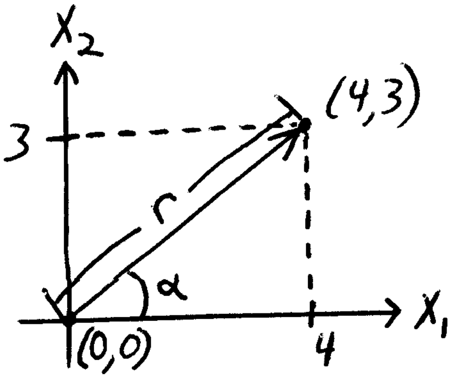
Math 17B
Vogler

Vectors in $\mathbb{R}^2$

Consider the point $(4,3)$ in the Cartesian Plane.



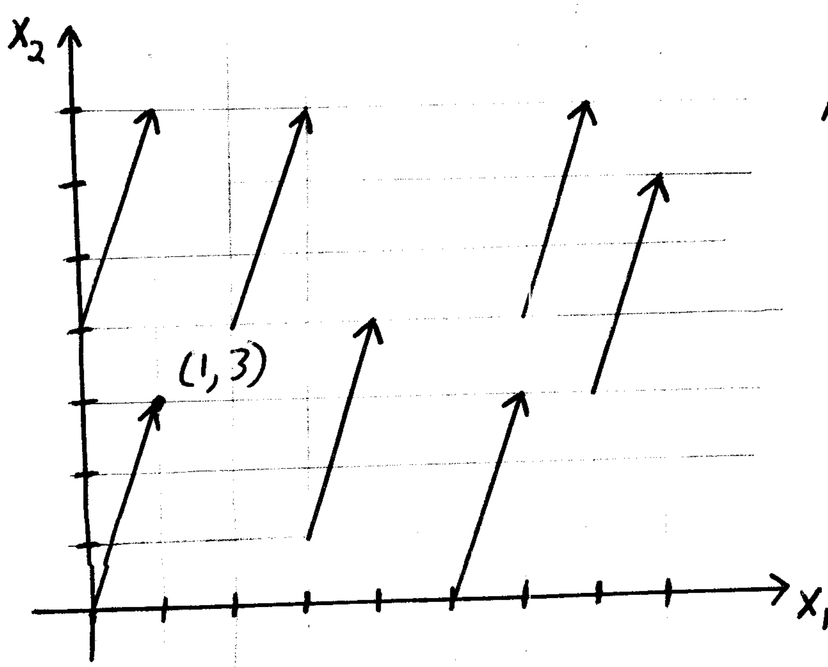
It's distance from the origin is
$$r=\sqrt{4^2+3^2}=5,$$
and forms an angle α, where
$$\tan\alpha=\frac{3}{4} \text{ or } \alpha=\arctan\frac{3}{4}.$$

Defn A 2×1 matrix is called a vector in $\mathbb{R}^2$.

Ex The following are vectors, $\begin{bmatrix}4\\-1\end{bmatrix}$, $\begin{bmatrix}0\\0\end{bmatrix}$, $\begin{bmatrix}1\\0\end{bmatrix}$, $\begin{bmatrix}3/4\\\sqrt{2}\end{bmatrix}$.

Notes: 1) A vector can also be defined by an arrow with length (magnitude) and direction (angle).

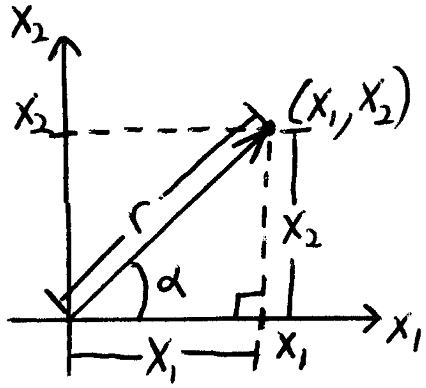
2) Any two vectors with the same length and direction are considered equal.



All 7 of these vectors are equal to the vector
$$\vec{x}=\begin{bmatrix}1\\3\end{bmatrix}$$

Defn

Consider the vector determined by the point (x_1, x_2)



The Cartesian form of this vector is: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

We also know from Trig. that $\boxed{r = \sqrt{x_1^2 + x_2^2}}$ and $\boxed{\alpha = \arctan\left(\frac{x_2}{x_1}\right)}$.

Moreover, we have

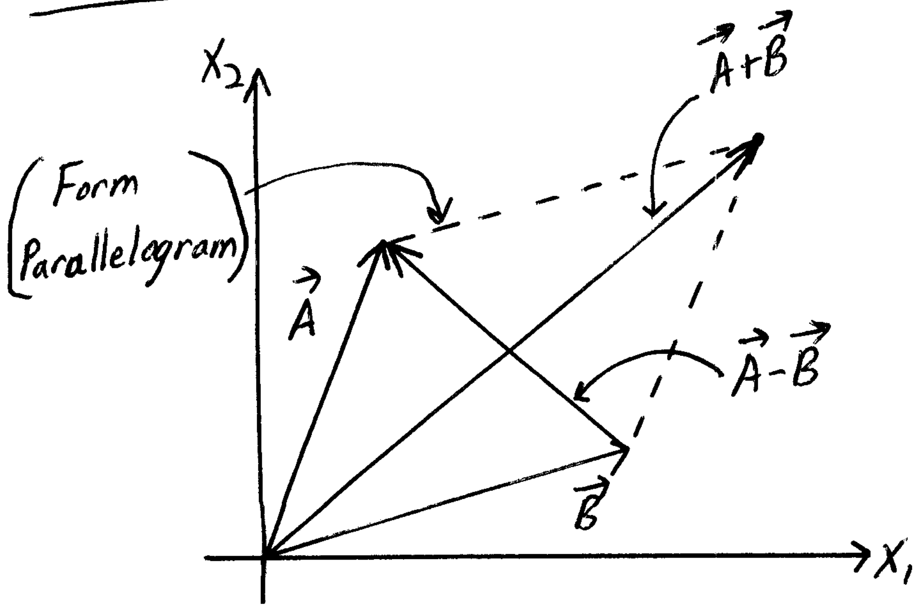
$\cos\alpha = \frac{x_1}{r} \Rightarrow \boxed{x_1 = r\cos\alpha}$ and $\sin\alpha = \frac{x_2}{r} \Rightarrow \boxed{x_2 = r\sin\alpha}$.

Then, a second form of a given vector is

Polar form: $\begin{bmatrix} r\cos\alpha \\ r\sin\alpha \end{bmatrix}$,

where r is magnitude and α is direction.

The Geometry of the Vectors $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$



Let $\vec{A}, \vec{B}$ be two vectors where $\vec{A} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ & $\vec{B} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

The resulting vectors $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ are

$$\vec{A}+\vec{B} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}$$

$$\vec{A}-\vec{B} = \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \end{bmatrix}$$