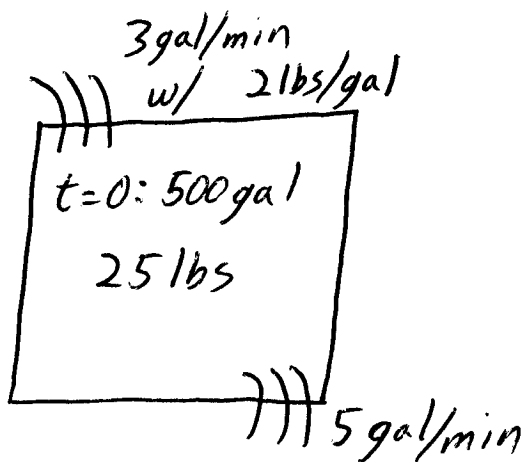


Math 17B
Vogler

Mixture Problem Handout
Solution to Example 2



a) $t=1 \text{ min}: 500 - 2(1) = 498 \text{ gal}$

$t=10 \text{ min}: 500 - 2(10) = 480 \text{ gal}$

$t=50 \text{ min}: 500 - 2(50) = 400 \text{ gal}$

$t \text{ min}: 500 - 2t \text{ gal}$

For empty tank, we must have

$500 - 2t = 0 \Rightarrow t = 250 \text{ min}$

b) $\frac{ds}{dt} = (\text{Rate In}) - (\text{Rate Out})$

$= \frac{2 \text{ lbs}}{\text{gal}} \frac{3 \text{ gal}}{\text{min}} - \frac{S \text{ lbs}}{500 - 2t \text{ gal}} \frac{5 \text{ gal}}{\text{min}}$

$\Rightarrow \boxed{\frac{ds}{dt} = 6 - \frac{5S}{500 - 2t} \text{ and } S(0) = 25 \text{ lbs}}$

(First-Order Linear D.E.)

$\Rightarrow \frac{ds}{dt} + \frac{5}{500 - 2t} S = 6 \left\{ \begin{array}{l} \mu = e^{\int \frac{5}{500 - 2t} dt} = e^{-\frac{5}{2} \ln(500 - 2t)} \\ = e^{\ln(500 - 2t)^{-5/2}} = (500 - 2t)^{-5/2} \end{array} \right.$

$\Rightarrow (500 - 2t)^{-5/2} \frac{ds}{dt} + (500 - 2t)^{-5/2} \frac{5}{500 - 2t} S = 6(500 - 2t)^{-5/2}$

$\Rightarrow D[(500 - 2t)^{-5/2} S] = 6(500 - 2t)^{-5/2}$

$\Rightarrow (500 - 2t)^{-5/2} S = \int 6(500 - 2t)^{-5/2} dt$
 $= 6 \cdot \frac{1}{2} \cdot \frac{-2}{-3} (500 - 2t)^{-3/2} + C$

$$\Rightarrow (500-2t)^{-5/2} S = 2(500-2t)^{-3/2} + C$$

$$\Rightarrow S = 2(500-2t) + (500-2t)^{5/2} C$$

$$\text{For } t=0, S=25 \Rightarrow 25 = 1000 + 500^{5/2} C$$

$$\Rightarrow C = \frac{-975}{500^{5/2}}$$

$$\text{Thus, } \boxed{S(t) = 1000 - 4t - \frac{975}{500^{5/2}} (500-2t)^{5/2}}$$

$$c) t = 10 \text{ min: } S \approx 79.6 \text{ lbs}$$

$$t = 100 \text{ min: } S \approx 328.1 \text{ lbs}$$

$$t = 200 \text{ min: } S \approx 182.6 \text{ lbs}$$

$$t = 240 \text{ min: } S \approx 39.7 \text{ lbs}$$

d) Optional (The only way a problem like this appears on the exam is as an Extra Credit one)