) is unstable.
   c) If \( \lambda = 0 \) and either \( g''(\hat{y}) > 0 \) or \( g''(\hat{y}) < 0 \),
      then \( \hat{y} \) is semi-stable.

Ex) Find and classify (stable/unstable) the equilibria
    for \( \frac{dN}{dt} = N^2 - 3N - 4 \)
    using a) graphical approach, b) analytical approach

(Solve for roots)
\[
g(N) = N^2 - 3N - 4 = (N - 4)(N + 1) = 0
\]
\[
\Rightarrow \begin{cases} N = 4 \text{ and } N = -1 \end{cases} \text{ are equilibria.}
\]

a) \[
\begin{array}{c c c}
\text{Stable} & \rightarrow & N = -1 \quad \text{Unstable} \\
\text{Stable} & \rightarrow & N = 4 \quad \text{Unstable}
\end{array}
\]

b) \( g'(N) = 2N - 3 \)
   i) \( g'(-1) = 2(-1) - 3 = -5 < 0 \),
      so \( N = -1 \) is \text{Stable}
   ii) \( g'(4) = 2(4) - 3 = 5 > 0 \),
      so \( N = 4 \) is \text{Unstable}